Exercício 7 d-g (Cap. 4 do livro)

d) Calcule a média e a variância de X e Y.

$D = \{(1,0),(1,1),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(3,3)\}$

$D_X = \{1,2,3\}$

$D_Y = \{0,1,2,3\}$

Média

$E(X) = \mu_X = \sum_{(x,y)\in D} x f_{X,Y}(x,y) = (1+1+2+2+2+3+3+3+3) \times \frac{1}{9} = \frac{20}{9}$

ou

$E(X) = \mu_X = \sum_{x\in D_X} x f_X(x) = 1 \times \frac{2}{9} + 2 \times \frac{3}{9} + 3 \times \frac{4}{9} = \frac{20}{9}$

$E(Y) = \mu_Y = \sum_{(x,y)\in D} y f_{X,Y}(x,y) = (0+1+0+1+2+0+1+2+3) \times \frac{1}{9} = \frac{10}{9}$

ou

$E(Y) = \mu_Y = \sum_{y\in D_Y} x f_Y(y) = 0 \times \frac{3}{9} + 1 \times \frac{3}{9} + 2 \times \frac{2}{9} + 3 \times \frac{1}{9} = \frac{10}{9}$

Variância

$Var(X) = \sigma_X^2 = E(X^2) - \mu_X = \sum_{x\in D_X} x^2 f_X(x) - \mu_X = (1^2 \times \frac{2}{9} + 2^2 \times \frac{3}{9} + 3^2 \times \frac{4}{9}) - (\frac{20}{9})^2 = 0,6173$

$Var(Y) = \sigma_Y^2 = E(Y^2) - \mu_Y = \sum_{y\in D_Y} y^2 f_Y(y) - \mu_Y = (0^2 \times \frac{3}{9} + 1^2 \times \frac{3}{9} + 2^2 \times \frac{2}{9} + 3^2 \times \frac{1}{9}) - (\frac{10}{9})^2 = 0,9877$

e) Obtenha $E(Y|X=x)$. Interprete o valor para $x = 2$

$E(Y|X=x) = \sum_{y\in D_Y} y f_{Y|X=x}(y)$

– $E(Y|X=1) = \sum_{y\in D_Y} y f_{Y|X=1}(y)$

$f_{Y|X=1}(y) = \frac{f_{X,Y}(1,Y)}{f_X(1)} \qquad y = 0,1$

$f_{Y|X=1}(0) = \frac{f_{X,Y}(1,0)}{f_X(1)} = \frac{1}{2}$

$f_{Y|X=1}(1) = \frac{f_{X,Y}(1,1)}{f_X(1)} = \frac{1}{2}$

$E(Y|X=1) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}$

– $E(Y|X=2) = \sum_{y\in D_Y} y f_{Y|X=2}(y)$

$f_{Y|X=2}(y) = \frac{f_{X,Y}(2,Y)}{f_X(2)} \qquad y = 0,1,2$

$f_{Y|X=2}(0) = \frac{f_{X,Y}(2,0)}{f_X(2)} = \frac{1}{3}$
$f_{Y|X=2}(1) = \frac{f_{X,Y}(2,1)}{f_X(2)} = \frac{1}{3}$
$f_{Y|X=2}(2) = \frac{f_{X,Y}(2,2)}{f_X(2)} = \frac{1}{3}$

$E(Y|X=2) = 0 \times \frac{1}{3} + 1 \times \frac{1}{3} + 2 \times \frac{1}{3} = 1$

– $E(Y|X=3) = \sum_{y \in D_Y} y f_{Y|X=3}(y)$

$f_{Y|X=3}(y) = \frac{f_{X,Y}(3,Y)}{f_X(3)} \qquad y = 0,1,2,3$

$f_{Y|X=3}(0) = \frac{f_{X,Y}(3,0)}{f_X(3)} = \frac{1}{4}$
$f_{Y|X=3}(1) = \frac{f_{X,Y}(3,1)}{f_X(3)} = \frac{1}{4}$
$f_{Y|X=3}(2) = \frac{f_{X,Y}(3,2)}{f_X(3)} = \frac{1}{4}$
$f_{Y|X=3}(2) = \frac{f_{X,Y}(3,2)}{f_X(3)} = \frac{1}{4}$

$E(Y|X=3) = 0 \times \frac{1}{4} + 1 \times \frac{1}{4} + 2 \times \frac{1}{4} + 3 \times \frac{3}{2}$

Sabendo que foram recebidas 2 máquinas pelo comerciante, o número médio de vendas mensalmente é de 1 máquina.

f) Determine a média e a variância do número de máquinas que ficam por vender mensalmente.

Z : variável aleatória que representa o número de máquinas que ficam por vender, mensalmente.

$Z = X - Y \quad (z = 0,1,2,3)$

A função probabilidade da variável aleatória Z foi determinada na alínea c)

$E(Z) = \mu_Z = \sum_{z \in D_Z} z f_Z(z) = 0 \times \frac{1}{3} + 1 \times \frac{1}{3} + 2 \times \frac{2}{9} + 3 \times \frac{1}{9} = \frac{10}{9}$

$Var(Z) = \sigma_Z^2 = E(Z^2) - \mu_Z = \sum_{z \in D_Z} z^2 f_Z(z) - \mu_Z = (0^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} + 2^2 \times \frac{2}{9} + 3^2 \times \frac{1}{9}) - (\frac{10}{9})^2 = 0,9877$

g) Determine o coeficiente de correlação entre X e Y.

$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$

$Cov(X,Y) = \sigma_{X,Y} = E(XY) - E(X)E(Y) = \sum_{(x,y) \in D} xy f_{X,Y}(x,y) - \mu_X \mu_Y =$

$= (0 \times 1 + 1 \times 1 + 2 \times 0 + 2 \times 1 + 2 \times 2 + 3 \times 0 + 3 \times 1 + 3 \times 2 + 3 \times 3)\frac{1}{9} - \frac{20}{9} \times \frac{10}{9} = \frac{25}{81} = 0,3086$

$Var(X)$ e $Var(Y)$ foram determinadas na alínea a)

$\rho_{X,Y} = \frac{0,3086}{\sqrt{0,6173 \times 0,9877}} = 0,3953$