Coordinator: Matthijs Oosterveen

## General Guidelines

- You may use a calculator;
- You may not use a programmable calculator;
- You may not use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up unto 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.


## 1. Production (7 points)

Let $y=x_{1}^{\alpha} x_{2}^{\beta}$ be a production function, where $y$ is the output and $x_{1}$ and $x_{2}$ are the two inputs.
1.1 Discuss under what values of $\alpha$ and $\beta$ this production function is Decreasing, Constant, and Increasing Returns to Scale (DRTS, CRTS, and IRTS).

Assume that $\alpha=\beta=\frac{1}{2}$ for the following three questions.
1.2. Sketch the isoquant for producing the output level $y=y_{0}$ :

$$
\left\{\left(x_{1}, x_{2}\right) \text { in } R_{+}^{2} \mid x_{1}^{1 / 2} x_{2}^{1 / 2}=y_{0}\right\}
$$

1.3. Find the Technical Rate of Substitution (TRS) of the production function.
1.4. Use the formula of the TRS, and/or the figure of the isoquant, to discuss how the TRS depends upon the level of $x_{1}$ and $x_{2}$. What is the economic intuition for this relationship?

## 2. Profit (6 points)

Consider a firm that uses two inputs $x_{1}$ and $x_{2}$. The prices for the two inputs are denoted by $w_{1}$ and $w_{2}$, respectively. Imagine that you observe the following data for the firm across two months:

| Month | $\boldsymbol{w}_{\mathbf{1}}$ | $\boldsymbol{w}_{\mathbf{2}}$ | $\boldsymbol{x}_{\mathbf{1}}$ | $\boldsymbol{x}_{\mathbf{2}}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 4 | 2 | 20 | 40 |
| 2 | 2 | 4 | 25 | 20 |

2.1. Can you test the Weak Axiom of Cost Minimization (WACM) for this firm? If yes, test if WACM holds. If not, why not?
2.2. Briefly explain the Envelope theorem. You may use words and/or equations.

## 3. Costs (7 points)

Consider a firm that produces output $y$ while using two inputs $x_{1}$ and $x_{2}$. The prices for the two inputs are $w_{1}=w_{2}=4$. The production function is given by $y=x_{1}^{1 / 2} x_{2}^{1 / 2}$.
3.1 Find the factor demand functions and the minimum costs to produce $y=y_{0}$.
3.2 Find the Lagrange multiplier for the constrained cost minimization problem above. Give the Lagrange multiplier an economic interpretation.

