2 Duality and Sensitivity Analysis

1. An individual, among many others, strongly invested in real estate-based funds, and finally managed to recover 25 thousand m.u. (monetary units) which he intends to invest during a certain period of time. After the past experience, his goal is to minimize the risk, however he would like to achieve a minimum return of 2 thousand m.u. at the end of the time period. The characteristics of the financial products, which he ponders to include in his portfolio, made him formulate the following LP problem, where x_i represents the amount ($\times 10^3$ m.u.) to be invested on product $i = 1, 2$.

$$\begin{aligned} \min z &= x_1 + 2x_2 \\ \text{s.t. } & x_1 + x_2 \leq 25 \quad \text{(budget constraint)} \\ & 0.5x_1 + 0.8x_2 \geq 2 \quad \text{(return constraint)} \\ & x_1, x_2 \geq 0 \end{aligned}$$

   (a) Solve graphically the given problem. Present and interpret the optimum value of the decision and slack (auxiliary) variables.

   (b) Write the dual and determine its optimal solution (only the decision variables). Note that you can take advantage of the above solution and resolution

2. In factory Choco, three new types of chocolate bars are going to be made for the food industry. Each bar is made of sugar and chocolate only. The table below shows the quantity of sugar and chocolate (in kg) and the profit (in monetary units) of each chocolate bar.

Bar	Sugar (kg/bar)	Chocolate (kg/bar)	Profit (m.u./bar)
Type 1	1	2	3
Type 2	1	3	7
Type 3	1	1	5

The factory has 50 kg of sugar and 100 kg of chocolate available. To formulate the problem, we define variables x_j, representing the number of chocolate bars Type j to make, where $j = 1, 2, 3$. Answer to the following questions using, when needed, the Solver/Excel to find the solution of the LP problems.

   (a) For which unit profit values of chocolate bars of Type 2 does the current solution remains optimal? Which will be the optimal solution in case the unit profit is 13 m.u.?

   (b) Is it worth considering an increase in the availability of sugar?

   (c) For which amount of sugar is the set of basic variables in the optimal solution the same?

   (d) Is it worth considering an increase in the availability of chocolate?

   (e) For which amount of chocolate is the set of basic variables in the optimal solution the same?

(f) If the amount of sugar available was of 60 kg, which would be the total profit of these products? Which should be the production plan that Choco should apply in these conditions?

(g) Repeat the previous question for an availability of sugar of 40 kg and 30 kg.

3. A humanitarian organization intends to plan a program for medicines distribution in two regions located in the Great Lakes area of Africa. For strategic and security purposes it is possible to use 3 airports from which, by land routes, the supply of the two regions will take place. Considering that the transportation cost of medicaments to the airports should be minimized, insuring, in each of the two regions, a minimum number of people is contemplated by the program, the following LP model has been formulated:

$$\begin{aligned} \min z &= 40x_1 + 18x_2 + 30x_3 \quad \text{(in m.u.)} \\ \text{s.t. } & 4x_1 + x_2 + x_3 \geq 250 \quad \text{(thousands of people)} \\ & 4x_1 + 3x_2 + 6x_3 \geq 350 \quad \text{(thousands of people)} \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

where x_j represents the tons of medicaments to be shipped to airport $j = 1, 2, 3$.

(a) Obtain the optimal solution for the problem using the Solver/Excel.

(b) Write a short report presenting the problem's solution, referring the value of the dual decision variables as well as its meaning.

(c) Consider that the number of thousands of people to be contemplated in the first region is 350 instead of 250, which will be the new cost for the program?

(d) Determine the changes in the solution if airport 1 cannot receive more than 40 ton. of medicaments.

Some solutions

2.1 $x^* = (0, 25)$, $z^* = 50 = w^*, y^* = (2, 0)$.

2.2 $x^* = (0, 25, 25)$, $z^* = 300 = w^*, y^* = (4, 1)$.

2.3 $x^* = (57.5; 0; 20)$, $z^* = 2900 = w^*, y^* = (6, 4)$.