



Macroeconomics II

Lecture 06 (1) March 2020

Harrod-Domar model
Criticism by Solow



Theoretical Lecture 6

Harrod-Domar model. The criticism by Solow

- the hypotheses of the Harrod-Domar (H-D) model;
- the H-D model: definition, behaviour and reduced-form equation;
- criticisms of the hypotheses and unstable equilibrium of the H-D model;
- reactions and solutions by R. Solow.

Reading:

Harrod, R.F. (1939), An Essay in Dynamic Theory, *The Economic Journal*, vol. 49, No 193, pp. 14-33



Roy Harrod (1900 – 1978)



Evsey Domar (1914-1997)



Sir Roy F. Harrod
(1900-1978)

English economist, born in London. He studied at Oxford and, apart from a short break for service in World War II, and again as adviser to the International Monetary Fund in the early 1950s, he remained there, at Christ Church, throughout his career (1922-1967). He wrote the official biography of Keynes (1951), and wrote also on philosophy and logic as well as economics. He made several other discoveries more or less in parallel with others (the marginal revenue curve, the long-run envelope of short-run average cost curves, the theory of imperfect competition, the multiplier-accelerator model, and the IS-LM model). At last, another discovery brought him recognition: Harrod's "Essay in Dynamic Theory" (1939). The idea, which marked the beginning of the modern theory of growth, had also been developed by Domar, but at least Harrod got his name on the model this time. He was personal advisor to Winston Churchill during World War II.

American economist. Born in Lodz, Russia (now Poland), he was raised and educated in Harbin, Manchuria, but moved permanently to the United States in 1936 and completed his studies there at the UCLA, Michigan, and Harvard, where he got his Ph.D. in 1947. He taught at several universities, including Johns Hopkins, before moving to MIT in 1958. He made contributions in at least three main areas of economics: economic growth, comparative economics, and economic history. His work on economic growth began with his 1944 model on government debt, which considered how economic growth can lighten the burden of the government debt. His major claim to fame, however, was in developing, parallel to Roy Harrod, a dynamic-equilibrium growth model (1946) as a way of extending the Keynesian demand-determined equilibrium into the long run.

Evsey D. Domar
(1914-1997)



Keynesian analysis in the models of economic growth

Roy Harrod (1939) and Evsey Domar (1946)

“Harrod and Domar expressed the dynamic relationship (*the effect of capital accumulation on growth*) in a simple equation, which neatly formalized, simplified and summarized the essence of almost 200 years’ theorizing about economic growth” (Gylfason, 2003, p. 25)

Economic growth depends on three factors:

Savings rate of the households ($S = s.Y$)

Capital-output ratio ($v = K/Y$, or $= \Delta K/\Delta Y$)

Depreciation rate (δ)

The hypotheses of the Harrod-Domar (H-D) model

1. The technology

production function with complimentary factors (i.e., non-substitutable), with two (2) production factors: capital and labour

technology with constant coefficients

$Y = A \cdot K = (1/v) \cdot K$, $v = K/Y$ (capital-output ratio) **constant**; $A = 1/v$ is capital productivity;

K (stock, €), Y (flow, €/year)

K/L constant (L not explicit in the model)

K and L grow at the same rate

Y/L constant (the growth rate of labour productivity is 0)

Y and L grow at the same rate

there is **no technological progress**

The hypotheses of the Harrod-Domar (H-D) model (cont.)

2. Full employment

productive capacity of the economy is fully used; for a given K ,

$$Y = (1/v) \cdot K$$

is the maximum GDP that the economy may create, given the hypotheses (potential output) and provided the technology is adequate (the convenient capital-output ratio) – otherwise, full employment is not reached

3. The economy is closed

4. Savings

total savings is a constant proportion of GDP

$$S = s \cdot Y$$

5. Equilibrium

$$I = S$$

Harrod-Domar model: behaviour of the economic actors

households/consumers

(eq. 1) $S = sY$,

s is savings rate

firms/producers

(eq. 2) $K = vY$

v is the capital-output ratio

definition of investment

(eq. 3) $I = \Delta K + \delta K$; δ being the depreciation rate, and I gross investment, ΔK net investment

Condition of equilibrium

(eq. 4) $S = I$

Harrod-Domar model

4 equations

4 variables

K stock variable

Y, I, S flow variables

3 parameters

v, s, δ

Harrod-Domar model: the reduced-form equation

Given the numerical value for v , and

$$K = vY \text{ (eq. 2),}$$

we obtain

$$\Delta K = v \cdot \Delta Y$$

and then, considering that $S = sY$ (eq. 1) and $I = \Delta K + \delta K$ (eq. 3)
and $S = I$ (eq. 4), then:

$$\Delta Y/Y = (s/v) - \delta$$

Harrod-Domar model:

$$r^w(Y) = \Delta Y/Y = s/v - \delta$$

where $r^w(Y)$ is the **warranted** (“*garantida*”) growth rate of GDP, that is, the highest growth rate that the economy may reach, given the hypotheses, namely given the amount of capital K.

unstable equilibrium

it is assumed that productive capacity of the economy is fully used;

if productive capacity is not fully used, the economy shall no longer be in equilibrium;
and the economy does not move into an equilibrium;

if the producers decide to produce less than the maximum, given K, then $v^e > v$ (the relation K/Y^e becomes bigger than v), and then $Y^e < Y$; then, $r(Y^e) < r^w(Y)$; what then? does the economy move into the equilibrium Y? According to H-D, **no!** the model does not explicit any economic behaviour that predicts this move; the economy deviates more and more from Y.

A summary of the logic of the model

Then, a simple circular logic:

$$\Delta+S \rightarrow \Delta+I \rightarrow \Delta+K \rightarrow \Delta+Y \rightarrow \Delta+S \dots$$

Increase in savings – investment – capital stock –
GDP - savings

Or, **growth of Y may be increased by further savings and/or efficiency of capital**

Harrod-Domar model (Summary)

$Y = A \cdot K$ (role of capital accumulation; technology with constant coefficients)

equivalent to

$K = v \cdot Y$ or $\Delta K = v \cdot \Delta Y$ (v is capital-output ratio),

$S = s \cdot Y$ (savings behaviour)

$I = \Delta K + \delta K$ (definition of Investment)

$I = S$ (macroeconomic equilibrium: “synthetic” Keynesian approach)

It comes (*“a simple equation, which neatly formalized the essence of almost 200 years’ theorizing about economic growth”*):

$$\Delta Y/Y = s/v - \delta$$

But an unstable equilibrium in the H-D model

The H-D model does not have any endogenous variable which may act as an adjustment variable in the model: if some exogenous change occurs that affects the economic equilibrium, there is no adjustment mechanism to reach the equilibrium again. The economic system deviates from the equilibrium.

Knife-edge equilibrium:

$$r^w(Y) = \Delta Y/Y = s/v - \delta, \text{ or } g_y \text{ warranted growth rate}$$

a. if decisions on production < feasible production with the existing stock of capital
 $v^{ef} > v$ (reduction of the efficiency of the capital)

$r(Y^e) < r^w(Y)$ (actual growth rate < warranted growth rate)

disequilibrium and unemployment?

b. if decisions on production > feasible production with the existing stock of capital
 $v^{ef} < v$ (increase of the efficiency of the capital)

disequilibrium and inflation?



Therefore, problems of interpretation:

Does the H-D model represent historical processes?

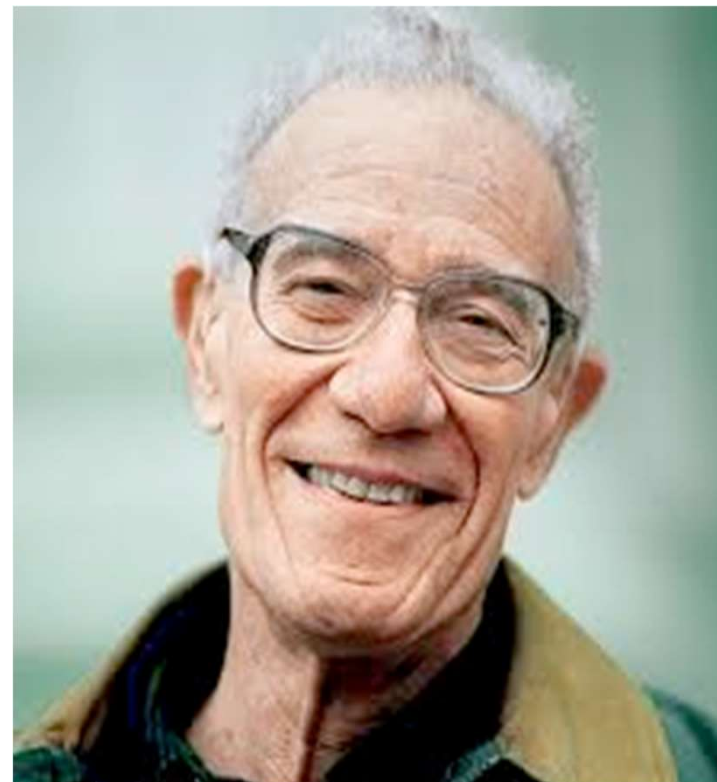
Does the H-D model represent the scope of possible choices in economic policy?

Is equilibrium an advantage or a disadvantage in this model?

The critical interpretation by R. Solow:

“a certain discomfort that I felt with their work. (...) Harrod and Domar seemed to be answering a straightforward question: when is an economy capable of steady growth at a constant level? They arrived by noticeable routes, at a classical simple answer: the national saving rate (the fraction of income saved) has to be equal to the product of the capital output ratio and the rate of growth of the (effective) labor force. Then and only then could the economy keep its stock of plant and equipment in balance with its supply of labor, so that steady growth could go on without the appearance of labor shortage on one side or labor surplus and growing unemployment on the other side. **They were right about that general conclusion.**”

But...



criticisms to the hypotheses of H-D model

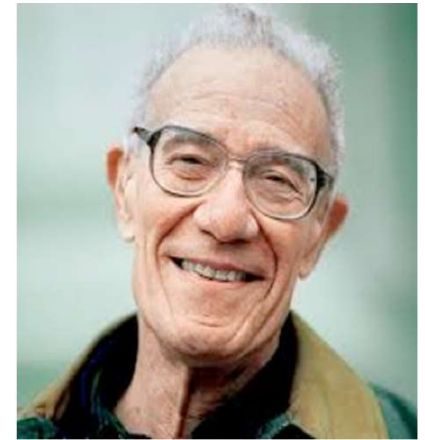
The savings rate (*preferences*), the growth rate of labour supply (*demographic and sociological*) and the capital-output ratio (*technology*) are constants, “facts of nature”?

They are independent. They may change but independently: they are not related to each other. **“In that case (...) the possibility of steady growth would be a miraculous stroke of luck”** (Solow)

critique of the interpretation

doubling the saving rate originates the doubling of the GDP growth rate?

No. “doubling the *ex ante* saving rate would **not** double the *ex post* saving rate unless something were taking care of the *ex ante* investment rate at the same time”.



Reaction by R. Solow ...

“That was the spirit in which I began tinkering with the theory of economic growth, trying to improve on the Harrod-Domar model. I can not tell you why I thought first about replacing the constant capital-output (and labor-output) ratio by a richer and more realistic representation of the technology”

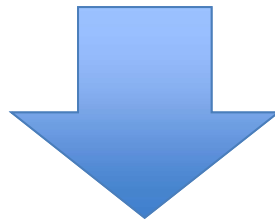
Suggests an alternative, the neoclassical production function (with technological flexibility);

to describe the equilibrium of the economic system as “equilibrium paths” (to specify the endogenous mechanisms of internal adjustments with transition into the equilibrium), since he suggests that s , v and δ cannot be independent.

... and solutions by R. Solow

technological flexibility through the **substitutability** of production factors (not complementarity); therefore, the adjustment occurs within the productive process

v (capital-output ratio) is not a constant but, instead, becomes a variable (internal adjustment)



The Solow-Swan model