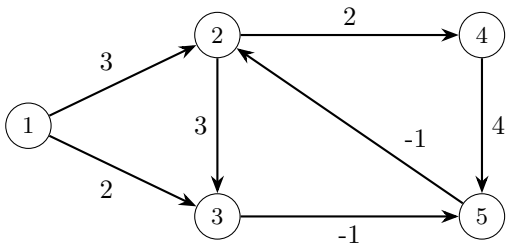
Mathematical Programming II

Chapter 3: Additional Exercises

Academic Year 2026/2027

Lisbon School of Economics & Management
Universidade de Lisboa

1. Consider the following graph:



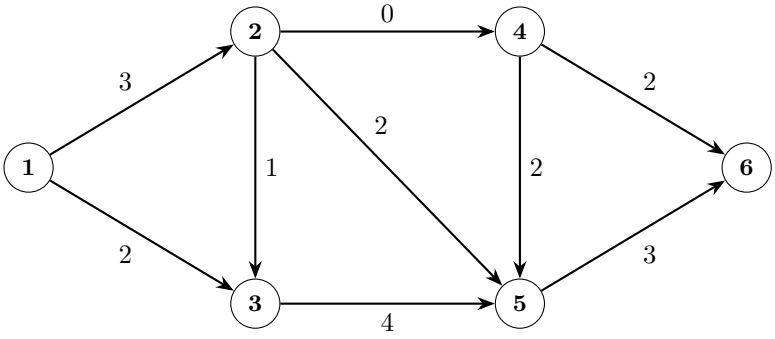
   (a) Consider the following vectors of dimension 5 and check whether they satisfy the Bellman–Ford conditions. Also determine which of them give the costs of shortest paths.

$$u = (0, 0, 1, 0, 1); \quad v = (0, 0, 2, 2, 1); \quad w = (0, 1, 2, 3, 1).$$

   (b) Consider the graph obtained from the graph in the figure by changing "4" to "-4". Use an appropriate algorithm to determine, if possible, the shortest paths from node 1 to all other nodes.

   (c) Now consider a new graph obtained from the graph in the figure by changing both "-1" values to "1". Use an appropriate algorithm to determine the shortest paths from node 1 to all other nodes.

2. Consider the following graph:



   (a) Use an appropriate algorithm to determine the shortest paths from node 1 to all other nodes.

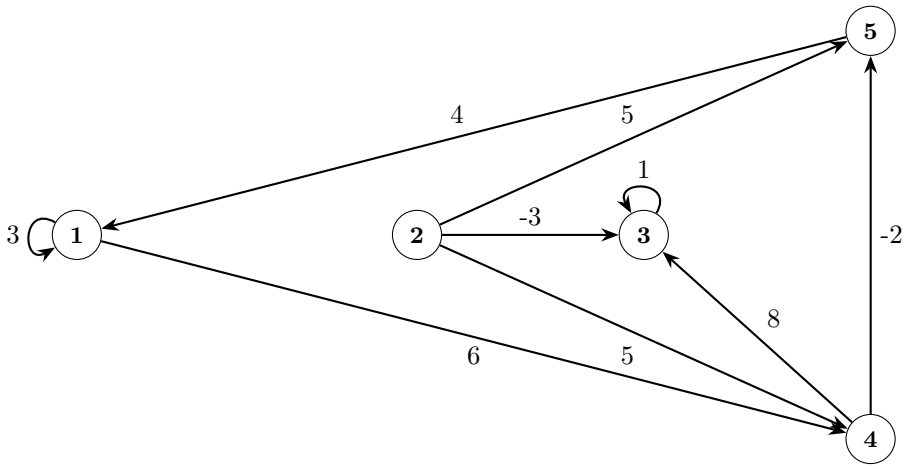
   (b) Repeat the previous part, modifying the graph by assigning costs -4 and -2 to arcs $(3, 5)$ and $(5, 4)$, respectively.

3. State the Bellman–Ford conditions for finding the longest paths from source s to all nodes of graph $G = (V, A)$.

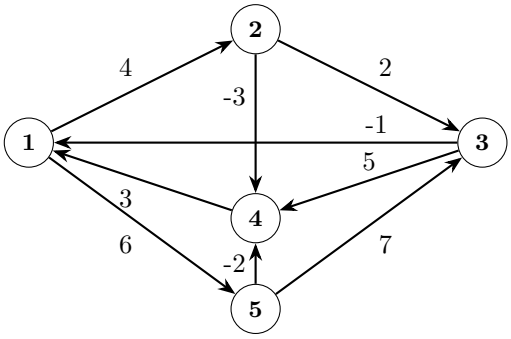
4. Consider the problem of finding paths with the fewest arcs from one node to all other nodes. Show how to solve this problem using the algorithms studied.

5. Consider a graph $G = (V, A)$ and two disjoint subsets S and T of V. We wish to find a shortest path between a node in S and a node in T. Show how to formulate this as a shortest-path problem between two nodes.

6. Use Floyd's algorithm to determine the shortest-path distance between every pair of nodes in the following graph:



7. Use Floyd's algorithm to determine the shortest paths and their respective costs between every pair of nodes in the following graph.



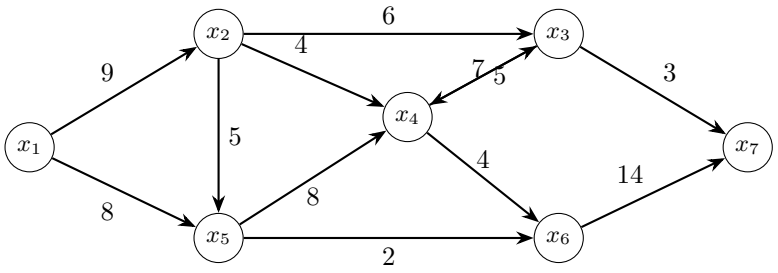
8. Consider the following matrix of arc costs for a graph $G = (V, A)$, where the node set is $V = \{1, 2, 3, 4\}$ and the arc set is $A = X^2$:

$$\begin{bmatrix} 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & 3 \\ 3 & 4 & 0 & 1 \\ 3 & -2 & 0 & 0 \end{bmatrix}$$

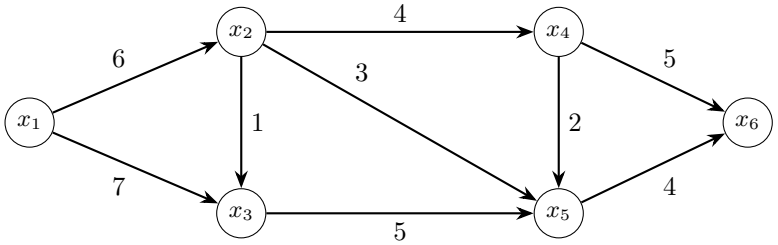
Identify a negative-cost directed cycle.

9. Determine the maximum flow from s to t in the following graphs using the Ford–Fulkerson algorithm (the value next to each arc represents its maximum flow capacity):

(a) $s = x_1$, $t = x_7$



(b) $s = x_1$, $t = x_6$



(c) In each case, determine a minimum-capacity cut separating s from t.

(d) We wish to increase the maximum flow value from s to t by 10 units by changing the capacities of as few arcs as possible. The capacity of any arc can be increased by at most 5 units. Which arc capacities should be increased? Justify your answer.