ISEG - Lisbon School of Economics and Management

Additional Exercises
2^{nd} Semester of 2019/2020

May 29, 2020

1. Let (X, Y) be a continuous random variable such that

$$f_{X,Y}(x, y) = \begin{cases} x + y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & otherwise \end{cases}.$$

   (a) Compute $\rho_{X,Y}$.

   (b) Compute $E(X|Y = y)$.

2. Let (X, Y) be a discrete random variable such that

$$f_{X,Y}(x, y) = \begin{cases} \frac{x-y}{a}, & (x, y) \in \{(0, -1), (2, -1), (1, 0), (2, 0), (2, 1), (3, 1)\} \\ 0, & otherwise \end{cases}.$$

   (a) Find a.

   (b) Compute $cov(X, Y)$.

   (c) Compute $E(X|Y = y)$.

   (d) Compute $E(E(X|Y))$.

3. Assume that X_1 is independent of X_2 and

$$X_i \sim NB(K_i, p), \text{ with } i = 1, 2.$$

   Prove that $X_1 + X_2 \sim NB(K_1 + K_2, p)$.

   Hint: Compute the moment-generating function of $X_1 + X_2$.

4. A factory produces 1000 industrial machines every month. From the past experience, the managers of the factory expect that 30 machines have a failure. An external quality commission check, every month, the quality of the machines produced. To do this, they select and analyze a random sample of 20 machines (without replacement).

(a) What is the probability that the external quality commission finds out 1 machine with failures?

(b) What is the probability that the external quality commission finds out more than 80 machines with failures in 135 months? Assume independence if you think it is necessary.