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Macroeconomics II

Lecture 05

Macro production function
Growth accounting

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Theoretical Lecture 5

The macroeconomic production function. Growth accounting.

- **factors of economic growth;**
- **physical capital; labour/human capital; technological progress;**
- **macroeconomic production function: concept;**
- **production function with complimentary factors (as in Harrod-Domar model);**
- **neoclassical production function with substitutable factors: properties;**
- **technological progress: Hicks, Solow and Harrod neutrality;**
- **growth accounting: the contribution of factors of economic growth.**

Reading

Jones & Vollrath (2013), *Introduction to Economic Growth*, Norton, pp. 44-50;

Acemoglou (2007), pp. 40-41 + pp. 82-83

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major questions

what causes GDP to growth? And what causes rates of growth to differ?

factors of economic growth

NR, K, L, A how to formulate? how to measure them? how to evaluate their contributions to growth?

NR Natural Resources
•are they really required for growth?
•*The Limits to Growth* (Meadows, 1972); resources depletion;
•The "Dutch disease": "bad" effects of wealth natural resources

K stock of physical capital
how to measure? the "Cambridge controversy"

L labour force, active population
how to measure? how to deal with its heterogeneity? from L to "human capital"

A technological progress
where is it embodied? physical capital, labour?

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Physical Capital (K)

K stock variable; measurement evaluated by relevant market prices

K_t all capital goods (equipments) that are installed in a country, in the beginning of period t, that are available for production of goods and services in that economy

$K_{t+1} = K_t + I_t – d.K_t$

I_t Gross Investment; a flow variable (new capital goods during period t)

$d.K_t$ Depreciation, d being the depreciation rate

Labour (L)

active population: all people aged above 15, who are available to work on producing goods and services (some are employed; others are unemployed).

stock variable: with reference to a moment

L Labour force (active population)
N resident population

L = N . (L/N)

L/N labour force participation rate (depends on policy decisions and individual decisions)

two related concepts

Y/N GDP per capita
Y/L labour productivity

One has: Y/N = Y/L . L/N

remember r(Y/N) = r(Y/L) + r(L/N)

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from Labour to "Human Capital"

heterogeneity of Labour (1 engineer + 1 unskilled worker = 2 workers?)

qualitative dimension of labour (school learning and training matters)

The concept of Human capital **(and discussion)**: a stock of knowledge available for producing goods and services

All knowledge and skills embodied in the active population, which result from school learning, professional training and on-the-job training

$H = h . L$ (where H = Human Capital; L = Labour)

where h is the average number, *per active person*, of hours spent on schooling and training.

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Production function

Microeconomics

Production year t Q_t

Production factors year t $X_1, X_2, \dots X_n$

$$Q_t = f(X_1, X_2, \dots, X_n)$$

Aggregate/Macroeconomics

Output year t $Y_t = GDP_t$

Production factors year t K_t, L_t

$$Y_t = F(K_t, L_t)$$

$$Y(t) = F(K(t).L(t))$$

Substitutable Factors (the same value of Y may be obtained with different combinations of production factors);

Complementary Factors (any value of Y will be obtained with one, and just one, combination of production factors).

technological progress

means the improvement of efficiency of the production factors (labour, capital, or both)

embodied in physical capital

this technological progress means that the new equipment is more modern and more efficient than the old ones

K -> a.K

embodied in "human capital", or labour

it is progress in knowledge, which means that the most recent graduates are more efficient that the older ones, even with the same number of hours of schooling and training

h.L -> c.h.L

desimbodied technological progress

it is not embodied either in physical capital or in human capital: progress in management, better working of the production processes, of the institutions of the whole society, etc

Neoclassical Production Function

Y(t) = F (K(t), L(t), A(t))

where A(t) is the technology.

We may skip (t) and use for simplification

Y = F(K, L, A)

Assumptions:

- the factors are substitutable (the relation of K with L is not a constant)
- there is technological progress.
- a set of properties for this function, which determines its functional form.

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1. Constant returns to scale

$F(\lambda K, \lambda L, A) = \lambda.F(K,L,A)$, for all $\lambda > 0$

(homogeneous function degree 1 in K and L)

2. Marginal productivities are positive and decreasing

$\delta F/\delta K > 0$, $\delta^2 F/\delta K^2 < 0$

$\delta F/\delta L > 0$, $\delta^2 F/\delta L^2 < 0$

3. Essenciality

$F(0,L) = F(K,0) = 0$

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Techological progress

How to insert technological progress in the production function?

As a shift of the production function. A(t) is a ***shifter*** of the production function.

How? It depends on the type of technological progress. Three possible views:

1. Disembodied technological progress (or **Hicks neutrality**):

F (K, L, A) = A . F(K, L)   total productivity rises

2. Technological progress is **embodied in Capital** (***capital-augmenting*** or **Solow neutrality**) F(K, L, A) = F(A.K, L) the new equipments are more efficient

3. Technological progress is **embodied in Labour** (***labour augmenting*** or **Harrod neutrality**)   F(K,L,A) = F(K, A.L)   labour is more productive

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Production function: Cobb-Douglas

… by Charles **Cobb** (mathematician) and Paul **Douglas** (economist), in 1927

$$Y(t) = A(t)\, K(t)^{\alpha}\, L(t)^{1-\alpha}$$

with $0 < \alpha < 1$

properties of the Cobb-Douglas production function

α = $\delta Y(t)/\delta K(t) \,.\, K(t)/Y(t)$ partial elasticity of Y relative to K

$(1 - \alpha)$ = $\delta Y(t)/\delta L(t) \,.\, L(t)/Y(t)$ partial elasticity of Y relative to L

economic interpretation of **α** and **$(1 - \alpha)$**

$Y = w.L + r.K$

if (neoclassical assumption) **r** = marginal productivity of capital and **w** = marginal productivity of labour, we get:

$\alpha = r.K/Y$ and **$1-\alpha = w.L/Y$**

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Production function: Cobb-Douglas

$$Y(t) = A(t)\, K(t)^{\alpha}\, L(t)^{1-\alpha} \qquad 0 < \alpha < 1$$

We get the following relations among the growth rates of the several variables involved (remember properties growth rates):

$$r(Y) = r(A) + \alpha . r(K) + (1-\alpha) . r(L)$$

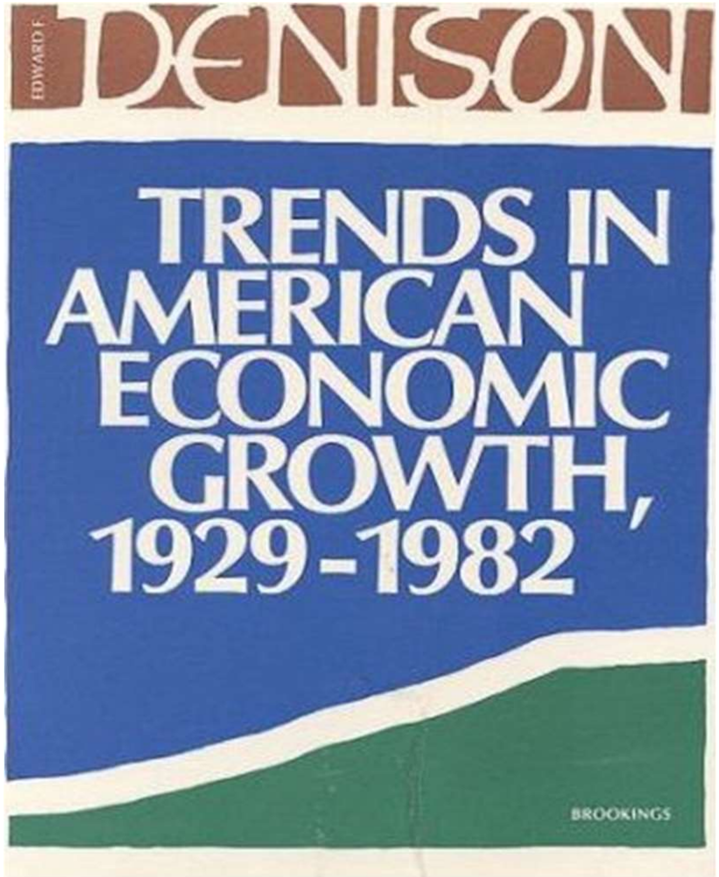
where
r(A) rate of growth of the **Total Factor Productivity** (TFP), that is, the difference between output growth and the weighted sum of the growth of factor inputs, where the weights are the factor shares. It is also known as Solow residual.

Problems with the Solow residual and the Cobb-Douglas function

How can a statistical residual explain a major economic fact?

It may be called **Total Factor Productivity** (TFP), but it is a statistical artifact.

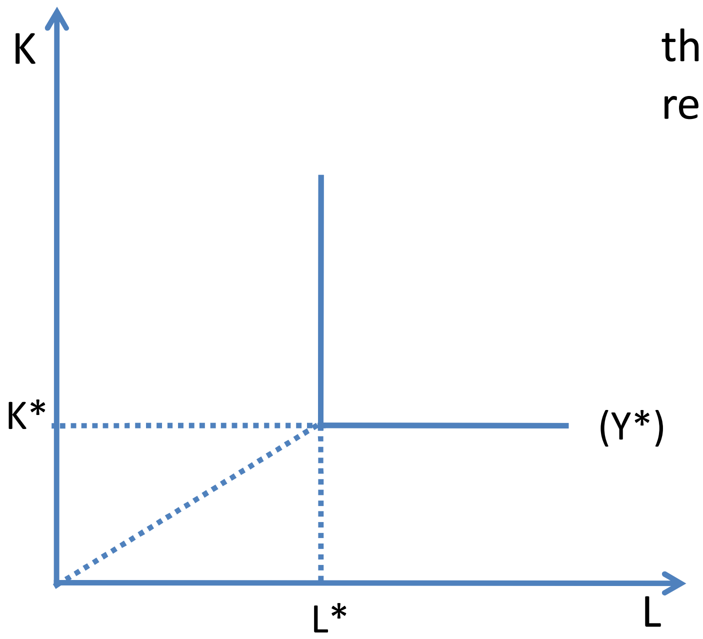
Can this “explain” 60% of growth? That is the result obtained by Edward Denison in “Trends in American Economic Growth 1929-1982”



Leontief production function

$Y = F(K, L) = min\{AK, BL\}$ with **complementary factors**

Coefficients (productivities) are constant (A e B)



the production of 1 unit of output requires 1/A units de K and 1/B units of L

productivity of K: A un Y / un K

productivity of L: B un Y / un L

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Technology with constant coefficients

Y = AK (is enough; it is assumed an implicit efficient relation to L)

Making use of the capital stock K, the firms generate output Y and, therefore

use (1/B)Y units of labour, that is:

use (1/B)AK units of labour (a constant relation between K and L)

K = (1/A)Y

K = vY (equivalent to Y = AK)

v is the **capital output ratio** v **= 1/A**

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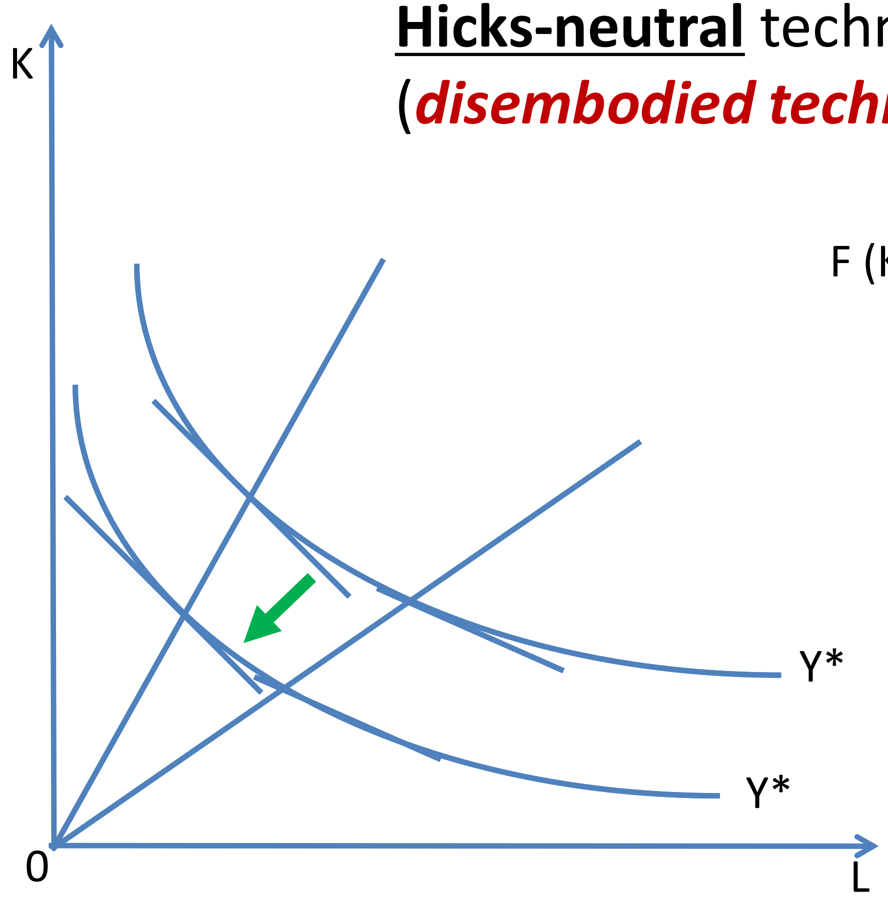
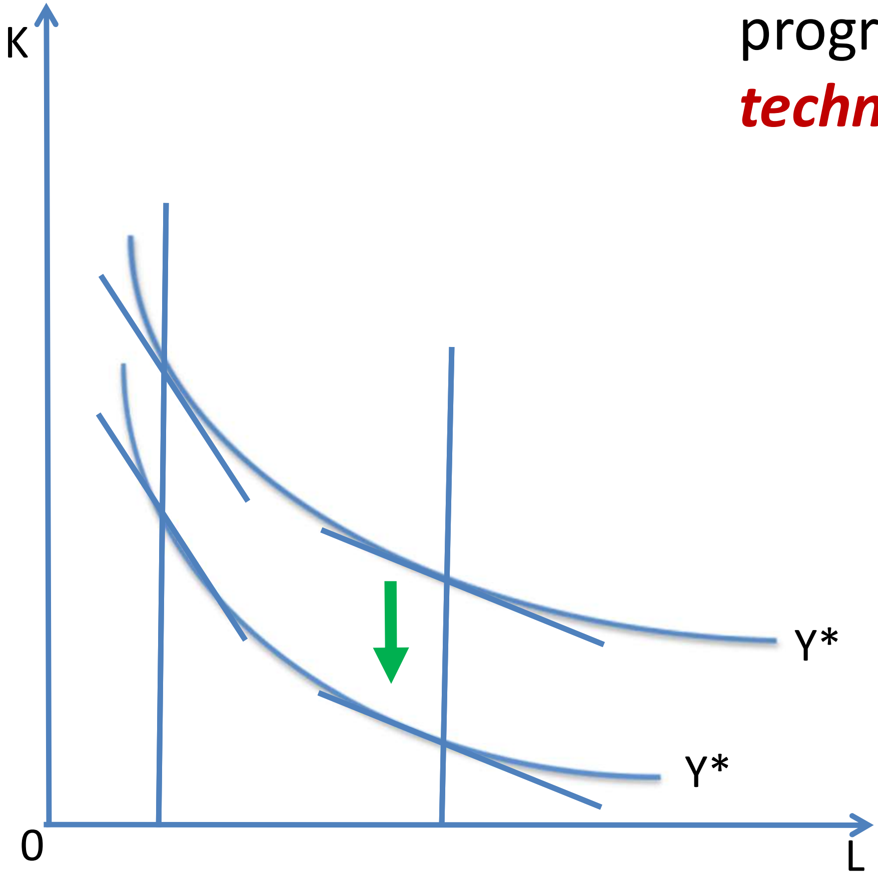
<u>Hicks-neutral</u> technological progress
(***disembodied technological progress***)

F (K, L, A) = A . F(K, L)



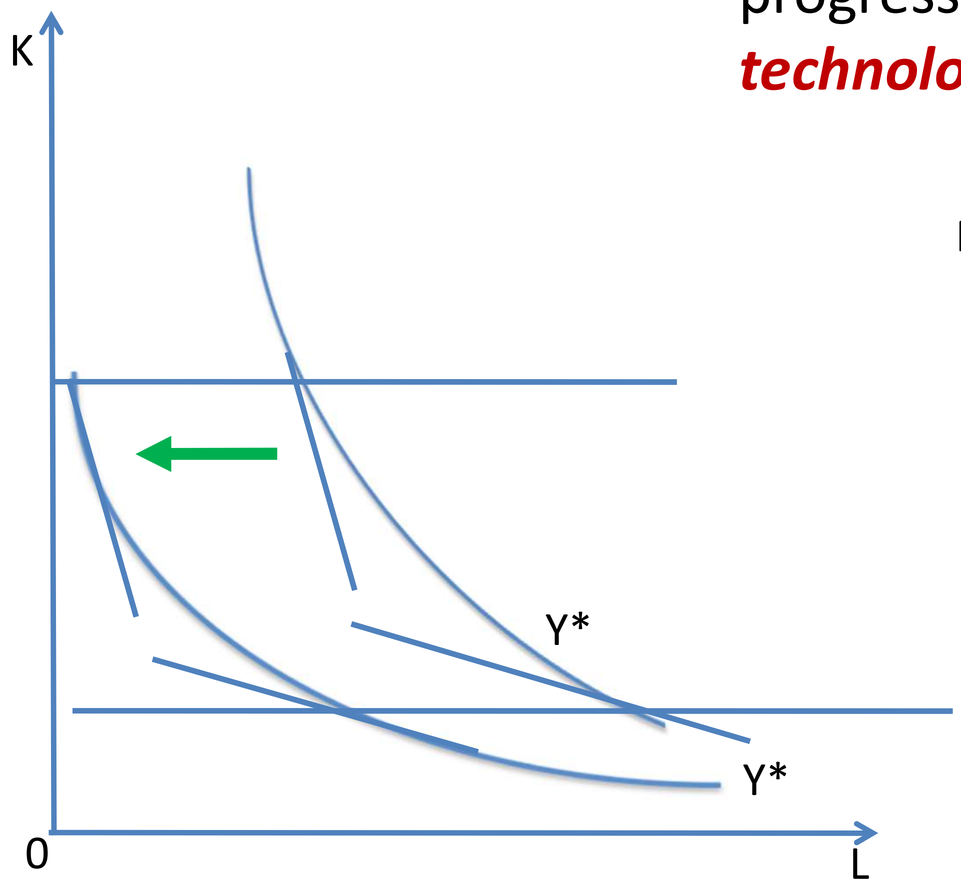
Solow-neutral technological progress (***capital augmenting technological progress***)



F(K, L, A) = F(A.K, L)

is equivalent to the economy having <u>more physical capital</u>

the isoquants shift in a way that they have constant slope at a <u>given labour-output ratio</u>



Harrod-neutral technological progress (***labour augmenting technological progress***)

F(K,L,A) = F(K, A.L)

is equivalent to the economy having <u>more labour</u>

the isoquants shift in a way that they have constant slope at a <u>given capital-output ratio</u>