Lisbon School of Economics & Management
Universidade de Lisboa

Undergraduate Degree in Finance
Subject: Financial Markets
Date: 22/05/2025
Time to complete the exam: 2:15 hours

I (7,5/20)

Consider an investor who bought a share of a company listed in a stock exchange, at a price of 200€.

1. Assuming a discount rate of 5% and a dividend of 8€, what is the dividend growth rate that provides the stock price mentioned, according to the Gordon model. **(1,5/20)**
2. Assuming the investor had 10000€ available to invest in shares of this company at the initial price mentioned (200€), how many shares could the investor buy if borrowing was allowed, with a margin of 20%. **(1,5/20)**
3. Assuming that the stock price is in equilibrium according to the CAPM, what would be the equation of the Security Market Line (SML) if:
   - the expected market return = 25%
   - the risk-free rate = 3% **(1,5/20)**
4. Compute and interpret the β of these shares, if:
   - the standard-deviation of the market returns = 30%
   - the covariance between the stock and the market returns = 0,18 **(1,5/20)**
5. Compute and interpret the expected return of the stock over the SML, using the information provided and the value obtained for β in the previous question (in case you haven't solved the previous question, please assume a β = 1,5).

II (5,0/20)

A portfolio manager is intending to build an efficient portfolio based on 2 different stocks, A and B, characterized by the following parameters:

			Covariance	
	$E[R_i]$	σ_i	A	B
A	0.15	0.15		
B	0.25	0.30	+0.04	
Risk Free	0.03			

1) What is the expected return and the standard deviation of a portfolio composed by 35% of Stocks A and 65% of Stocks B? **(2.0/20)**
2) What is the most efficient portfolio exclusively composed by stocks A and B, assuming that short sales and borrowing and lending at the risk free rate are allowed, with the risk-free rate = 3%? **(3,0/20)**

III (7,5/20)

Considering a Treasury Bond with a price equal to 980, a Principal of 1000€, coupon rate of 1,8% and maturity of 2 years, along with the following information about the average (annualized) interest rates in the Euro money market for different maturities on the last 12th May:

Maturities (months)	Interest Rates (%)
1	2,114
3	2,124
6	2,111
9	2,068
12	2,024

1. Compute the yield-to-maturity of the bond and interpret how and why does it compare to the coupon rate. (2,5/20)
2. Compute and interpret the Duration and the Modified Duration of the bond (in case you haven't computed the yield in the previous question, assume a value of 2%). (2,5/20)
3. Compute the forward interest rate for a time-to-settlement of 6 months and a maturity of 3 months, explaining its financial meaning and also interpreting it according to the explanatory theories of the term structure of interest rates (2,5/20)

Formulas

Gordon Model: $P_0 = \frac{D_1}{k-g}$

Margins:

Long Purchase: $Margin = \frac{Market\ Value\ of\ Assets - Amount\ borrowed}{Market\ Value\ of\ Assets}$

Short Sales: $Margin = \frac{Market\ Value\ of\ Assets - Market\ Value\ of\ Securities\ sold\ short}{Market\ Value\ of\ Securities\ sold\ short}$

CAPM:

Security market line: $\overline{R}_i = R_F + \beta_i(\overline{R}_M - R_F)$

Beta: $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$

Portfolio of 2 assets:

Expected return: $\overline{R}_P = E(R_P) = E\left(\sum_{i=1}^{2} X_i R_i\right)$

Variance of the return: $\sigma_P^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_{12}$

System of equations to compute the portfolio allocation when short sales and lending and borrowing at the risk-free rate are allowed:

$$
\begin{aligned}
\overline{R}_1 - R_F &= Z_1\sigma_1^2 + Z_2\sigma_{12} + Z_3\sigma_{13} + \cdots + Z_N\sigma_{1N} \\
\overline{R}_2 - R_F &= Z_1\sigma_{12} + Z_2\sigma_2^2 + Z_3\sigma_{23} + \cdots + Z_N\sigma_{2N} \\
\overline{R}_3 - R_F &= Z_1\sigma_{13} + Z_2\sigma_{23} + Z_3\sigma_3^2 + \cdots + Z_N\sigma_{3N} \\
&\vdots \\
\overline{R}_N - R_F &= Z_1\sigma_{1N} + Z_2\sigma_{2N} + Z_3\sigma_{3N} + \cdots + Z_N\sigma_N^2
\end{aligned}
$$

Bonds:

Price (P): $P = \sum_{n=1}^{N} \frac{C_n}{(1+y)^n} + \frac{M}{(1+y)^N}$ ⇔(for a 2-year bond) $\mathrm{P}y^2 + (2\mathrm{P} - \mathrm{C}_1)y + (\mathrm{P} - \mathrm{C}_1 - \mathrm{C}_2 - \mathrm{M}) = 0$

Solution of a 2nd order equation $ax^2 + bx + c = 0$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\mathrm{a} = \mathrm{P}$
$\mathrm{b} = (2\mathrm{P} - \mathrm{C}_1)$
$\mathrm{c} = (\mathrm{P} - \mathrm{C}_1 - \mathrm{C}_2 - \mathrm{M})$

being C_n the coupons to be paid in the *n*-th periods, *N* the residual maturity *M* the redemption or par value and *y* the yield-to-maturity.

Duration: $D = \frac{\sum_{n=1}^{T} n \cdot ce^{-yn} + T \cdot FVe^{-yT}}{P} = 1 \cdot \frac{ce^{-y}}{P} + 2 \cdot \frac{ce^{-2y}}{P} + 3 \cdot \frac{ce^{-3y}}{P} + \cdots + T \cdot \frac{ce^{-yT}}{P} + T \cdot \frac{FVe^{-yT}}{P}$

$D = -\frac{1}{P} \cdot \frac{\Delta P}{\Delta y}$

Modified Duration: $\frac{1}{(1+y)} D = MD$

Forward rate: ${}_m f_n = \left[\frac{(1+s_{m+n})^{m+n}}{(1+s_m)^m}\right]^{\frac{1}{n}}$-1 , with *s* representing the spot rate for a given maturity.