Lecture notes, exercises, and study suggestions

Lecture 1 — February 19

1. Make yourself familiar with some time series application such as ITSM, EViews, or R.
2. Get some of the Wei's book data sets at the author's website. Plot some of the data series and check whether they present a trend, seasonality, cycles and noise.
3. Read carefully Wei's book definition of stochastic processes and make sure you understand that time series can be seen as models or realizations of stochastic processes in which we are interested in the sequence of X_t variables or x_t observations as functions of t.
4. Make sure you understand the concepts of *n-dimensional distribution function*, *strict stationarity* and *second-order stationarity*.
5. Review the properties of γ_k and ρ_k.

Lecture 2 — February 26

1. Review the concept and computation of ϕ_{kk}.
2. Do the following exercises from Wei's book: 2.1; 2.3(c); 2.7(a), (b), and (c) with Excel; 2.7(d) and (e) with an appropriate computer program.
3. By using the appropriate system of equations, compute ϕ_{22} for a process such that $\gamma_0 = 4, \gamma_1 = 2, \gamma_2 = 1$.
4. Do the same for a process such that $\gamma_0 = 4, \gamma_1 = 2, \gamma_2 = 1/2$
5. Make sure you understand the concepts of white noise, iid and iin processes
   - Give one example of a process $a_t \sim \text{wn}(0, \sigma^2)$ which is not iid
   - Give one example of a process $\epsilon_t \sim$ iid which is not $\text{wn}(0, \sigma^2)$

6. Make sure you understand what is an *ergodic in mean* process and that a sufficient condition for this property is that $\rho_k \to 0$.

7. Review Bartlett's formula and understand that you can construct bands for testing wether $\rho_k = 0$ or $\phi_{kk} = 0$ on the form $\pm 1.96/\sqrt{n}$

8. Do exercise 2.9 from Wei's book.

Lecture 3 — March 4

1. Review MA(∞) representation of a process, also called Wold representation

2. Do exercise 3.1 from Wei's book: parts (a) and (b) manually, and parts (c) and (d) with appropriate software.

3. Recall that $B^i X_t = X_{t-i}$ and $\nabla = (1 - B)$. Express the following in terms of X_i:

   - BX_{t-2}
   - $\nabla^2 X_t$
   - $(1 - B^2)X_t$
   - $BX_t - B^{-2}X_t$
   - $(1 - B^s)X_t$

4. Using Excel, simulate an AR(1) process. Try different values of ϕ, do the time series plot of the generated series and the scatter plot of Z_t against Z_{t-1}. Try to compare and interpret the pictures.

5. Make sure you know how to derive the ACF of an AR(1) process.

6. Make sure you understand what is an invertible process and what is a stationary process and the corresponding conditions in terms of the Wold representation and the AR(∞) representations.

Lecture 4 — March 18 and 20

1. Derive the ACVF and the ACF of the process $Z_t = a_t + \frac{1}{2}a_{t-1} + a_{t-2}$ with $a_t \sim \text{iin}(0, 1)$, and check your results against ITSM or other appropriate software.

2. Make sure you are able to derive theoretically the ACF and the ACVF of AR(1), AR(2), MA(1), and MA(2) processes by computing the expected values $\text{E}(Z_t Z_{t+k})$

3. Make sure you understand why $\rho_k = 0$ for $k > q$ for MA(q) processes.

4. Make sure you understand the AR-MA duality and are able to compute ψ_j or π_j weights by using the method of undetermined coefficients (or by inverting the polynomials). See examples on section 3.3, pages 55-56, of Wei's 2nd edition, or go through the worked example in the file `Inverting MA-AR example.pdf`.

5. Consider the ARMA process Z_t that follows the equation:

$$Z_t - \frac{1}{4}Z_{t-1} = a_t - \frac{5}{6}a_{t-1} - \frac{1}{6}a_{t-2}$$

   and write it in inverted form, i.e., as an AR(∞) process. Show the numeric values of ψ_j for $j = 1, 2, 3, 4$.

6. Solve exercises 3.2, 3.6, 3.8 (a) and (c), 3.12, and 3.15 (a) and (b). You can use an appropriate software to check the results.

7. By using an appropriate software, work out the exercise 3.16.

Lecture 5 — March 25

1. Make sure you understand the differences between a trend-stationary process, i.e., a process you can transform to a stationary one by removing a deterministic trend, and a difference stationary process, i.e., a process you can transform to a stationary one by differencing it.

2. Make sure you understand what is a random walk process, with and without drifts.

3. Make sure you understand what's an ARIMA process.

4. Solve exercises 4.1 and 4.5.

5. Referring to the previous exercise make sure you understand the following concepts:

   - under-differencing, i.e, not differencing a process when you need to do so for it to become stationary, therefore observing an
   - AR unit root – non-stationarity (non-causality)
   - over-differencing, i.e, differencing a process when you don't need to do so for it to become stationary and so introducing a
   - MA unit root – non-invertibility

Verify that overdifferencing increases the variance

6. By using an appropriate software, work out the exercise 4.3.

7. Make sure you understand what it means for a time series to be nonstationary in variance or variability.

8. Make sure you understand the idea behind the Box-Cox transformation.

9. With the appropriate software, reproduce what we showed in the last slide relative to Chapter 4 with the Airline data or Wei's ww06.dat series. I.e., plot (a) the original data (b) the data resulting from applying Box-Cox or simply logs (c) the differenced data. Do it again, but difference first and log later and observe that by the resulting plot that you didn't achieve stationarity both in mean and variance.

Lecture 6 — April 1

1. Make sure you understand the distinction between a stochastic and a deterministic trend, and what's appropriate to make stationary each of these two types of series.

2. Make sure you understand what is a unit root that makes the series nonstationary and a unit root that makes a series noninvertible.

3. Exercise: Consider two time series Z_t and Y_t that follow the following processes:

$$\begin{aligned} Z_t &= 50 + 2t + a_t \\ Y_t &= Y_{t-1} + 2 + a_t, \quad Y_0 = 50 \end{aligned}$$

   (a) By using some software, simulate one run of each series with 100 observations and with $a_t \sim$ wn(0,1)

   (b) Do the same but now with $a_t \sim$ wn(0,10)

   (c) When are trend stationary and difference stationary observations easier to distinguish, in the first or in the second case?

   (d) Verify that you can make both series stationary by applying the differencing operator $\nabla = (1 - B)$ to each of them.

   (e) What is the variance of ∇Z_t and what is the variance of ∇Y_t? Do it both theoretically and with the simulated series.

4. Review the slides and understand the differences among the three Dickey-Fuller or Augmented Dickey Fuller (ADF) types of unit root tests.

5. Solve exercise 9.4a.

Lecture 7 — April 15–17

1. Know how to write a SARIMA model as an ARIMA model with restrictions on the parameters. Try out a practical example with ITSM.

2. Understand the difference between stationarity and nonstationarity in seasonality—SARIMA vs. SARMA—and try out a few examples with observed time series.

3. Try out a few models for the Airplane data.

4. Solve exercises 8.1 and 8.7 (Series ww26.dat) from Wei's book.

5. Consider the process Y_t that follows an $ARIMA(1,1,1)(1,0,0)_4$

   (a) Without using the operators B nor ∇, write in equation form in terms of the observations Y_t, Y_{t-1} $\ldots$, the noise $a_t, a_{t-1}\ldots$, and the corresponding parameters ϕ_i, Φ_i, θ_i, Θ_i, as applicable.

   (b) If you estimate this model with a nonseasonal $ARIMA(p, d, q)$ model, what are the p, d, q orders and what are the restrictions you should apply to the coefficients during the estimation.

Lecture 8 — April 22

1. Please review how the Wold representation allows the computation of forecasts in terms of past noise and allows the computation of the forecasting variance.

2. Review the slides *5. Forecasting.* Review the example on forecasting an ARMA(2,1) and solve exercises 5.2 and 5.5 (a) and (c) in Wei's book.

3. Review the first part of slides *7. Estimation and Model Selection.*

4. In these slides, pay attention to the unit root ADF tests in EViews outputs.