



LISBON  
SCHOOL OF  
ECONOMICS &  
MANAGEMENT  
UNIVERSIDADE DE LISBOA

# Macroeconomics II

## Lecture 08 (March 2020)

Romer model

Extensions



## Theoretical Lecture 08

### The Romer model

- the model of endogenous growth by Romer: main assumptions;
- production function;
- production of new ideas and productivity of research (externality due to duplication; the spillover effect of research);
- economic growth in the *steady state*;
- long-run effect of research policy.

### Reading

Jones, C., Vollrath, D. (2013), Introduction to Economic Growth, Norton, ch. 5, pp. 97-119.

### Classical

Romer, P. (1990), "Endogeneous Technological Change", *Journal of Political Economy*, 98, October 1990: S71-S102

# exogenous models of economic growth: main concepts

**Solow** model: exogenous model of economic growth

the concept of steady state; there are endogenous mechanisms that drive the economy towards the steady state:  
substitutable production factors;  
capital accumulation is characterized by diminishing returns (declining marginal product of each unit of capital).

in steady state, the economy grows due to exogenous factors (technological progress at the rate “ $g$ ”; population at the rate “ $n$ ”), which are not “explained” by the model;

in the Solow model, economic policy has only temporary effects on the rate of economic growth; **it has no long-run effects on the rate of economic growth.**

## growth theory since the 1950s

**neoclassical growth models** (Solow, etc), in the 1950s and 1960s:

savings (= investment) rate is exogenous (it is crucial to explain the steady state income level) (there is a social planner who decides?)

technological progress is exogenous (it is crucial to explain the long-run per capita growth rate) (does the technological progress come from heaven?)

they are called **exogenous growth models**

**unsatisfactory!** -> two theoretical reactions towards **endogenous growth models**:

growth models with **endogenous savings** (mid 1960s)

Cass (1965) and Koopmans (1965), that go back to Ramsey (1928), Young (1928), Knight (1944)

growth models with **endogenous technological progress** (mid 1980s)

Arrow (1962), Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991)

## growth theory in the 1970s

in between the two theoretical contributions (1970s): a great vacuum of growth economic theory of about 15 years!

main emphasis on short-term analysis in the 1970s (real business cycles, rational expectations, general equilibrium models, etc);

very much technical approaches and very little empirical applications on economic growth;

development economists emerged with great emphasis on the study of growth in less developed countries;

**growth** economics vs. **development** economics

## The “new” models of endogenous growth

The models of exogenous growth are not adequate to explain economic growth if the long-run growth rate of GDP is explained by (endogenous) technological progress

In the 1970s and up to the mid-1980s, macroeconomics was focused mainly in short term issues.

By mid-1980s some economists made theoretical work devoted to fill gaps in the explanatory models of growth; for instance Romer (1986, 1987) deals with technological progress as an endogenous variable (the process of generation of new ideas, research and innovation, R&D);

**These models tend to consider as well population growth as an endogenous variable** (dependent on GDP per capita)



## endogenous growth models: the role of technological progress

difficulty of incorporation of technological progress in neoclassical models because competitive assumptions cannot be made regarding the creation of new ideas (imperfect competition/ideas as quasi-public goods)

main contribution of **Paul Romer**:

**Romer, P.** (1986). “Increasing returns and long-run growth”. *Journal of Political Economy*, 90: 6 (Dec.), 1257-1278

Romer, P. (1990). “Endogenous technological change”. *Journal of Political Economy*. 98:5 (October), Part II, S71-S102

author of the incorporation of R&D theories and imperfect competition into the growth models. That was the reason for the **Nobel 2018**.

## endogenous growth models: the role of technological progress (cont.)

incorporation of Schumpeterian ideas of progress (as “creative destruction”)

+ new lines of research:

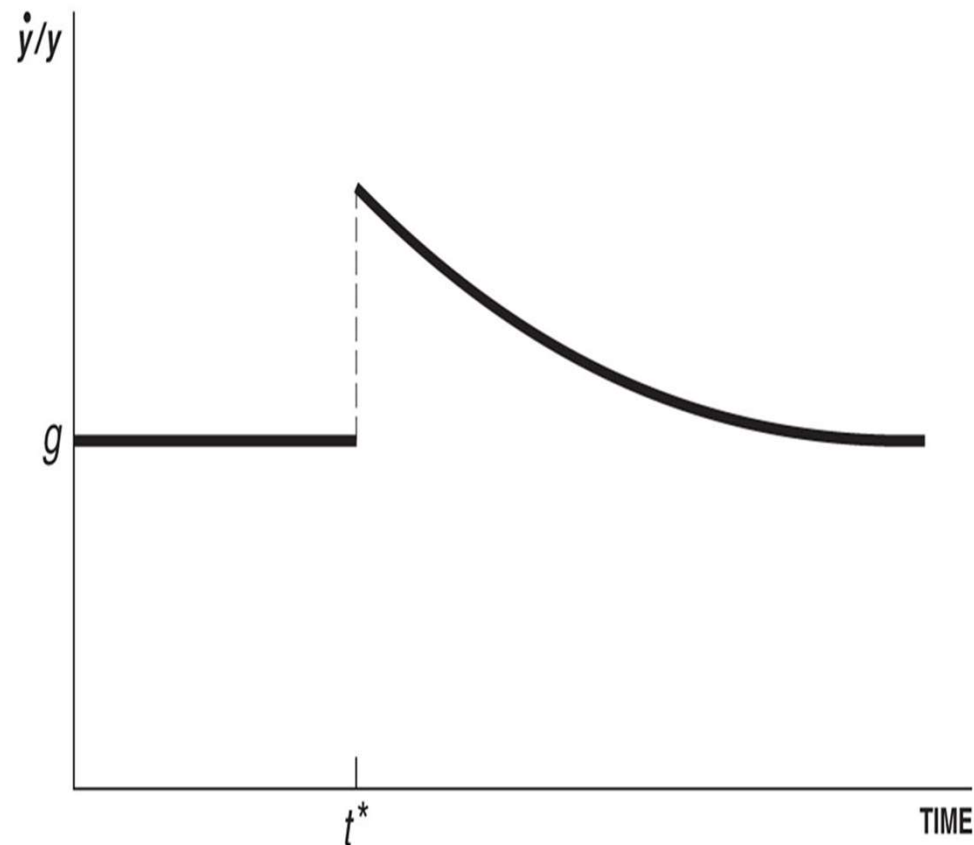
Diffusion of technology and its role in economic growth (a line of empirical research in progress);

Notice the relevance of FDI and its growth in the last decades





FIGURE 2.12 THE EFFECT OF AN INCREASE IN INVESTMENT ON GROWTH

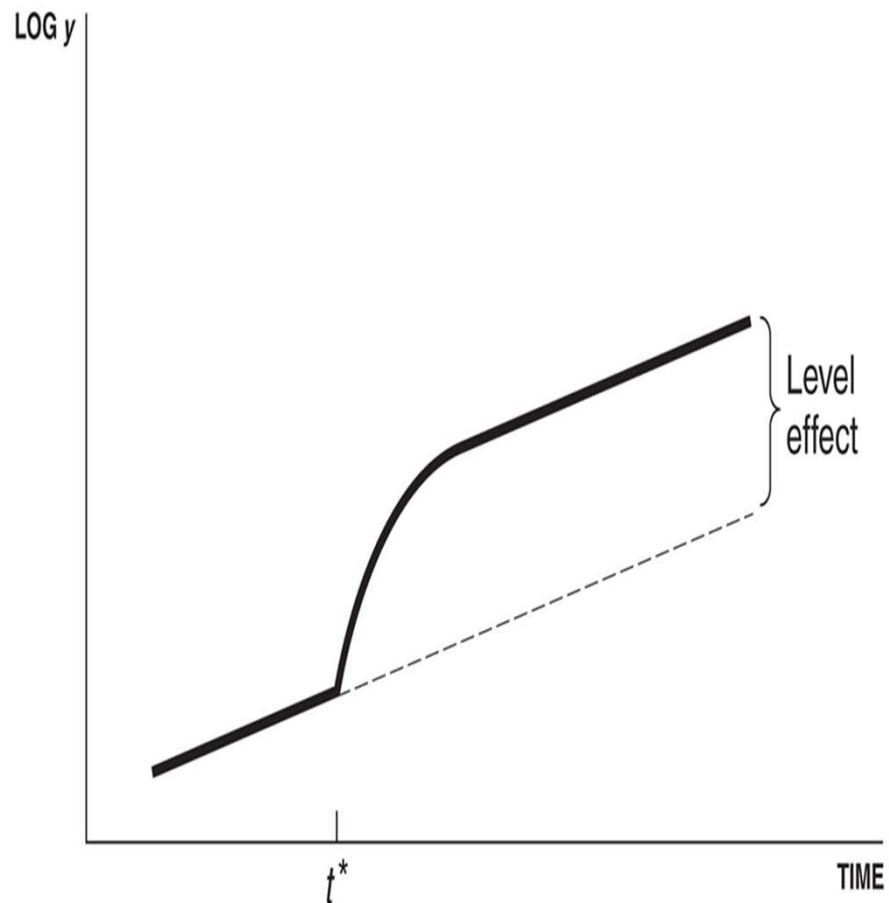


### Solow model:

the rise of the investment rate  
has no effect on the long-run  
growth rate of GDP *per capita*



FIGURE 2.13 THE EFFECT OF AN INCREASE IN INVESTMENT ON  $y$



**Solow model:**

the rise of the investment rate

**has positive effect** on the long-run

level of GDP *per capita*

## The Romer model

To endogenize the technological progress: **research as an economic activity** (R&D)

production function:

$$^{(1)} Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$$

**A(t)** – level of technology in the economy, which is measured by the stock of ideas accumulated until the present; the model intends to explain the growth of A(t).

for a given level of technology A, the production function has constant returns to scale in K e  $L_Y$  (Labour in the production of final goods)

A is an input for production (*stock* of ideas: i.e., the use of *patents*)  
The production function has increasing returns to scale in K,  $L_Y$  and A

## the generation of the inputs: physical capital (K), labour (L) and ideas (A)

### physical capital

$$(2) \quad dK/dt = s_k Y - \delta K$$

### labour

$$(3) \quad (dL/dt)/L = n \quad n \text{ is exogenous}$$

$$(4) \quad L = L_Y + L_A \quad \text{labour in the production of final goods } (L_Y) \text{ and in research } (L_A)$$

$$(5) \quad L_A/L = s_R \quad s_R \text{ constant; exogenous? endogenous?}$$

$A(t)$ , the growth of the stock of ideas is endogenous

$$dA/dt = \theta^* \cdot L_A$$

$dA/dt$  is the evolution of the discovery of new ideas

$\theta^*$  productivity of research (the rate of producing new ideas by the researchers)

$L_A$  number of researchers

$\theta^* = \theta(A)$ , the productivity of researchers is a function of the **stock of ideas**

increasing function? a large set of accumulated ideas facilitates the discovery of new ideas; positive spillover (what has been discovered facilitates new ideas);

decreasing function? Given a large set of accumulated ideas, it becomes more difficult to discover “new” ideas (since so much is already known ...)

$\theta^* = \theta \cdot A^\Phi$  rate at which new ideas are produced, with  $\Phi > 0$

(increasing) or  $\Phi < 0$  (decreasing)

**$\theta^* = \theta(LA)$** , the productivity of the researchers is a function of the **number of researchers**

a large number of researchers facilitates the creation of research networks and then strengthens the ability of each research and his/her team to make more and better research ( $\lambda > 1$  below)

**externality** associated to duplication: some ideas may be not new ideas, since they may have been already discovered/or being discovered simultaneously by other researchers ( $\lambda < 1$  below)

**$dA/dt = \theta \cdot L_A^\lambda \cdot A^\Phi$**  using  $L_A^\lambda$  because the productivity of research depends on the number of researchers looking for new ideas

$\lambda < 1$  (duplication or repetition effect), or  $\lambda > 1$

$\Phi < 0$  (decreasing with A), or  $1 > \Phi > 0$  (increasing with A, spillover effect)

## The Romer model

$$(1) \quad Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$$

$$(2) \quad dK/dt = s k_Y - \delta K$$

$$(3) \quad (dL/dt)/L = n$$

$$(4) \quad L = L_Y + L_A$$

$$(5) \quad L_A/L = s_R$$

$$(6) \quad dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

## growth of the economy in *steady state*

Assuming  $s_R$  constant, the growth of GDP per capita in *steady state* is explained by the technological progress (as in Solow model):

$$g_y = g_k = g_A$$

What is (and what explains) the rate of technological progress,  $g_A$ ?

**Reminder**: this growth rate is endogenous in the model!

$$dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

$$(dA/dt)/A = \theta \cdot (L_A^\lambda \cdot A^\Phi)/A = \theta \cdot L_A^\lambda \cdot A^{\Phi-1}$$

or 
$$g_A = \theta \cdot L_A^\lambda \cdot A^{\Phi-1}$$



## the rate of technological progress in *steady state*

$$g_A = \theta \cdot L_A^\lambda A^{\Phi-1} = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

In the *steady state*  $g_A$  is constant: growth rate of the numerator = growth rate of denominator

$$\lambda \cdot (dL_A/dt)/L_A = (1 - \Phi) \cdot (dA/dt)/A$$

In the *steady state*  $(dL_A/dt)/L_A = n$  (growth rate of population)

$$\lambda \cdot n = (1 - \Phi) \cdot g_A$$

**Technological progress**

$$g_A = \lambda \cdot n / (1 - \Phi)$$

$g_A$  is explained by the parameters of the production function of ideas ( $\lambda$  e  $\Phi$ ) and the growth rate of population ( $n$ )

$$g_A = \lambda \cdot n / (1 - \Phi)$$

## interpretation

let the special case

$\lambda = 1$  and  $\Phi = 0$  (for an easier explanation)

then:

$dA/dt = \theta \cdot L_A$ , from equation (6)

If  $L_A$  is constant, in each period  $\theta \cdot L_A$  new ideas emerge. Then the growth rate of  $A$ ,  $g_A$ , is **decreasing** (the stock rises by equal amounts, so that the growth rate decreases). The alternative to get a non-decreasing growth  $g_A$  is to prevent the decrease of the number of researchers. **This requires the population to rise and explains  $n$  in the equation above.**

## **conclusion**

If the population does not increase, **economic growth will not happen**, even keeping research activity and technological progress in the economy.

## Effects of Economic Policy

Economic policy may have effect on long-run economic growth?

example of an economic policy measure: **incentives to research,**  
creating/increasing research subsidies; **rise of  $s_R$**

$$(dA/dt)/A = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

let us assume that  $\lambda = 1$  and  $\Phi = 0$

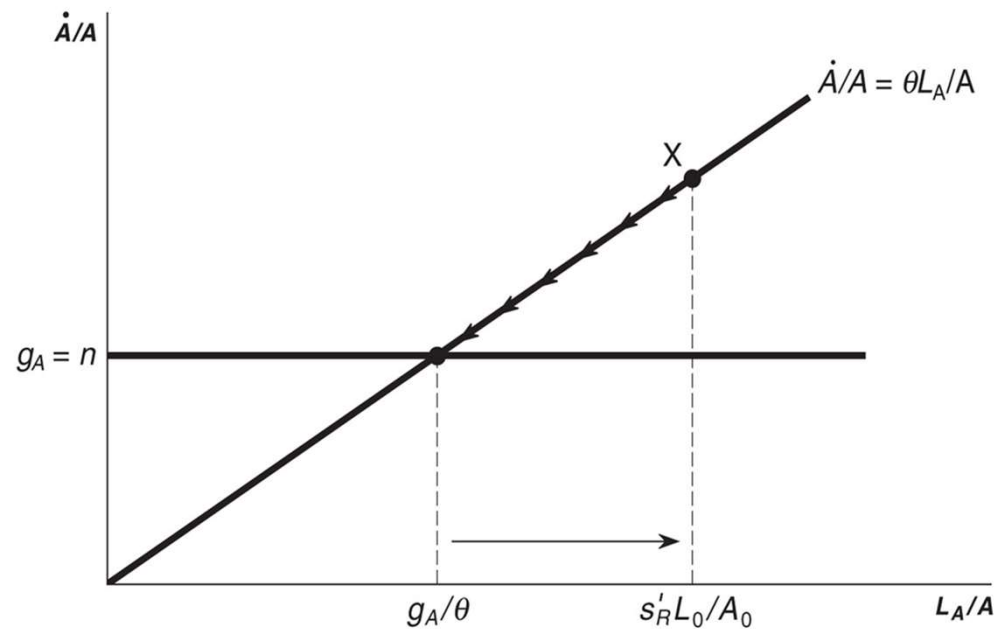
then:

$$(dA/dt)/A = \theta \cdot L_A / A = \theta \cdot s_R \cdot L / A$$

We know that in steady state, with  $\lambda = 1$  and  $\Phi = 0$ ,  $g_A = n$

What is the effect of a policy of incentives to research by rising  $s_R$ ?

**FIGURE 5.1 TECHNOLOGICAL PROGRESS: AN INCREASE IN THE R&D SHARE**



$s'_R > s_R$        $s_R$  increased (number of researchers in I&D)

$s'_R \cdot L_0 > s_R \cdot L_0$ , the number of ideas increases;

technological progress > population growth ( $n$ )

$\Rightarrow L_A/A$  decreases

$\Rightarrow g_A$  decreases

the economy returns to the previous *steady state*

*steady state:*  $\theta \cdot s_R \cdot L_0/A_0 = g_A \Rightarrow s_R \cdot L_0/A_0 = g_A/\theta$

FIGURE 5.2  $\dot{A}/A$  OVER TIME

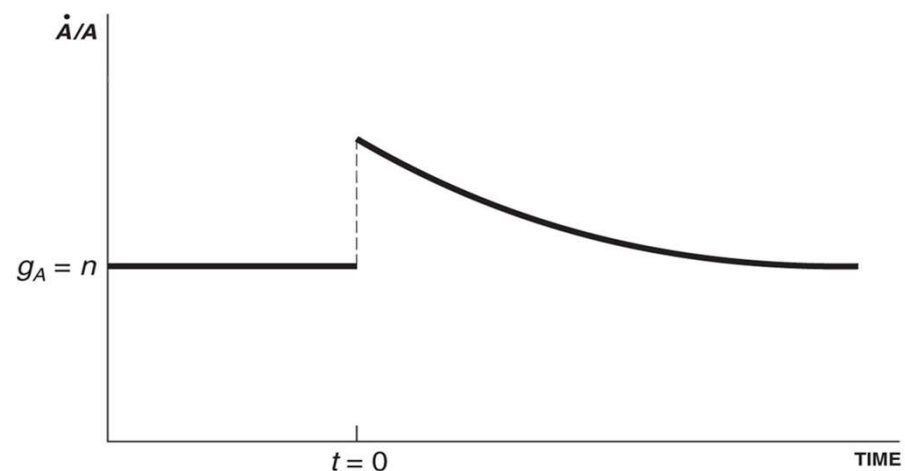
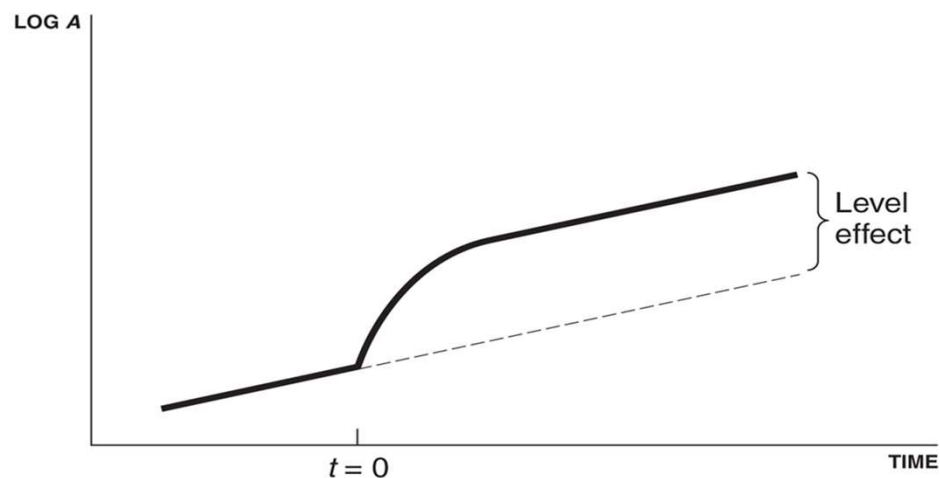


FIGURE 5.3 THE LEVEL OF TECHNOLOGY OVER TIME



A permanent increase of  $s_R$  increases the technological progress (and economic growth) only in a temporary way, not permanently in the long-run.

But it increases permanently (in the long-run) the level of technology.

## technological progress and growth

(to remind)

growth accounting: growth of factors and of TFP (total factor productivity)

growth of TFP through technological progress

technological progress in the models of exogenous growth

growth in steady state: rate  $g$

rate  $g$  is **exogenous**

technological progress in the models of endogenous growth

rate  $g$  is **endogenous**

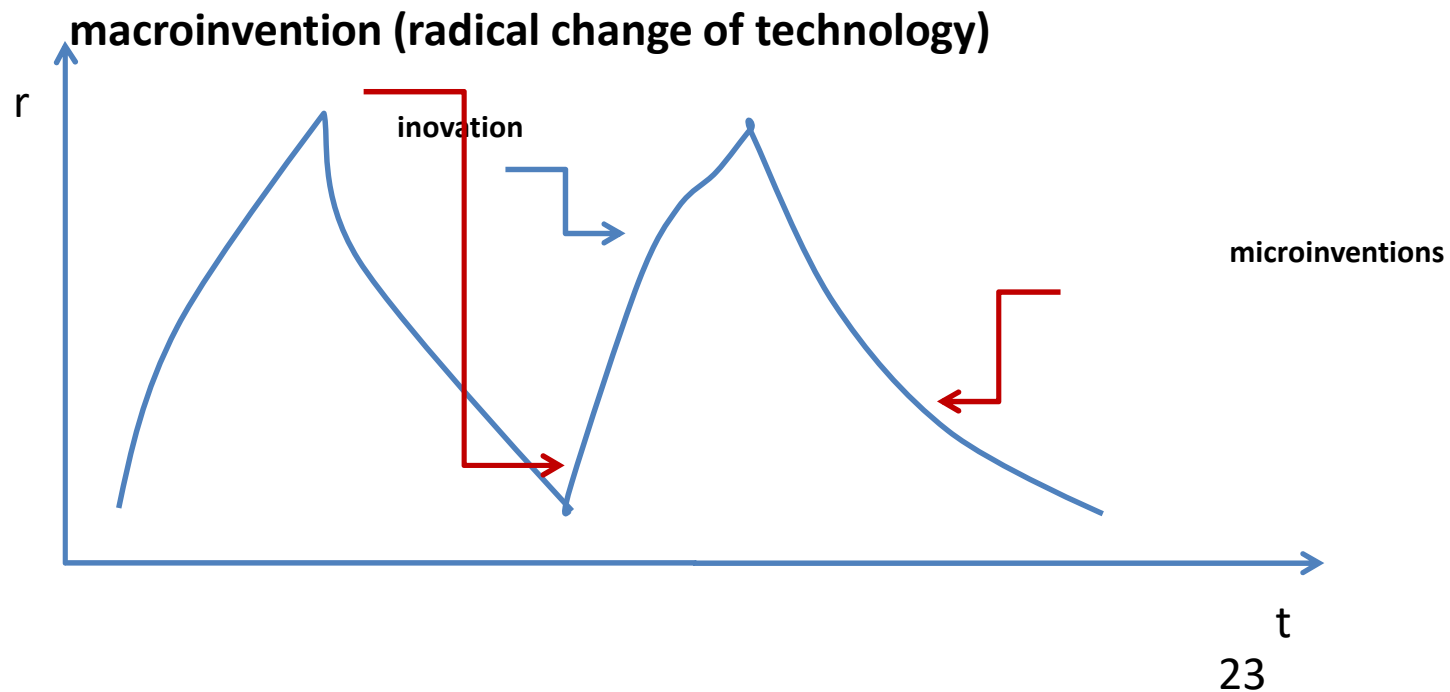
technological progress is “explained” by the working of the economy  
(it is an output of a sector of economic activity)



# Facts and concepts, technological progress: invention and innovation

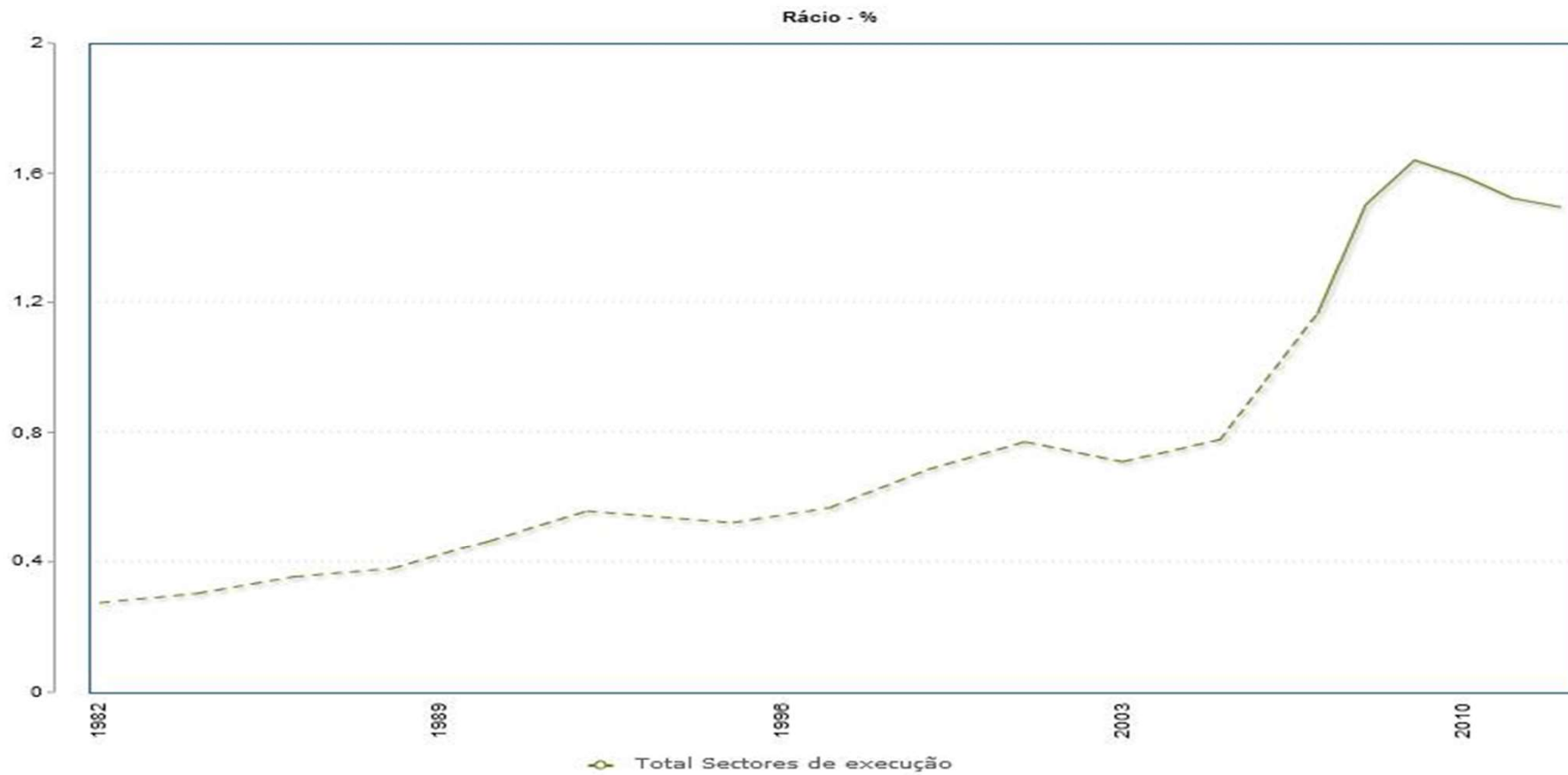
invention: discovery of new ideas

innovation: implementation of the new ideas in the economic activity





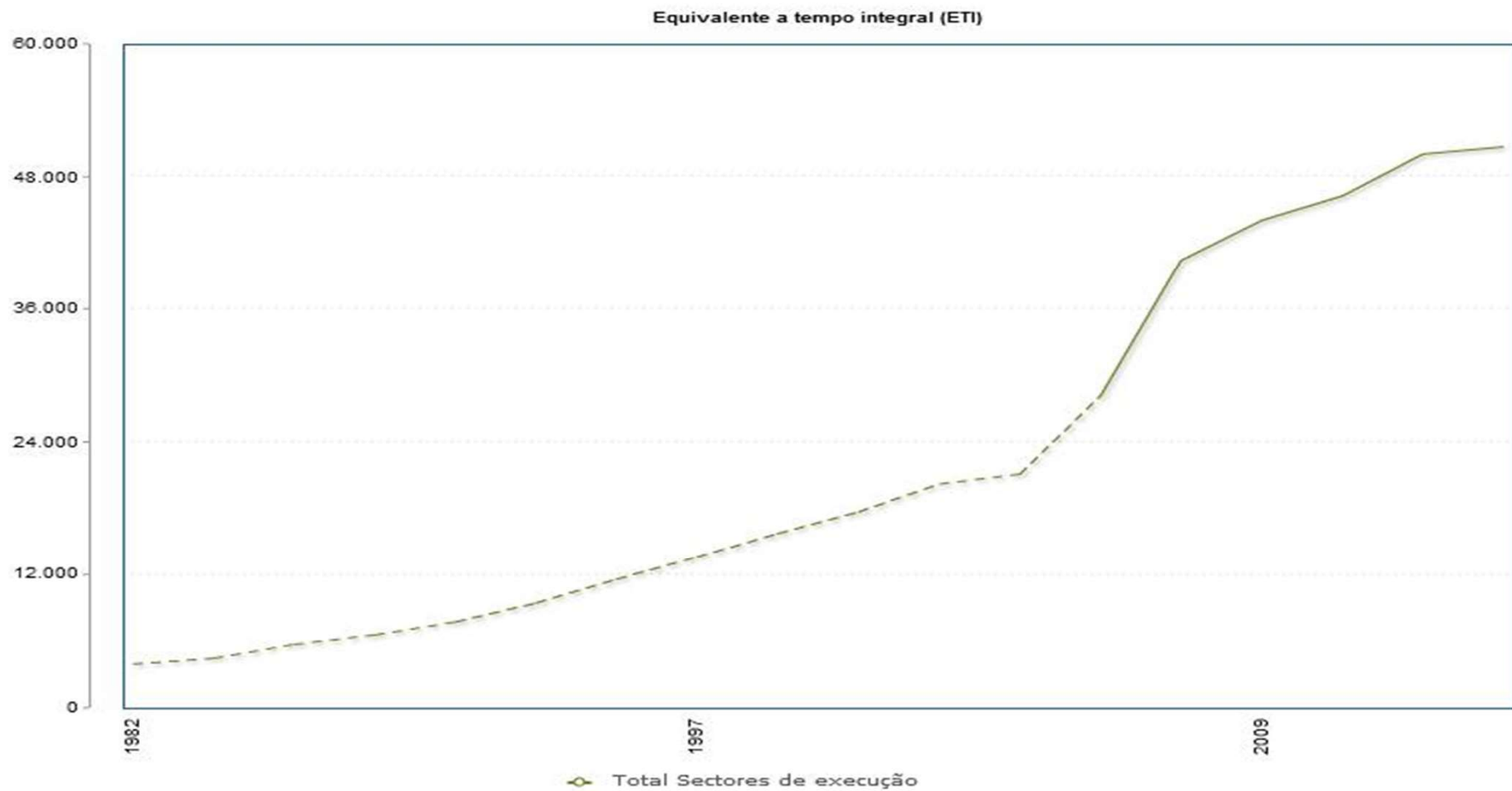
## PORTUGAL: Expenditure on R&D as % of GDP





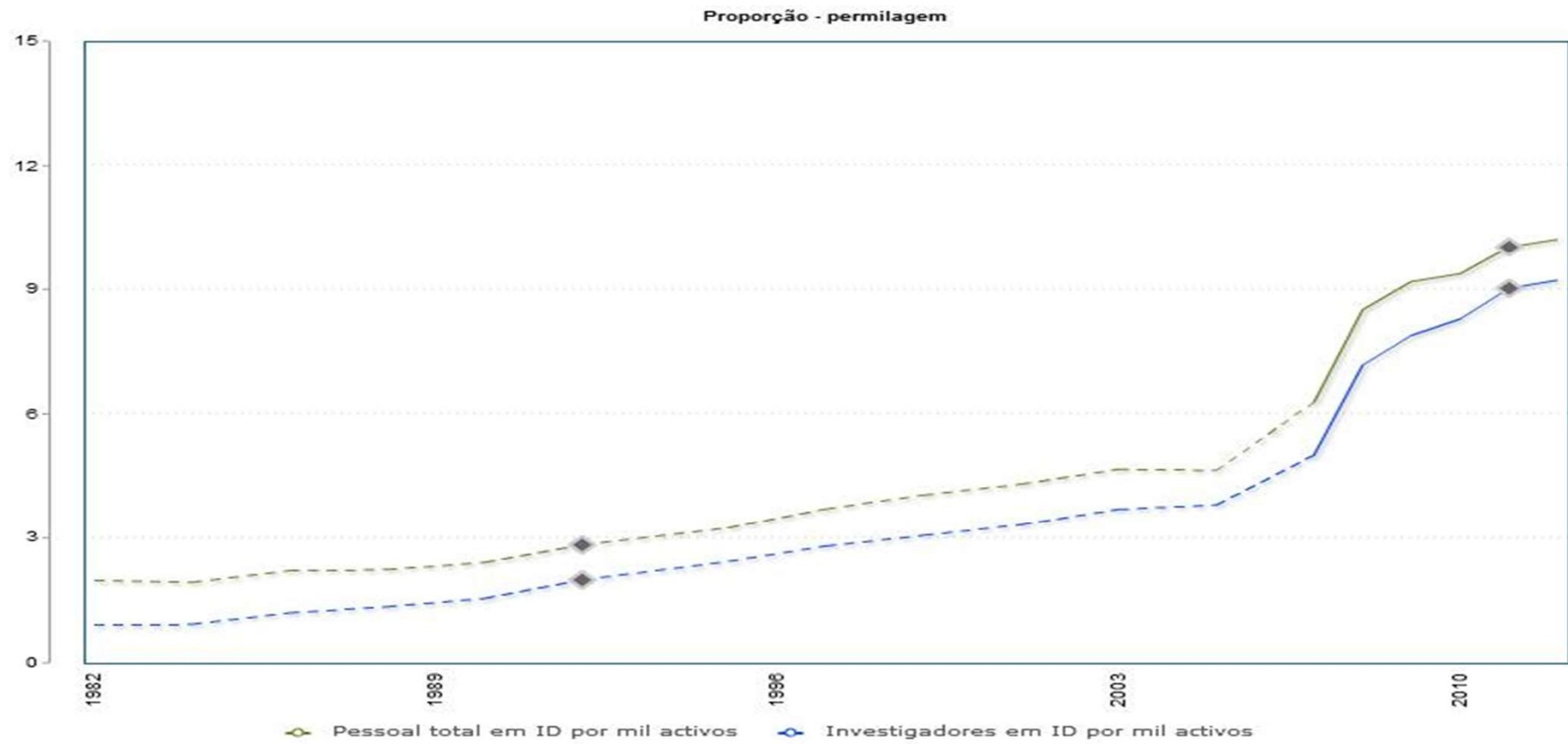


## PORTUGAL: Nr of researchers (Full Time Equivalent) in R&D



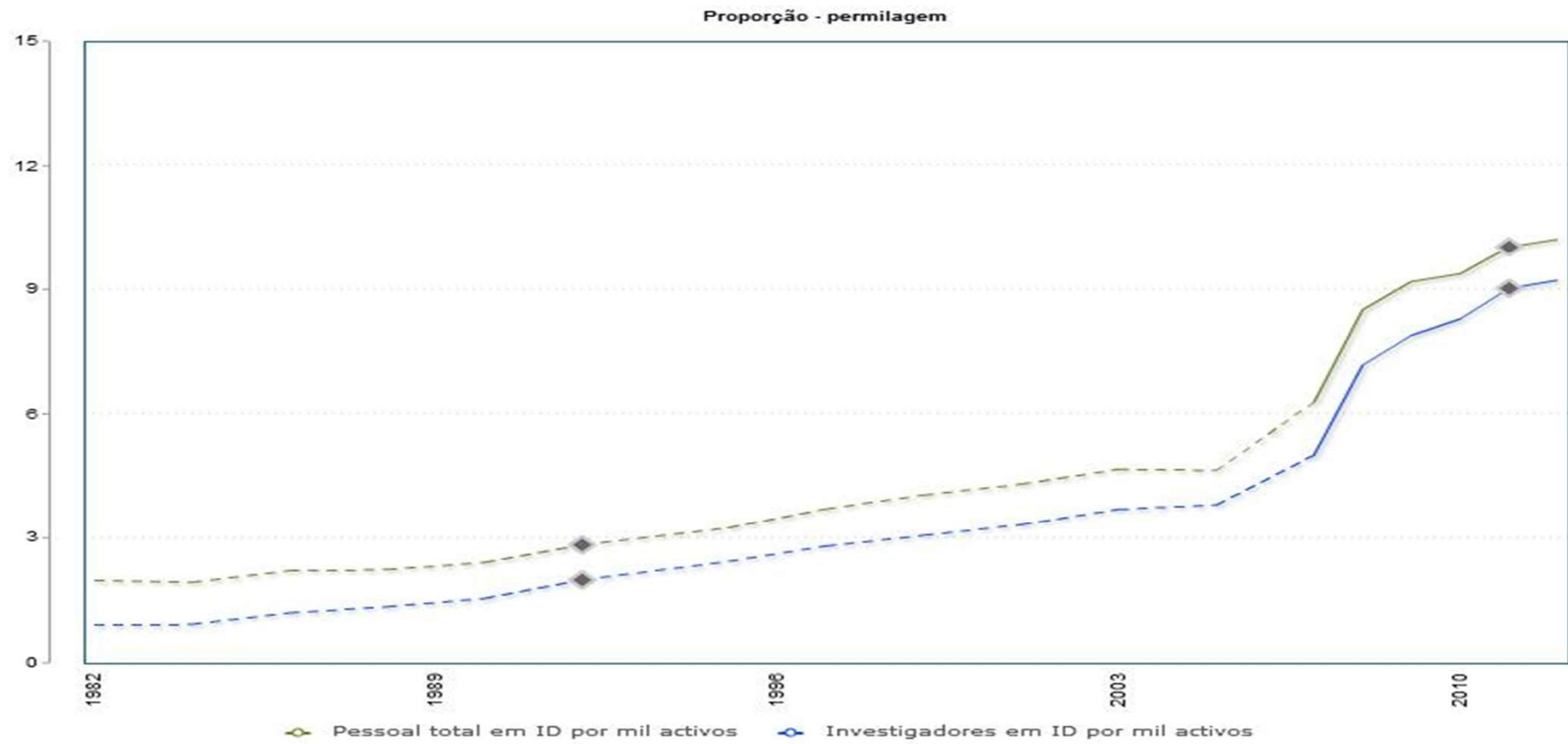


## Nr of workers in R&D as % of active population



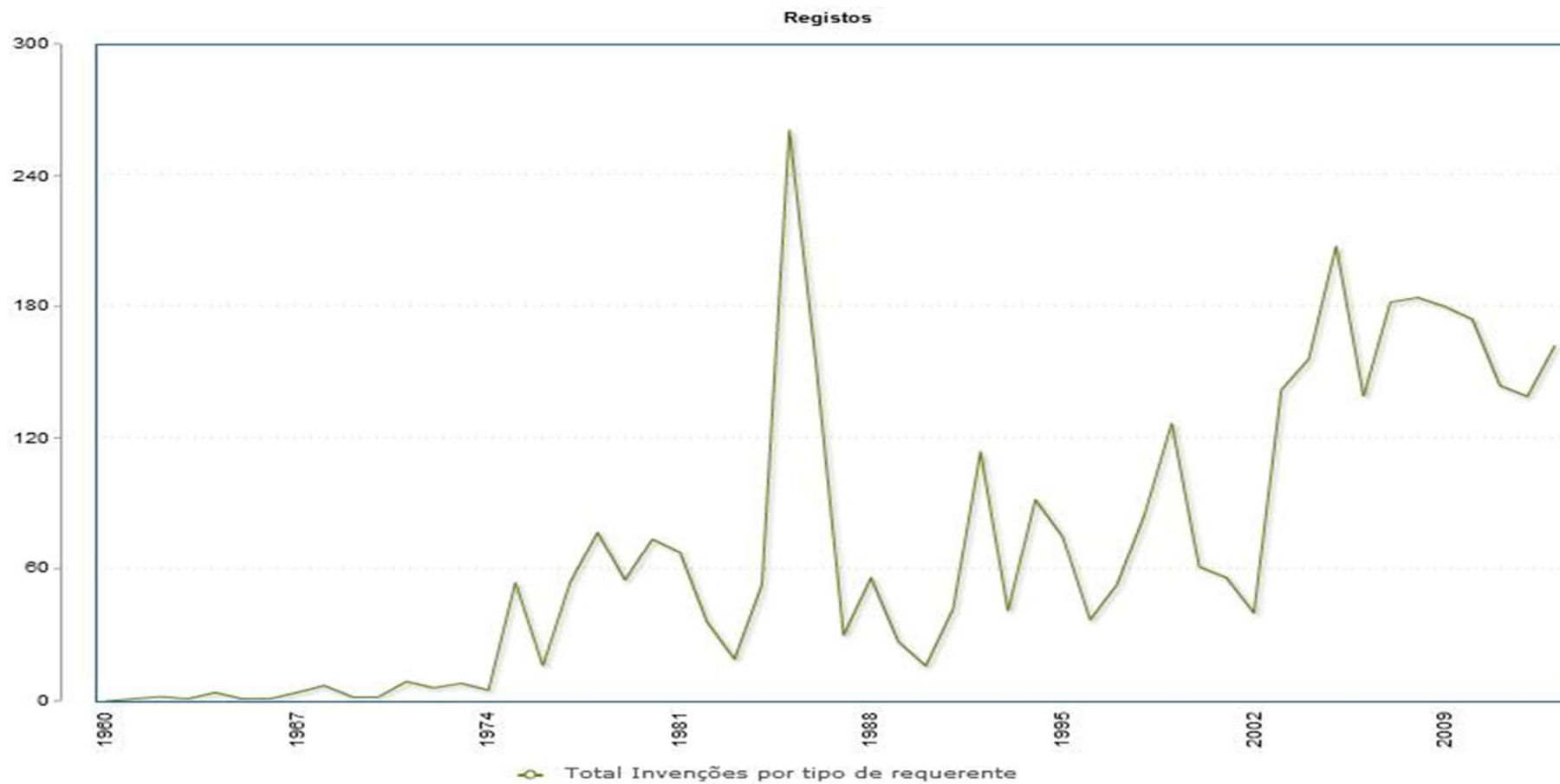


## Nr of workers in R&D as % of active population





## Nr of inventions/patents issued in Portugal



## The Romer model (summary)

to endogenize the technological progress:

- **production function** (it has ideas/“*knowledge*” as a production factor)
- equations to describe the creation of inputs (including the ideas/“*knowledge*”)



(1)  $Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$

(2)  $dK/dt = s_k Y - \delta K$

(3)  $(dL/dt)/L = n$

(4)  $L = L_Y + L_A$

(5)  $L_A/L = s_R$

(6)  $dA/dt = \theta^* \cdot L_A$

(7)  $\theta^* = \theta \cdot$

$A^\Phi$

$dA/dt = \theta L^\lambda_A \cdot A^\Phi$

p.f. constant returns to scale in K and  $L_Y$ ;  
increasing returns in K,  $L_Y$  and A.

**A(t)** *stock* of knowledge

(nr of ideas invented until moment t)

$\theta^*$  productivity of research (nr. of new ideas  
produced per researcher)

$0 < \lambda < 1$  externality associated with duplication

$\Phi > 0$  positive knowledge spillover in research

some of the ideas created by a  
researcher may be not new ( $\lambda < 1$ );  
but may also exist a network  
effect ( $\lambda > 1$ )

much of what has been discovered  
so far facilitates the generation of  
new ideas (positive spillover); a large  
set of accumulated ideas becomes more  
difficult to discover "new" ideas (negative  
spillover)

