Models for Nonnegative Outcomes

Continuous Outcomes and Count Data Log-Linear and Exponential Regression Models Poisson and Negative Binomial Models Panel Data Models

9 odels for Nonnegative Outcomes
• Intinuous Outcomes and Count Data
• Nonnegative outcomes can be:
• Continuous: $Y \in [0, +\infty[$
• Examples: prices, wages,... Models for Nonnegative Outcomes Continuous Outcomes and Count Data

- - Continuous: $Y \in [0, +\infty[$
		- Examples: prices, wages,…
	-
- **Properties (CONTE A)**
 Properties (CONTE A)
 Properties: Continuous: $Y \in [0, +\infty[$

 Examples: prices, wages,...

 Discrete (counts): $Y \in \{0, 1, 2, 3, ...\}$

 Examples: patents applied for by a firm in a year, times in a year,...
- Nonnegative outcomes can be:
• Continuous: $Y \in [0, +\infty[$

 Examples: prices, wages,...

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 Examples: patents applied for by a firm in a year, times someone is arrested

in a year, because: ■Continuous: $Y \in [0, +\infty[$

- Continuous: $Y \in [0, +\infty[$

- Examples: prices, wages,...

■Discrete (counts): $Y \in \{0, 1, 2, 3, ...\}$

- Examples: patents applied for by a firm in a year, times someone is arrested

in a year, At least close to the lower bound of Y , it does not make sense to assume containing the lower bound of Y , it does not the most suitable option

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At l - Examples: prices, wages,...
Discrete (counts): $Y \in \{0,1,2,3,...\}$
- Examples: patents applied for by a firm in a year, times someone is a
in a year,...
ear regression models are not the most suitable option
cause:
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$$
ln(Y_i) = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + u_i
$$

- Assumption: $E(u_i|x) = 0$
- $\begin{split} \text{y-1} & \text{index} \ \text{y-2} & \text{and} \ \text{Exponential Regression Models} \ \text{log-linear regression model:} \ \text{by} & \text{log-linear regression model:} \ \end{split}$
 $\begin{split} ln(Y_i) &= \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + u_i \ \text{and} \ \text{d} & \text{d} & \text{d} \text{is} \ \text{by} & \text{if} \ \text{by$ unbounded: $Y \in]0, +\infty[\implies \ln(Y) \in]-\infty, +\infty[$ **og-linear regression model:**
 $ln(Y_i) = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + u_i$

• Assumption: $E(u_i|x) = 0$

• With this transformation, the dependent variable becomes

unbounded: $Y \in]0, +\infty[\Rightarrow ln(Y) \in]-\infty, +\infty[$

• However, two new prob $\ln(\text{Y}_i) = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + u_i$

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However, two new problems arise:

The log-linear mod
- -
- $ln(Y_i) = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + u_i$
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bounded: $Y \in]0, +\infty[\Rightarrow ln(Y) \in]-\infty, +\infty[$
wever, two new problems arise:
The log-linear model is not defined f $ln(Y_i) = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + u_i$

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unbounded: $Y \in]0, +\infty[\Rightarrow ln(Y) \in]-\infty, +\infty[$

However, two new problems arise:

The log-linear model is u_i

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solutions

, and not in the

the latter directly

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of relatively sumption: $E(u_i|x) = 0$
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bounded: $Y \in]0, +\infty[\Rightarrow \ln(Y) \in]-\infty, +\infty[$
wever, two new problems arise:
The log-linear model is not defined for $Y = 0$; adding
value to Y or dropping zer : 0

on, the dependent variable becomes
 $\infty[\Rightarrow \ln(Y) \in]-\infty, +\infty[$

plems arise:

iot defined for $Y = 0$; adding a small constant

eros are not in general good solutions

sesting in the original scale, \hat{Y}_i , and not in sumption: $E(u_i|x) = 0$

th this transformation, the dependent variable becomes

bounded: $Y \in]0, +\infty[\Rightarrow \ln(Y) \in] -\infty, +\infty[$

wever, two new problems arise:

The log-linear model is not defined for $Y = 0$; adding a small const assumptions are dependent variable becomes

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value to Y or dropping zeros th this transformation, the dependent variabounded: $Y \in]0, +\infty[\implies \ln(Y) \in] -\infty$,
wever, two new problems arise:
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value to Y or dropping zeros are not in general go
Predic

Analogy Iodels for Nonnegative Outcomes

Dg-Linear and Exponential Regression Models

Exponential regression model:
 $Y = exp(x'\beta + u)$
 $E(Y|X) = exp(x'\beta)$ Models for Nonnegative Outcomes Log-Linear and Exponential Regression Models

$$
Y = exp(x'\beta + u)
$$

$$
E(Y|X) = exp(x'\beta)
$$

- Assumption: $E(e^u|x) = 1$
- Advantages:
	- \bullet \widehat{Y}_i is always nonnegative
- r Nonnegative Outcomes

and Exponential Regression Models

mtial regression model:
 $Y = exp(x'\beta + u)$
 $E(Y|X) = exp(x'\beta)$

mption: $E(e^u|x) = 1$

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edictions are obtained directly in the original scale

y **Prediction are obtained Models**
 Prediction are obtained directly in the original scale, without requiring
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 $Y = exp(x'\beta + u)$
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sumption: $E(e^u|x) = 1$

vantages:
 \hat{Y}_i is always nonnegative

Predictions are obtained directly in the origin

any retransformations

tial effects:
 $\Delta X_j = 1 \Longrightarrow \Delta E$ $Y = exp(x'\beta + u)$
 $E(Y|X) = exp(x'\beta)$

• Assumption: $E(e^u|x) = 1$

• Advantages:

• \hat{Y}_i is always nonnegative

• Predictions are obtained directly in the origin

any retransformations

• Partial effects:
 $\Delta X_j = 1 \Longrightarrow \Delta E(Y|X) = \beta_j e x$
-

$$
\Delta X_j = 1 \Longrightarrow \Delta E(Y|X) = \beta_j exp(x'\beta)
$$

-
- $E(Y|X) = exp(X^t|X)$

Simplion: $E(e^u|x) = 1$

Advantages:

 \hat{Y}_i is always nonnegative

 Predictions are obtained directly in the original scale, without requiring

any retransformations

Partial effects:
 $\Delta X_j = 1 \Rightarrow \Delta E(Y|$ \bullet β_i can be interpreted as a semi-elasticity, since: mption: $E(e^{ax}|x) = 1$

ntages:

s always nonnegative

edictions are obtained directly in the original scale, without

y retransformations

all effects:
 $\Delta X_j = 1 \Rightarrow \Delta E(Y|X) = \beta_j exp(x'\beta)$

e sign of the effect is given by the sig $\Delta f = 100 \frac{E(V|V)}{E(V|V)}$, i.e. $\Delta \Delta f = 1 \implies 90\Delta E(V|V) = 100P_1$

Models for Nonnegative Outcomes Poisson and Negative Binomial Models

- Monnegative Outcomes
• Assumptions and estimation methods according to the type of
• Assumptions and estimation methods according to the type of
• Continuous response: nonnegative outcome: S for Nonnegative Outcomes

In and Negative Binomial Models

Assumptions and estimation methods according to the

Internative continuous response:

- Assumption: only $E(Y|X)$; estimation: QML

- Count data - two alternativ Solary and Negative Dutcomes

Assumptions and estimation methods according to the type

- Assumption: only $E(Y|X)$; estimation: QML

- Count data - two alternatives:

- Assumption: only $E(Y|X)$; estimation: QML

- Assumpt • Assumptions and estimation methods according to the type of

• Continuous response:

• Continuous response:

– Assumption: only $E(Y|X)$; estimation: QML

• Count data - two alternatives:

– Assumption: only $E(Y|X)$; est
	- -
	- -
		-
- **nonnegative outcome:**

 Continuous response:

 Assumption: only $E(Y|X)$; estimation: QML

 Count data two alternatives:

 Assumption: only $E(Y|X)$; estimation: QML

 Assumption: $E(Y|X)$ and $Pr(Y = j|X)$; estimat

Thr
	- **Poisson**
	- **Negative Binomial 1**
	- **Negative Binomial 2**

Models for Nonnegative Outcomes Poisson and Negative Binomial Models

Tools for Nonnegative Outcomes

\noisson and Negative Binomial Models

\nPoisson regression model:

\n
$$
Y_i \sim Poisson(\lambda_i) \implies Pr(Y_i = y | x_i) = \frac{e^{-\lambda_i} \lambda_i^y}{y!}
$$

where $\lambda_i = E(Y|X) = exp(x' \beta)$

- sson and Negative Binomial Models

oisson regression model:
 $Y_i \sim Poisson(\lambda_i) \Rightarrow Pr(Y_i = y | x_i) = \frac{e^{-\lambda_i} \lambda_i y}{y!}$

where $\lambda_i = E(Y | X) = exp(x' \beta)$

Fistimation methods: ML (only count data) or QML, since the

Poisson distribution belongs t sson regression model:
 $Y_i \sim Poisson(\lambda_i) \Longrightarrow Pr(Y_i = y | x_i) = \frac{e^{-\lambda_i} \lambda_i^y}{y!}$

where $\lambda_i = E(Y | X) = exp(x'\beta)$

Estimation methods: ML (only count data) or QML, since the

Poisson distribution belongs to the linear exponential family

By de
- $Y_i \sim Poisson(\lambda_i) \Rightarrow Pr(Y_i = y | x_i) = \frac{y_i}{y!}$
where $\lambda_i = E(Y | X) = exp(x' \beta)$
Estimation methods: ML (only count data) or QML, since the
Poisson distribution belongs to the linear exponential family
By definition, $E(Y | X) = Var(Y | X)$ (equidispers where $\lambda_i = E(Y|X) = exp(x'\beta)$
Estimation methods: ML (only count data) or QML, since the
Poisson distribution belongs to the linear exponential family
By definition, $E(Y|X) = Var(Y|X)$ (equidispersion), which
may be a strong assumpt

- - NEGBIN1: $Var(Y|X) = (1 + \delta)E(Y|X)$ ML estimation
	- NEGBIN2: $Var(Y|X) = [1 + \delta E(Y|X)]E(Y|X)$ it belongs to the **•** NEGBIN1: $Var(Y|X) = (1 + \delta)E(Y|X)$ - ML estimation

	NEGBIN2: $Var(Y|X) = [1 + \delta E(Y|X)]E(Y|X)$ - it belon

	linear exponential family, enabling estimation by both

	count data) and QML

	NEGBIN1: nbreg $YX_1 ... X_k$, dispersion(constant)

	N NEGBIN2: $Var(Y|X) = [1 + \delta E(Y|X)]E(Y|X)$ - it belongs
linear exponential family, enabling estimation by both M
count data) and QML

NEGBIN1: nbreg $YX_1 ... X_k$, dispersion(constant)
NEGBIN2 (ML): nbreg $YX_1 ... X_k$, dispersion(mean)

linear exponential family, enabling estimation by both ML (only

count data) and QML

NEGBIN1: nbreg Y_{A_1} ... X_k , dispersion(constant)

NEGBIN2 (ML): nbreg Y_{X_1} ... X_k , dispersion(mean)

NEGBIN2 (QML): nbreg $Y_{$ **Stata** NEGBIN1: nbreg $YX_1 \ldots X_k$, dispersion(constant) NEGBIN2 (ML): nbreg $YX_1 \ldots X_k$, dispersion(mean) NEGBIN2 (QML): nbreg $YX_1 ... X_k$, dispersion(mean) robust

Models for Nonnegative Outcomes Panel Data Models

Base model:

$$
E(Y_{it}|x_{it}, \alpha_i) = exp(\gamma_i + x_{it}'\beta) = \alpha_i exp(x_{it}'\beta)
$$

Models for Nonnegative Outcomes		
Base model:	.	Continuous / count data:
$E(Y_{it} x_{it}, \alpha_i) = exp(\gamma_i + x_{it}'\beta) = \alpha_i exp(x_{it}'\beta)$		
. Count data:		
$Pr(Y_{it} = y x_{it}, \alpha_i) = \frac{e^{-\lambda_{it}} \lambda_{it} y}{y!}$		
$\lambda_i = E(Y_{it} x_{it}, \alpha_i) = \alpha_i exp(x_{it}'\beta)$		
Pooled estimator:	.	Based on the cross-sectional assumption $E(Y_{it} x_{it}) = exp(x_{it}'\beta)$
Produces consistent estimators only if $E(\alpha_i x_{it}) = 1$		
Does not require the Poisson distributional assumption		
Using a robust vec controls for both overdispersion and time dependence		
poisson $Y_{X_1} \dots X_k$, $vec(\text{cluster cluster cluster})$		

- Based on the cross-sectional assumption $E(Y_{it}|x_{it}) = exp(x_{it}^{\prime} \beta)$
-
-
- dependence

Models for Nonnegative Outcomes
Panel Data Models
Random Effects Poisson Estimator:
• Assumptions:
• K. $\approx Poisson(\lambda, \lambda)$ Models for Nonnegative Outcomes Panel Data Models

andom Effects Poisson Estimator:

• $Y_{it} \sim Poisson(\lambda_{it})$

• $\lambda_i = E(Y_{it}|x_{it}, \alpha_i) = \alpha_i exp(x_{it}'\beta)$

• $log(\alpha_i) = \gamma_i \sim Gamma(1, \eta)$

• Resulting model:

• NEGBIN2-type model

• Estimation method: ML

- - $Y_{it} \sim Poisson(\lambda_{it})$
	- \bullet $\lambda_i = E(Y_{it} | x_{it}, \alpha_i) = \alpha_i exp(x'_{it} \beta)$
	- $log(\alpha_i) = \gamma_i \sim Gamma(1, \eta)$
- - NEGBIN2-type model
	-

- Assumptions:

 $Y_{it} \sim Poisson(\lambda_{it})$

 $\lambda_i = E(Y_{it}|x_{it}, \alpha_i) = \alpha_i exp(x_{it}'\beta)$

 $log(\alpha_i) = \gamma_i \sim Gamma(1, \eta)$

Resulting model:

 NEGBIN2-type model

 Estimation method: ML

 $E(Y_{it}|x_{it}) = exp(x_{it}'\beta)$, which implies that the Poo

consistent u ■ $E(Y_{it}|x_{it}) = exp(x_{it}'\beta)$, which implies $\exp(x'_{it}\beta)$
 $\pi(1,\eta)$
 $\frac{\text{Stad}}{\text{xtpoisson }YX_1 \dots X_k, \text{re}}$
 \ldots , which implies that the Pooled estimator is

m effects of this type $Y_{it} \sim Poisson(\lambda_{it})$
 $\lambda_i = E(Y_{it}|x_{it}, \alpha_i) = \alpha_i exp(x_{it}'\beta)$
 $log(\alpha_i) = \gamma_i \sim Gamma(1, \eta)$

sulting model:

NEGBIN2-type model

Estimation method: ML
 $E(Y_{it}|x_{it}) = exp(x_{it}'\beta)$, which implies that the Pooled estimator is

consistent under random
- $log(\alpha_i) = \gamma_i \sim Gamma(1, \eta)$

 Resulting model:

 NEGBIN2-type model

 Estimation method: ML

 $E(Y_{it}|x_{it}) = exp(x'_{it}\beta)$, which implies that the Pooled es

consistent under random effects of this type

 Alternative model: assum \sim 2) and produces $(Y_{it}|x_{it}) = exp(x'_{it}\beta)$ but has no close form solution $\begin{array}{l} \text{Stata} \\ \text{xtpoisson } YX_1 \dots X_k \text{, re} \end{array}$ nich implies that the Pooled estimator is
ffects of this type
es $log(\alpha_i) = \gamma_i \sim N(0, \sigma^2)$ and $\begin{array}{l} \text{at} \\ \text{at} \end{array}$ but has no close form solution

Fixed Effects Estimators:

• Fixed Effects Estimators:

• Fixed effects Poisson estimator (three equivalent versions):

• Pooled estimator with individual effects

• Estimator conditional on $\sum_{t=1}^{T} Y_{it}$, with $\sum_{t=1$ Models for Nonnegative Outcomes Panel Data Models

- -
	- $T_{t=1}$ Y_{it} , with $\sum_{t=1}^{T} Y_{it} \neq 0$ $t=1$ \mathbf{i} it \mathbf{t} $\mathbf{0}$
- **Example 15 for Nonnegative Outcomes

I Data Models

d Effects Estimators:

Fixed effects Poisson estimator (three equivalent version

Pooled estimator with individual effects

Pestimator conditional on** $\sum_{t=1}^{T} Y_{it}$ **, Example 15 for Nonnegative Outcomes

I Data Models

d Effects Estimators:

Fixed effects Poisson estimator (three equivalent versic

Pooled estimator with individual effects

Estimator conditional on** $\sum_{t=1}^{T} Y_{it}$ **, wi Example 18 Solution Matel State Models**
 Coast Models
 Coast Externators:
 Pooled estimator with individual effects
 Pooled estimator with individual effects
 Pooled estimator conditional on $\sum_{t=1}^{T} Y_{it}$ **, wi** 1984) ixed Effects Estimators:

• Fixed effects Poisson estimator (three equivalent verefore and the pooled estimator with individual effects

• Estimator conditional on $\sum_{t=1}^{T} Y_{it}$, with $\sum_{t=1}^{T} Y_{it} \neq 0$

• Quasi-mea G EFFECTS ESTIMITIONS.

Fixed effects Poisson estimator (three equivalen

Fooled estimator with individual effects

Fooled estimator conditional on $\sum_{t=1}^{T} Y_{it}$, with $\sum_{t=1}^{T} Y_{it} \neq 0$

Cluasi mean-differenced GMM Fixed effects Poisson estimator (three equivale

• Pooled estimator with individual effects

• Estimator conditional on $\sum_{t=1}^{T} Y_{it}$, with $\sum_{t=1}^{T} Y_{it} \neq 0$

• Quasi mean-differenced GMM estimator (Hausman,

1984) Fultra Estimator conditional on $\sum_{t=1}^{T} Y_{it}$, with $\sum_{t=1}^{T} Y_{it} \neq 0$

Fultra Quasi mean-differenced GMM estimator (Hausman, Hall and Griliches, 1984)
 Cuasi-differences GMM estimator:

Fultra Chamberlain (1992
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	-

Models for Nonnegative Outcomes
Panel Data Models
Fixed effects Poisson estimator:
• May be derived using the three equivalent versions
• Pooled estimator with individual effects:
• Adds individual dummies, associated to **Foolels for Nonnegative Outcomes

ixed effects Poisson estimator:**

• May be derived using the three equivalent versions

• Pooled estimator with individual effects:

• Adds individual dummies, associated to the γ_i 's
 Models for Nonnegative Outcomes Panel Data Models

-
- - Adds individual dummies, associated to the γ_i 's
- **Example 18 for Nonnegative Outcomes

1 Data Models

deffects Poisson estimator:**

May be derived using the three equivalent versions

Pooled estimator with individual effects:

Adds individual dummies, associated to the **Example 15 for Nonnegative Outcomes

1 Data Models

d effects Poisson estimator:**

May be derived using the three equivalent versions

"

"Ooled estimator with individual effects:

"Adds individual dummies, associated to
- incidental parameters problem)

 The quasi mean-difference equivalent versions

 Pooled estimator with individual effects:

 Adds individual dummies, associated to the γ'_s

 As in linear models, β is consistentl ed effects Poisson estimator:

May be derived using the three equivalent versions

Pooled estimator with individual effects:

• Adds individual dummies, associated to the γ_i 's

• As in linear models, β is consistent • As in linear models, β is consistently estimated even in short panels (no
incidental parameters problem)

• The quasi mean-differenced GMM estimator is based on the

following moment condition:
 $E\left(Y_{it} - \frac{\lambda_{it}}{\overline{\lambda$

$$
E\left(Y_{it}-\frac{\lambda_{it}}{\overline{\lambda}_i}\overline{Y}_i\middle|\,x_{it}\right)=0,
$$

where $\lambda_{it} = exp(x_{it}'\beta)$ and $\bar{\lambda}_i = \frac{1}{T}\sum_{t=1}^{T} \lambda_{it}$

Stata xtpoisson $Y X_1 ... X_k$, fe

Models for Nonnegative Outcomes Panel Data Models

Models for Nonnegative Outcomes

\nPauli Data Models

\nQuasi-differences GMM estimator:

\n\n- Chamberlain (1992):
\n- $$
E\left(\frac{\lambda_{i,t-1}}{\lambda_{i,t}}Y_{it} - Y_{i,t-1} \middle| x_{it}\right) = 0
$$
\n- Woodridge (1997):
\n- $$
E\left(\frac{Y_{it}}{\lambda_{it}} - \frac{Y_{i,t-1}}{\lambda_{i,t-1}} \middle| x_{it}\right) = 0
$$
\n

$$
E\left(\frac{Y_{it}}{\lambda_{it}} - \frac{Y_{i,t-1}}{\lambda_{i,t-1}}\middle| \mathcal{X}_{it}\right) = 0
$$

 $E\left(\frac{x_{i,t-1}}{\lambda_{i,t}}Y_{it}-Y_{i,t-1}\Big|\,x_{it}\right)=0$

• Wooldridge (1997):
 $E\left(\frac{Y_{it}}{\lambda_{it}}-\frac{Y_{i,t-1}}{\lambda_{i,t-1}}\Big|\,x_{it}\right)=0$

• In both cases the explanatory variables do not need to be

strictly exogenous, so these estimators are par Wooldridge (1997):
 $E\left(\frac{Y_{it}}{\lambda_{it}} - \frac{Y_{i,t-1}}{\lambda_{i,t-1}}\middle| x_{it}\right) = 0$

In both cases the explanatory variables do not need to be

strictly exogenous, so these estimators are particularly useful

in dynamic models Wooldridge (1997):
 $E\left(\frac{Y_{it}}{\lambda_{it}} - \frac{Y_{i,t-1}}{\lambda_{i,t-1}}\middle| \ x_{it}\right) = 0$

In both cases the explanatory variables do no

strictly exogenous, so these estimators are pain dynamic models