Mathematical Programming II

Chapter 1: Goal Programming

Outline

Outline

Motivation

Example (Adapted from Mourão et al.):

The PMat factory produces two products, P1 and P2, in two departments, S1 and S2. The departments have weekly capacities of 60 machine-hours and 40 labor-hours, respectively. Producing one unit of P1 requires 3 machine-hours in S1 and 1 labor-hour in S2, and yields a profit of 5 monetary units. Producing one unit of P2 requires 2 machine-hours in S1 and 2 labor-hours in S2, and yields a profit of 2 monetary units. In addition, at least 10 units of P2 must be produced.

The PMat CEOs have different goals. CEO A wants to achieve a minimum profit of 75 monetary units, whereas CEO B wants the weekly machine-hours used in Department S1 to be at most 50, in order to reduce machine maintenance costs.

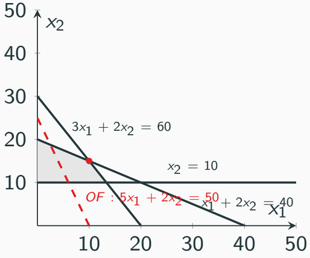
Formulate the problems of CEO A and CEO B as linear programming models, assuming that CEO A aims to maximize profit and CEO B aims to minimize the machine-hours used in Department S1, and solve them graphically.

Motivation

- Decision variables: x_i - number of units of product Pi to be produced weekly, $i \in \{1, 2\}$

CEO A

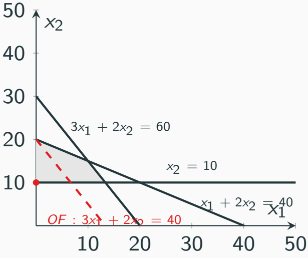
$$\begin{aligned} \text{Maximize } & 5x_1 + 2x_2 \\ \text{Subject to: } & 3x_1 + 2x_2 \leq 60 \\ & x_1 + 2x_2 \leq 40 \\ & x_1, x_2 \geq 0 \end{aligned}$$



$x^* = (10, 15)$ and $z^* = 80$ monetary units (uses 60 machine-hours in S1).

CEO B

$$\begin{aligned} \text{Minimize } & 3x_1 + 2x_2 \\ \text{Subject to: } & 3x_1 + 2x_2 \leq 60 \\ & x_1 + 2x_2 \leq 40 \\ & x_1 \geq 0,\ x_2 \geq 10 \end{aligned}$$



$x^* = (0, 10)$ and $z^* = 20$ units (yielding a profit of 20 monetary units).

Motivation

- Which one is the best solution?
  - Each CEO has a different preference.
  - The objective-function values are expressed in different units.
  - The solutions are not comparable.

- More that one conficting objecti.

Motivation

- Examples of problems with two or more conflicting objectives
  - Production planning: minimize costs vs. maximize resource utilization.
  - Project selection: maximize return vs. minimize risk.
  - Facility location: maximize service quality vs. minimize costs.
  - Distribution: minimize transportation costs vs. maximize the quantity of items distributed.
  - Energy supply: minimize costs vs. maximize the use of renewable energy.
  - Urban planning: minimize costs and environmental impacts vs. maximize population satisfaction.

Introduction

Several approaches can be used to deal with multiple-objective problems:

- Multiobjective Programming
  - More than one objective, but no goals attached.
- Goal Programming ★
  - More than one objective with a specific goal attached to each one.

Introduction

In goal programming, the decision maker identifies the goals to be achieved and specifies:

- Weights associated with the goals; or
  - Nonpreemptive Goal Programming
    The weights may be interpreted as penalties associated with failure to achieve the goals and reflect the decision maker's preferences regarding the relative importance of each goal.

- The priority order of the goals to be achieved.
  - Preemptive Goal Programming
    In this case, the goals are considered according to their priorities; that is, a goal is considered only after all higher-priority goals have been satisfied.

Introduction

Example (Adapted from Mourão et al.):

The PMat factory produces two products, P1 and P2, in two departments, S1 and S2. The departments have weekly capacities of 60 machine-hours and 40 labor-hours, respectively. Producing one unit of P1 requires 3 machine-hours in S1 and 1 labor-hour in S2, and yields a profit of 5 monetary units. Producing one unit of P2 requires 2 machine-hours in S1 and 2 labor-hours in S2, and yields a profit of 2 monetary units. In addition, at least 10 units of P2 must be produced.

The PMat CEOs have different goals. CEO A wants to achieve a minimum profit of 75 monetary units, whereas CEO B wants the weekly machine-hours used in Department S1 to be at most 50, in order to reduce machine maintenance costs.

Introduction

- Decision variables:

x_i - number of units of product Pi to be produced weekly, $i \in \{1, 2\}$

The goals that the CEOs want to achieve are:

CEO A goal: $5x_1 + 2x_2 \geq 75$ (profit of at least 75 monetary units)

CEO B goal: $3x_1 + 2x_2 \leq 50$ (use at most 50 machine-hours in S1)

- It is not possible to satisfy both goals. $\Longrightarrow$ Compromise solution.
- Goal programming finds a compromise solution by converting each goal into a flexible goal whose corresponding constraint may be violated if necessary.
- Goal programming seeks a compromise (satisficing) solution rather than an optimum solution to the original conflicting goals.

Introduction

How can goals be converted into constraints that may be violated?

General Idea:

Add nonnegative deviational variables to the goals, representing deviations below and above the right-hand side of each goal. Thus, the left-hand side of the goal may be either above or below its right-hand side.

Introduction

The CEOs' goals can equivalently be written as follows:

CEO A goal: $5x_1 + 2x_2 + s_1^- - s_1^+ = 75$ (profit ≥ 75 monetary units)

CEO B goal: $3x_1 + 2x_2 + s_2^- - s_2^+ = 50$ (m.h. in S1 ≤ 50)

The nonnegative variables s_1^-, s_1^+, s_2^-, and s_2^+ are the deviational variables.

Note: In each constraint, at most one of the two deviational variables can assume a positive value.

Introduction

- Consider CEO A's goal:

$$\text{Case } s_1^- > 0 \text{ and } s_1^+ = 0\text{:}\ 5x_1 + 2x_2 + s_1^- = 75 \implies 5x_1 + 2x_2 < 75.$$
$$\text{Case } s_1^- = 0 \text{ and } s_1^+ > 0\text{:}\ 5x_1 + 2x_2 - s_1^+ = 75 \implies 5x_1 + 2x_2 > 75.$$

If $s_1^- = 0$, CEO A's goal is achieved. $\implies$ We therefore want s_1^- to be as small as possible.

- Consider CEO B's goal:

$$\text{Case } s_2^- > 0 \text{ and } s_2^+ = 0\text{:}\ 3x_1 + 2x_2 + s_2^- = 50 \implies 3x_1 + 2x_2 < 50.$$
$$\text{Case } s_2^- = 0 \text{ and } s_2^+ > 0\text{:}\ 3x_1 + 2x_2 - s_2^+ = 50 \implies 3x_1 + 2x_2 > 50.$$

If $s_2^+ = 0$, CEO B's goal is achieved. $\implies$ We therefore want s_2^+ to be as small as possible.

Introduction

To obtain a compromise solution, we want to:

- Minimize s_1^-
- Minimize s_2^+

subject to the remaining constraints of the problem.

■ How can this problem be solved?

- Nonpreemptive Goal Programming
  - Only weights are associated with the goals.
- Preemptive Goal Programming
  - There is a priority relationship among the goals.

Outline

Introduction

Goal Programming Formulation

Nonpreemptive Goal Programming (or Weighted Goal Programming)

Preemptive Goal Programming (or Lexicographic Goal Programming)

Goal Programming Formulation

Part 1:

- Define all decision variables of the problem (except the deviational variables).
- Identify and formulate the goals to be achieved.

Part 2:

- Functional constraints of the problem.
- Sign restrictions – variable domains.
- Goal constraints – including the deviational variables.

Part 3:

- Objective function $\Longrightarrow$ Minimize the deviational variables associated with the goals.
  - Specify the weight of each goal, or
  - Specify the priority order among the goals.

Example

Example (Adapted from Mourão et al.):

The PMat factory produces two products, P1 and P2, in two departments, S1 and S2. The departments have weekly capacities of 60 machine-hours and 40 labor-hours, respectively. Producing one unit of P1 requires 3 machine-hours in S1 and 1 labor-hour in S2, and yields a profit of 5 monetary units. Producing one unit of P2 requires 2 machine-hours in S1 and 2 labor-hours in S2, and yields a profit of 2 monetary units. In addition, at least 10 units of P2 must be produced.

The PMat CEOs have different goals. CEO A wants to achieve a minimum profit of 75 monetary units, whereas CEO B wants the weekly machine-hours used in Department S1 to be at most 50, in order to reduce machine usage.

Formulate the PMat problem as a goal programming model.

Example

- Decision variables:

x_i - number of units of product Pi to be produced weekly, $i \in \{1, 2\}$

- Goals to be achieved:

$5x_1 + 2x_2 \geq 75$ (minimum profit of 75 monetary units)

$3x_1 + 2x_2 \leq 50$ (at most 50 machine-hours used in S1)

- Goal programming formulation:

$$
\begin{aligned}
\min Z &= s_1^- + s_2^+ \\
s.a: \ & 3x_1 + 2x_2 \leq 60 \\
& x_1 + 2x_2 \leq 40 \\
& x_2 \geq 10 \\
& 5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \\
& 3x_1 + 2x_2 + s_2^- - s_2^+ = 50 \\
& x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ \geq 0
\end{aligned}
$$

Note: In nonpreemptive goal programming, the objective function must include the weights. In preemptive goal programming, the different objectives and their priority order must be specified.

Outline

Nonpreemptive Goal Programming

Consider a problem with r goals, where the ith goal is:

$$\text{Minimize } G_i, \quad i \in \{1, \ldots, r\}$$

The **weighted objective function** that combines all goals is:

$$\text{Minimize } Z = \sum_{i=1}^{r} w_i G_i = w_1 G_1 + w_2 G_2 + \ldots + w_r G_r$$

The positive weights $w_1, w_2, \ldots, w_r$ reflect the decision maker's preferences regarding the relative importance of each goal.

- The larger the value of w_i, the greater the importance assigned to the goal.
- Example: If $w_i = 1$, all goals have equal importance.

Note: The determination of the weights is highly subjective.

How can a nonpreemptive goal programming problem be solved?

After formulating the weighted objective function, we obtain a linear programming problem with a single objective.

Which solution methods do you know for linear programming problems with a single objective?

- Simplex Method
  - Software for solving the simplex method – *solvers*

Example

Example (Adapted from Mourão et al.):

The PMat factory produces two products, P1 and P2, in two departments, S1 and S2. The departments have weekly capacities of 60 machine-hours and 40 labor-hours, respectively. Producing one unit of P1 requires 3 machine-hours in S1 and 1 labor-hour in S2, and yields a profit of 5 monetary units. Producing one unit of P2 requires 2 machine-hours in S1 and 2 labor-hours in S2, and yields a profit of 2 monetary units. In addition, at least 10 units of P2 must be produced.

The PMat CEOs have different goals. CEO A wants to achieve a minimum profit of 75 monetary units, whereas CEO B wants the weekly machine-hours used in Department S1 to be at most 50, in order to reduce machine usage.

Formulate the PMat factory problem as a goal programming model and solve it by assigning a weight of 2 to CEO A's goal and a weight of 1 to CEO B's goal.

Example

- Decision variables:

x_i - number of units of product Pi to be produced weekly, $i \in \{1, 2\}$

- Goals to be achieved:

$5x_1 + 2x_2 \geq 75$ (minimum profit of 75 monetary units) $\Longrightarrow$ Weight 2

$3x_1 + 2x_2 \leq 50$ (at most 50 machine-hours used in S1) $\Longrightarrow$ Weight 1

- Nonpreemptive goal programming formulation:

$$
\begin{aligned}
\min Z &= 2s_1^- + 1s_2^+ \\
s.a: \; & 3x_1 + 2x_2 \leq 60 \\
& x_1 + 2x_2 \leq 40 \\
& x_2 \geq 10 \\
& 5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \\
& 3x_1 + 2x_2 + s_2^- - s_2^+ = 50 \\
& x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ \geq 0
\end{aligned}
$$

Example

■ Step 1: Write the problem in augmented form (all constraints are equations and all variables are ≥ 0):

$$\begin{aligned}
\min Z = {} & 2s_1^- + 1s_2^+ \\
s.a: {} & 3x_1 + 2x_2 + x_3 = 60 \\
& x_1 + 2x_2 + x_4 = 40 \\
& x_2 - x_5 = 10 \\
& 5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \\
& 3x_1 + 2x_2 + s_2^- - s_2^+ = 50 \\
& x_1, x_2, x_3, x_4, x_5, s_1^-, s_1^+, s_2^-, s_2^+ \geq 0
\end{aligned}$$

■ Step 2: Determine an initial basic feasible solution (BFS):

Consider $(x_1, x_2, x_3, x_4, x_5, s_1^-, s_1^+, s_2^-, s_2^+) = (0, 10, 40, 20, 0, 55, 0, 30, 0)$.

Example

■ Step 3: Write the simplex tableau associated with the initial BFS and find the optimal solution:

	x_1	x_2	x_3	x_4	x_5	s_1^-	s_1^+	s_2^-	s_2^+	RHS
Z	10	0	0	0	4	0	-2	0	-1	110
x_3	3	0	1	0	2	0	0	0	0	40
x_4	1	0	0	1	2	0	0	0	0	20
x_2	0	1	0	0	-1	0	0	0	0	10
s_1^-	5	0	0	0	2	1	-1	0	0	55
s_2^-	3	0	0	0	2	0	0	1	-1	30

$CE = \max\{10, 4\} \rightarrow x_1$ becomes a BV

$CS = \min\{\frac{40}{3}, \frac{20}{1}, \frac{55}{5}, \frac{30}{3}\} = 10 \rightarrow s_2^-$ becomes an NBV

Example

	x_1	x_2	x_3	x_4	x_5	s_1^-	s_1^+	s_2^-	s_2^+	RHS
Z	0	0	0	0	-8/3	0	-2	-10/3	7/3	10
x_3	0	0	1	0	0	0	0	-1	1	10
x_4	0	0	0	1	4/3	0	0	-1/3	1/3	10
x_2	0	1	0	0	-1	0	0	0	0	10
s_1^-	0	0	0	0	-4/3	1	-1	-5/3	5/3	5
x_1	1	0	0	0	2/3	0	0	1/3	-1/3	10

$CE = \max\{\frac{7}{3}\} \rightarrow s_2^+$ becomes a BV

$CS = \min\{\frac{10}{1}, \frac{10}{\frac{1}{3}}, \frac{5}{\frac{5}{3}}\} = 3 \rightarrow s_1^-$ becomes an NBV

Example

	x_1	x_2	x_3	x_4	x_5	s_1^-	s_1^+	s_2^-	s_2^+	RHS
Z	0	0	0	0	-4/5	-7/5	-3/5	-1	0	3
x_3	0	0	1	0	4/5	-3/5	3/5	0	0	7
x_4	0	0	0	1	8/5	-1/5	1/5	0	0	9
x_2	0	1	0	0	-1	0	0	0	0	10
s_2^+	0	0	0	0	-4/5	3/5	-3/5	-1	1	3
x_1	1	0	0	0	2/5	1/5	-1/5	0	0	11

Since the objective-function row has no positive coefficients, the current solution is optimal.

Therefore, $(x_1, x_2, x_3, x_4, x_5, s_1^-, s_1^+, s_2^-, s_2^+)^* = (11, 10, 7, 9, 0, 0, 0, 0, 3)$

Example

Solution analysis:

- The solution obtained consists of producing 11 units of P1 and 10 units of P2.
- Since $Z^* > 0$, we can conclude that at least one goal was not achieved.
  - The profit obtained is 75 monetary units, so CEO A's goal is achieved exactly ($s_1^- = s_1^+ = 0$).
  - A total of 53 machine-hours are used in Department S1, so CEO B's goal is not achieved: machine usage exceeds the specified limit by 3 machine-hours ($s_2^+ = 3$).

Note: It is expected that CEO A's goal is the one achieved, since it was assigned the larger weight.

Outline

Preemptive Goal Programming

The decision maker ranks the goals according to their priorities. For a problem with r goals, the priorities can be written as follows:

Highest Priority

$$\text{Minimize } P_1 = p_1$$

$$\vdots$$

Lowest Priority

$$\text{Minimize } P_r = p_r$$

- where p_i, with $i \in \{1, \ldots, r\}$, represents a function of the deviational variables associated with goal i.

Preemptive Goal Programming

How can a preemptive goal programming problem be solved?

General Idea:

Optimize the (single) objectives in priority order, adding constraints so that the values obtained for higher-priority objectives cannot deteriorate.

- The solution procedure starts with the highest-priority objective and ends with the lowest-priority objective. A lower-priority solution must never degrade a higher-priority solution.

Solution Method

Identify the goals and rank them in priority order:

$$P_1 = p_1 > P_2 = p_2 > \ldots > P_r = p_r.$$

Set $i = 1$.

for all $i \in \{1, \ldots, r\}$ **do**

    Solve the problem that minimizes priority objective P_i and obtain p_i^*, the optimal value of the corresponding function of deviational variables p_i.

    Add the constraint $p_i \leq p_i^*$ to the problem.

    Set $i = i + 1$.

end for

Example

Example (Adapted from Mourão et al.):

The PMat factory produces two products, P1 and P2, in two departments, S1 and S2. The departments have weekly capacities of 60 machine-hours and 40 labor-hours, respectively. Producing one unit of P1 requires 3 machine-hours in S1 and 1 labor-hour in S2, and yields a profit of 5 monetary units. Producing one unit of P2 requires 2 machine-hours in S1 and 2 labor-hours in S2, and yields a profit of 2 monetary units. In addition, at least 10 units of P2 must be produced.

The PMat CEOs have different goals. CEO A wants to achieve a minimum profit of 75 monetary units, whereas CEO B wants the weekly machine-hours used in Department S1 to be at most 50, in order to reduce machine usage.

Formulate the PMat factory problem as a goal programming model and solve it assuming that CEO B's goal has higher priority than CEO A's goal.

Example

- Decision variables:

x_i - number of units of product Pi to be produced weekly, $i \in \{1, 2\}$

- Goals to be achieved:

$5x_1 + 2x_2 \geq 75$ (minimum profit of 75 monetary units) $\Longrightarrow$ Priority 2

$3x_1 + 2x_2 \leq 50$ (at most 50 machine-hours used in S1) $\Longrightarrow$ Priority 1

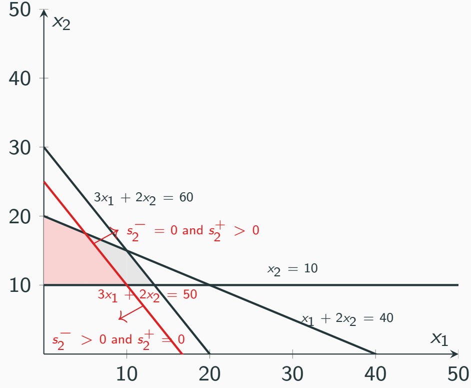
- Preemptive goal programming formulation:

$$
\begin{aligned}
\min Z &= P_1(s_2^+) + P_2(s_1^-) \\
s.a: &\ 3x_1 + 2x_2 \leq 60 \\
&\ x_1 + 2x_2 \leq 40 \\
&\ x_2 \geq 10 \\
&\ 5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \\
&\ 3x_1 + 2x_2 + s_2^- - s_2^+ = 50 \\
&\ x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ \geq 0
\end{aligned}
$$

Example – Graphical Solution

Problem minimizing priority objective P_1:

$$\min Z = s_2^+$$
$$s.a: 3x_1 + 2x_2 \leq 60$$
$$x_1 + 2x_2 \leq 40$$
$$x_2 \geq 10$$
$$3x_1 + 2x_2 + s_2^- - s_2^+ = 50$$
$$x_1, x_2, s_2^-, s_2^+ \geq 0$$



There are several solutions that achieve the Priority 1 goal $\Longrightarrow$ all solutions in the red region have $s_2^+ = 0$.

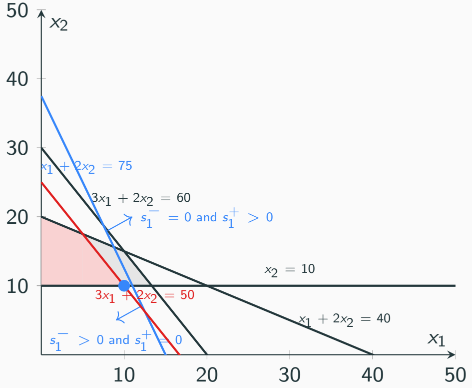
- Add the constraint $3x_1 + 2x_2 \leq 50$.
  - The feasible region of the next problem is the region shown in red.

Example – Graphical Solution

Problem minimizing priority objective P_2:

$$\min Z = s_1^-$$

$$\begin{aligned} s.a: \; & 3x_1 + 2x_2 \leq 60 \\ & x_1 + 2x_2 \leq 40 \\ & x_2 \geq 10 \\ & 3x_1 + 2x_2 \leq 50 \\ & 5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \\ & x_1, x_2, s_2^-, s_2^+ \geq 0 \end{aligned}$$



The best solution is: $(x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+) = (10, 10, 5, 0, 0, 0)$.

Example – Graphical Solution

The solution consists of producing 10 units of P_1 and 10 units of P_2 per week.

- The Priority 1 goal is achieved because $s_2^+ = 0$.
  - The number of machine-hours used in S1 is 50, which is equal to the maximum target value of 50 machine-hours.
- The Priority 2 goal is not achieved because $s_1^- = 5$.
  - The profit is 70 monetary units, which falls short of the required minimum of 75 monetary units.

Note: Higher-priority goals are expected to be the ones achieved.

Example – Simplex Method Solution

■ Step 1: Write the first problem in augmented form (all constraints are equations and all variables are ≥ 0):

$$\min Z = s_2^+$$
$$s.a: 3x_1 + 2x_2 + x_3 = 60$$
$$x_1 + 2x_2 + x_4 = 40$$
$$x_2 - x_5 = 10$$
$$3x_1 + 2x_2 + s_2^- - s_2^+ = 50$$
$$x_1, x_2, x_3, x_4, x_5, s_2^-, s_2^+ \geq 0$$

■ Step 2: Determine an initial basic feasible solution (BFS):

Consider $(x_1, x_2, x_3, x_4, x_5, s_2^-, s_2^+) = (0, 10, 40, 20, 0, 30, 0)$.

Example – Simplex Method Solution

■ Step 3: Write the simplex tableau associated with the initial BFS and find the optimal solution:

	x_1	x_2	x_3	x_4	x_5	s_2^-	s_2^+	RHS
Z	0	0	0	0	0	0	-1	0
x_3	3	0	1	0	2	0	0	40
x_4	1	0	0	1	2	0	0	20
x_2	0	1	0	0	-1	0	0	10
s_2^-	3	0	0	0	2	1	-1	30

Since the objective-function row has no positive coefficients, the current solution is optimal.

Example – Simplex Method Solution

■ Step 4: Add the constraint FO $\leq Z^*$:

$s_2^+ \leq 0 \iff 3x_1+2x_5+s_2^- -30 \leq 0 \iff 3x_1+2x_5+s_2^- \leq 30 \implies 3x_1 + 2x_5 + x_6 = 30$

and write the second problem in augmented form in terms of the optimal basis of the first problem:

$$
\begin{aligned}
\min Z &= s_1^- \\
s.a: 3x_1 + x_3 + 2x_5 &= 40 \\
x_1 + x_4 + 2x_5 &= 20 \\
x_2 - x_5 &= 10 \\
3x_1 + 2x_5 + x_6 &= 30 \\
5x_1 + 2x_2 + s_1^- - s_1^+ &= 75 \\
x_1, x_2, x_3, x_4, x_5, x_6, s_1^-, s_1^+ &\geq 0
\end{aligned}
$$

■ Step 5: Write the objective function and the new constraint in terms of the NBVs. BVs: x_3, x_4, x_2, x_6 e s_1^-.

Constraint 5: $5x_1 + 2x_2 + s_1^- - s_1^+ = 75 \iff 5x_1 + 2(10 + x_5) + s_1^- - s_1^+ = 75 \iff 5x_1 + 2x_5 + s_1^- - s_1^+ = 55$

FO: $Z_2 = s_1^- \iff Z_2 = 55 + s_1^+ - 5x_1 - 2x_2 \iff Z_2 + 5x_1 + 2x_5 - s_1^+ = 55$

Example – Simplex Method Solution

■ Step 6: Write the simplex tableau associated with the current BFS and find the optimal solution:

	x_1	x_2	x_3	x_4	x_5	x_6	s_1^-	s_1^+	RHS
Z	5	2	0	0	0	0	0	-1	55
x_3	3	0	1	0	2	0	0	0	40
x_4	1	0	0	1	2	0	0	0	20
x_2	0	1	0	0	-1	0	0	0	10
x_6	3	0	0	0	2	1	0	0	30
s_1^-	5	2	0	0	0	0	1	-1	55

$CE = \max\{5, 2\} = 5 \rightarrow x_1$ becomes a BV

$CS = \min\{\frac{40}{3}, \frac{20}{1}, \frac{30}{3}, \frac{55}{5}\} = 10 \rightarrow x_6$ becomes an NBV

Example – Simplex Method Solution

	x_1	x_2	x_3	x_4	x_5	x_6	s_1^-	s_1^+	RHS
Z	0	2	0	0	-4/3	-5/3	0	-1	5
x_3	0	0	1	0	0	-1	0	0	10
x_4	0	0	0	1	4/3	-1/3	0	0	10
x_2	0	1	0	0	-1	0	0	0	10
x_1	1	0	0	0	2/3	1/3	0	0	10
s_1^-	0	0	0	0	-4/3	-5/3	1	-1	5

All coefficients in the objective-function row are $\leq 0 \implies$ the current solution is optimal.

Notes on Goal Programming

- New paradigm: We determine a compromise (satisficing) solution rather than an optimum solution to the original conflicting goals.
- Each goal is converted into a constraint that may be violated through the use of deviational variables.

  ★ In a simplex iteration, at most one of the two deviational variables can assume a positive value.
- Solution methods:
  - Nonpreemptive goal programming: decision makers assign weights to the goals.
  - Preemptive goal programming: decision makers establish a priority order among the goals.

References

Mourão, M. C., Santiago Pinto, L., Simões, O., Valente, J., & Vaz Pato, M. (2019). *Investigação Operacional Exercícios e Aplicações*. Escolar Editora.

Taha, H. A. (2017). *Operations Research An Introduction*. Pearson Education Limited.