Decision Making and Optimization

Master in Data Analytics for Business

Cristina Requejo

Lisbon School
of Economics
& Management
Universidade de Lisboa

Linear Programming

Model or Formulation

Model or Formulation

Model or Formulation

A model or formulation is a schematic description of a system that takes into account its known properties and can be used to know it better, study its characteristics, and to better program the decisions.

To write a model we must answer the following questions:

- Which are the decisions to take?
- Under which constraints is the decision taken?
- Which is the appropriate criteria that allow an evaluation of the different alternatives of each decision?

Model or Formulation

A typical mathematical programming model or formulation is organized as follow

min or max	an objective function
subject to:	a set of constraints

with the objective function and constraints being mathematical functions written as a function of the decision variables,

Model or Formulation

defining the decision variables $x \in \mathbb{R}^n$, the mathematical programming model is

$$
\begin{array}{lll}
\min \text{ (or } \max) & z = f(x) & \\
\text{s. to:} & g_i(x) \leq b_i, \quad i \in I_1 & (1) \\
& g_i(x) \geq b_i, \quad i \in I_2 & (2) \\
& g_i(x) = b_i, \quad i \in I_3 & (3) \\
& & \\
& x_j \geq 0, \quad j \in J_1 & (4) \\
& x_j \leq 0, \quad j \in J_2 & (5) \\
& x_j \in \mathbb{R}, \quad j \in J_3 & (6)
\end{array}
$$

with

x_j are the decision variables, $x = (x_j)$ is a vector in $\mathbb{R}^n$,

$$n = |J| = |J_1 \cup J_2 \cup J_3|$$

z is the objective function to be optimised

(1), (2), (3) are the functional constraints, $m = |I| = |I_1 \cup I_2 \cup I_3|$

(4), (5) e (6) are the domain or sign constraints of the variables

Model or Formulation

Some types of mathematical programming models

in case of

- a Linear Programming (LP) model, the objective function and the constraints are all linear
- an Integer Linear Programming (ILP) model, all decision variables are integer
- a Non-Linear Programming (NLP) model, the objective function is nonlinear, the constraints can be linear or nonlinear

Model or Formulation

The General Form of a Linear Programming Model

$$\begin{aligned}
&\min \text{ or } \max && z = \sum_{j \in J} c_j x_j \\
&\text{s. t.:} && \sum_{j \in J} a_{ij} x_j \leq b_i, \quad i \in I_1 && (1) \\
& && \sum_{j \in J} a_{ij} x_j \geq b_i, \quad i \in I_2 && (2) \\
& && \sum_{j \in J} a_{ij} x_j = b_i, \quad i \in I_3 && (3) \\
& && x_j \geq 0, \quad j \in J_1 && (4) \\
& && x_j \leq 0, \quad j \in J_2 && (5) \\
& && x_j \in \mathbb{R}, \quad j \in J_3 && (6)
\end{aligned}$$

with parameters

c_j the cost coefficient of the decision variable $j \in J$ in the objective function

a_{ij} technological coefficient of the decision variable $j \in J$ in the constraint $i \in I$

b_i the right-hand-side (RHS) of the functional constraint $i \in I$

Model or Formulation

To write a model we must answer the following questions:

- Which are the decisions to take?

  define the decision variables

- Under which constraints is the decision taken?

  define the constraints

- Which is the appropriate criteria that allow an evaluation of the different alternatives of each decision?

  define the objective function

Lets model the challenge

The organic market *From Orchard to Basket* prepares weekly fruit baskets for its customers. There are two types of baskets:

- **Premium Basket (A)** – designed for juices and fruit salads.
- **Smart Basket (B)** – an economical option for daily snacks.

The warehouse has limited fruit stock:

- **Apples:** 10 units available.
- **Oranges:** 14 units available.

Each basket requires the following resources:

- Premium Basket (A): 1 apple and 2 oranges.
- Smart Basket (B): 1 apple and 1 orange.

The profits are:

- Premium Basket (A): 5 € per unit.
- Smart Basket (B): 3 € per unit.

Decide how many baskets A and B to prepare in order to maximize profit without exceeding the stock of fruit.