Lecture 3: Price Setting and Information Frictions

Cole, Chapter 10

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Our Model Meets Mr. Keynes

- Many people believe that increases in money
  - increases nominal demand
  - and this in turn increases output and employment
- This is a bit surprising since
  - if money doubles and prices all double then nothing real has changed
  - Money in the long-run seems neutral for this reason.
- Many argue that some element of "surprise" is important here.

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- There are several main types of evidence here:
- There is historical evidence like the long-depression in the UK after WWI and the rebound when inflation returned after the Great Depression.
- There is evidence from vector-auto-regressions (VARs):
- This is a standard sort of VAR equation

$$y_t = A + By_{t-1} + C(m_t - E_{t-1}m_t) + Dx_t + \epsilon_t$$

  where y_t endogenous non-policy, m_t policy, and x_t exogenous variables. We need to come up with some assumptions to determine money growth expectations and then estimate their effect.
  - Many ways to do this, and at least some suggest reasonably sizable and moderately persistent effects.
- The other is to directly date "surprise" changes in central bank policy and see what happened.
- This evidence led to the development of New Keynesian models.
- The simplest of these directly impose pricing-setting rigidities. So, that is what we are going to do.

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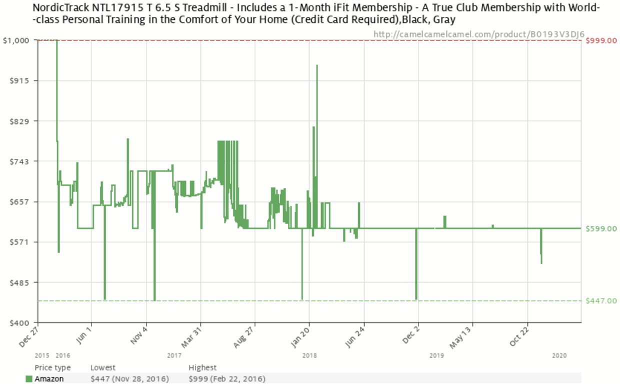
Before we jump off this cliff, some skeptical comments:

- Much of retail is on online and changing prices in response to demand seems much easier.
- Wages are more sticky, but employer/employee relationship complicated.
- Price data from Amazon



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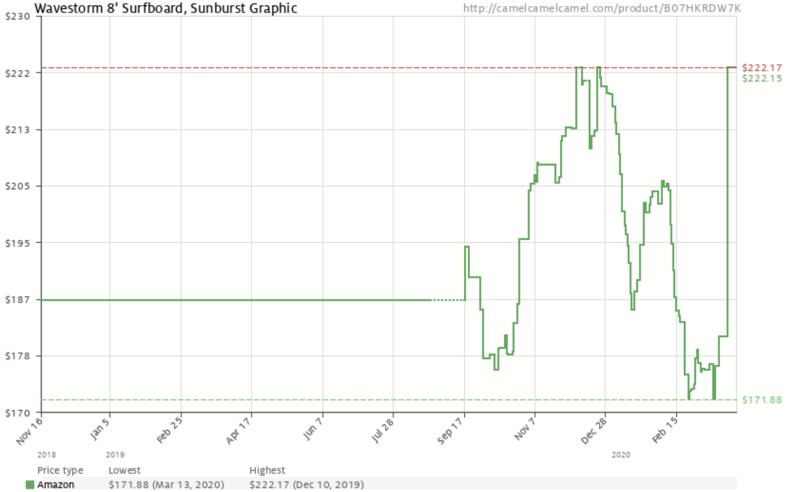
- Price data from Amazon



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Timing in each period

- Household i starts period with M and picks $P_t(i)$
- Seller and buyer split up and go to their respective markets
- The buyer spends M and seller works L_t to meet demand
- The seller and buyer come together in the asset market
- They jointly consume the consumption good
- The period ends

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This timing does several things:

- Allows money injections to affect nominal demand
- If HH does not know τ_t at the time it chooses $P_t(i)$ then money shocks can lead to surprise demand shocks
- This can in turn increase labor and output

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Step 1: Putting in price-setting

- We are going back to our original set-up with multiple goods indexed by $i \in I$, and produced by an individual producer.
- This producer chooses the price $P(i)$ at which she wants to sell her good i.
- So now they choose their price and meeting demand.
- We bring back our consumption aggregator over the different goods

$$C = \left\{ \frac{1}{\# I} \sum_{i \in I} C(i)^{\rho} \right\}^{1/\rho}$$

- The household's preferences are again given by $u(C)$.
- Given all of the prices, determine the demand for good i, $C(i)$.
- Then, production is $C(i)$, labor is $C(i)/Z$, and revenue is $P(i)C(i)$.

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Step 2: Deriving our Demand Function

- Start with a household's optimal expenditure problem: spending M to maximize C.

$$\mathbb{L} = \max_{C(i)} \min_{\lambda} \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{1/\rho} + \lambda \left\{ M - \sum_{i} P(i) C(i) \right\}.$$

F.O.C. is

$$\left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{1/\rho - 1} \frac{1}{\#I} C(i)^{\rho - 1} = \lambda P(i)$$

$$\Longrightarrow C^{1-\rho} \frac{1}{\#I} C(i)^{\rho - 1} = \lambda P(i).$$

Also,

$$\frac{\partial \mathbb{L}}{\partial M} = \lambda$$

so λ is value of money in terms of composite consumption

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- 1 unit of money will get me λ units of C
- A price index is a way of saying what something costs.
- A price index $\bar{P}$ for composite consumption says that C units of composite cost $\bar{P} \times C$ units of money.
- $M = \bar{P}C$ implies $dC = \frac{1}{\bar{P}} dM$
- This together with $\frac{\partial \mathbb{L}}{\partial M} = \lambda$ implies $\bar{P} = \lambda^{-1}$
- Inserting $\bar{P}^{-1}$ for λ in our first-order condition, we get that

$$\frac{1}{\#I} C(i)^{\rho-1} = \frac{P(i)}{\bar{P}} C^{\rho-1},$$

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- so

$$C(i) = \left[\#I \frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}} C \equiv D\left(\frac{P(i)}{\bar{P}}\right)$$

  which gives us a nice form for the demand function
- From this we can see the slope of the demand function is

$$\begin{aligned}
\frac{d}{dP(i)} D\left(\frac{P(i)}{\bar{P}}\right) &= C\frac{1}{\rho-1}\left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}-1}\left[\#I\frac{1}{\bar{P}}\right] \\
&= C\left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}-1}\left[\#I\frac{1}{\bar{P}}\right]\frac{1}{\rho-1}\frac{P(i)}{P(i)} \\
&= C\left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}-1}\left[\#I\frac{P(i)}{\bar{P}}\right]\frac{1}{\rho-1}\frac{1}{P(i)} \\
&= D\left(\frac{P(i)}{\bar{P}}\right)\frac{1}{\rho-1}P(i)^{-1}.
\end{aligned}$$

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- so

$$\frac{d}{dP(i)} D\left(\frac{P(i)}{\bar{P}}\right) = D\left(\frac{P(i)}{\bar{P}}\right) \frac{1}{\rho - 1} P(i)^{-1}.$$

- And it's elasticity is

$$\frac{dD\left(\frac{P(i)}{\bar{P}}\right)}{dP(i)} \frac{P(i)}{D\left(\frac{P(i)}{\bar{P}}\right)} = \frac{1}{\rho - 1}$$

- But what is $\bar{P}$?
- For everything to add up,

$$M = \bar{P}C = \sum_{i\in I} P(i)C(i) = \sum_{i\in I} P(i)\left[\#I\lambda P(i)\right]^{1/(\rho-1)} C$$

$$\implies \bar{P} = \sum_{i\in I} P(i)\left[\#I\frac{P(i)}{\bar{P}}\right]^{1/(\rho-1)} = \sum_{i\in I}\left[\#I\frac{P(i)P(i)^{\rho-1}}{\bar{P}}\right]^{1/(\rho-1}$$

$$= \sum_{i\in I}\left[\#I\frac{P(i)^{\rho}}{\bar{P}}\right]^{1/(\rho-1)} = \left[\frac{\#I}{\bar{P}}\right]^{1/(\rho-1)} \sum_{i\in I} P(i)^{\frac{\rho}{\rho-1}}$$

$$\implies \bar{P}^{\frac{\rho}{\rho-1}} = \left[(\#I)\right]^{1/(\rho-1)} \sum_{i} P(i)^{\frac{\rho}{\rho-1}}$$

$$\iff \bar{P} = \left\{\left[(\#I)\right]^{1/(\rho-1)} \sum_{i} P(i)^{\frac{\rho}{\rho-1}}\right\}^{\frac{\rho-1}{\rho}}$$

- So, now we know what $\bar{P}$ is too.

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- There's a units problem here since

$$C = \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{1/\rho}$$

implies that one unit of each good i leads to 1 unit of the composite good.

- So in total it takes $\#I$ units of the different goods to get the composite.
- Hence, when all prices $P(i) = P$, $\bar{P} = \#I \times P$. This is something we saw before and just need to adjust Z

$$PZ = \bar{P}Z/\#I.$$

- I'm going to ignore this small distinction going forward because it won't matter for anything.
- This could be simplified by assuming a continuum of firms in the interval $[0, 1]$.
- In this case $\bar{P} = \int_0^1 P(i)di = P$ if $P(i) = P$.

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Step 3: The HH's 2-period problem

$$\max_{\{C_t, P_t(i), M_{t+1}, B_{t+1}\}_{t=1,2}} u(C_1) - v\left(D\left(\frac{P_1(i)}{\bar{P}_1}\right)/Z_1\right)$$
$$+\beta\left[u(C_2) - v\left(D\left(\frac{P_2(i)}{\bar{P}_2}\right)/Z_2\right)\right]$$
$$+\beta^2 V(M_3, B_3) \text{ subject to}$$

$$\begin{aligned} M_t &\geq \bar{P}_t C_t \text{ and} \\ P_t(i) D\left(\frac{P_t(i)}{\bar{P}_t}\right) + [M_t - \bar{P}_t C_t] + B_t + T_t &\geq M_{t+1} + q_t B_{t+1} \end{aligned}$$

imposing the condition that the household must work enough to satisfy demand for its product given the price it sets and the overall price index $\bar{P}$.

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- Three of our first-order conditions are completely unchanged:

$$
\begin{aligned}
&\beta^{t-1} u'(C_t) - \bar{P}_t[\lambda_t + \mu_t] = 0 \\
&\implies \frac{\beta^{t-1}}{Z_t L_t} = \bar{P}_t[\lambda_t + \mu_t] \text{ in equilibrium with log preferences} \\
&\implies \beta^{t-1} = \bar{M}_t[\lambda_t + \mu_t] \text{ because the cia says } \bar{P}_t C_t = \bar{M}_t = \bar{P}_t Z_t L_t
\end{aligned}
$$

where $\bar{M}_t$ is whatever amount of money they have in the goods market.

$$
\begin{aligned}
&-\mu_t + \{\mu_{t+1} + \lambda_{t+1}\} = 0 \\
&\implies \mu_t = \beta^t / \bar{M}_{t+1}
\end{aligned}
$$

$$
-q_t \mu_t + \mu_{t+1} = 0
$$

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- The first-order condition for pricing is then given by

$$
\begin{aligned}
0 \;&=\; -\beta^{t-1} v'\left(D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t\right)\frac{1}{Z_t}\frac{d}{dP_t(i)}D\left(\frac{P_t(i)}{\bar{P}_t}\right) \\
&\quad +\mu_t\left[D\left(\frac{P_t(i)}{\bar{P}_t}\right)+P_t(i)\frac{d}{dP_t(i)}D\left(\frac{P_t(i)}{\bar{P}_t}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
0 \;&=\; -\beta^{t-1} v'\left(D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t\right)\frac{1}{Z_t}D\left(\frac{P_t(i)}{\bar{P}_t}\right)\frac{1}{\rho-1}P_t(i)^{-1} \\
&\quad +\mu_t\left[D\left(\frac{P_t(i)}{\bar{P}_t}\right)+D\left(\frac{P_t(i)}{\bar{P}_t}\right)\frac{1}{\rho-1}\right]
\end{aligned}
$$

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- Divide by $D\left(\frac{P_t(i)}{\bar{P}_t}\right)\frac{1}{\rho-1}$

$$0 = \beta^{t-1} v'\left(D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t\right)\frac{1}{Z_t}P_t(i)^{-1} - \rho\mu_t$$

- So, finally, we get

$$\beta^{t-1} v'\left(D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t\right)\frac{1}{\rho} = Z_t P_t(i)\mu_t.$$

- This condition replaces our f.o.c. for labor in our original model.
- At the same time, our f.o.c.'s for (composite) consumption, money and bonds are essentially unchanged.

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- Step 4: Reconstructing Our Fundamental Equation
- In equilibrium every household will have the same pricing rule, so $P_t(i) = P_t(i') = \bar{P}_t$.

Also $D\left(\frac{P_t(i)}{\bar{P}_t} = 1\right) / Z_t = L_t$,

- The cia constraint will be assumed to bind, so

$$\bar{P}_t = \bar{M}_t / Z_t L_t$$

- As a result, we can boil things down to a system of two equations, just as we did in the basic stochastic model.
- Things are so simple we can just substitute out for our multipliers. (Just like we did the first time through everything.)

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- Our labor equation

$$\beta^{t-1} v' \left(D \left(\frac{P_t(i)}{\bar{P}_t} \right) / Z_t \right) \frac{1}{\rho} = Z_t P_t(i) \mu_t .$$

Becomes

$$\beta^{t-1} v' \left(L_t \right) \frac{1}{\rho} = Z_t \bar{P}_t \{ \beta^t / \bar{M}_{t+1} \}.$$

- Since $\bar{P}_t = \bar{M}_t / Z_t L_t$ get our modified labor supply condition:

$$\frac{1}{\rho} v'(L(s_t)) L(s_t) = \frac{\beta}{(1 + \tau_t(s^t))}.$$

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- But wait a second, the labor equation implies that

$$L(s_t) = \left[\frac{\rho\beta}{(1+\tau_t(s^t))}\right]^{1/(1+\gamma)}$$

- This is a trivial variation on what we had before. As $\rho < 1$ it follows that in the steady state labor supply will be lower with price setting
- Money growth *lowers* labor and output. Thus our current model is *missing* something important
- The price level is given by $\bar{P}_t = \bar{M}_t / Z_t L_t$.
- So the change in prices

$$\frac{\bar{P}_{t+1}}{\bar{P}_t} = \frac{(1+\tau_t)/Z_{t+1}L_{t+1}}{1/Z_t L_t}$$

and the loss from rising prices coming from $1+\tau_t$ is unchanged.

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OK, Take II:

- A key problem is that money growth is coming after the goods market clears and hence generates no increase in current demand.
- Another key problem is that the time t money injection scales up P_{t+1} and hence lowers the return you earn from holding money between today and tomorrow.
- Now assume that the transfer occurs at the beginning of the period and before the shopper gets to the good market

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$$
\begin{aligned}
&\max_{\{C_t, P_t(i), M_{t+1}, B_{t+1}\}_{t=1,2}} u(C_1) - v\left(D\left(\frac{P_1(i)}{\bar{P}_1}\right)/Z_1\right) \\
&+\beta\left[u(C_2) - v\left(D\left(\frac{P_2(i)}{\bar{P}_2}\right)/Z_2\right)\right] \\
&+\beta^2 V(M_3, B_3) \text{ subject to}
\end{aligned}
$$

$$
\begin{aligned}
M_t + T_t &\geq \bar{P}_t C_t \text{ and} \\
P_t(i) D\left(\frac{P_t(i)}{\bar{P}_t}\right) + [M_t + T_t - \bar{P}_t C_t] + B_t &\geq M_{t+1} + q_t B_{t+1}.
\end{aligned}
$$

- This leaves *all* our f.o.c.'s unchanged from the first take.

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- Prices will be the equal again, and the cia will bind. Hence

$$\bar{P}_t = \frac{\bar{M}_t(1+\tau_t)}{Z_t L_t}.$$

  (Warning this seems like a very bad equation for us because we want sticky prices.)

- If we take into account that $D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t = L_t$ and make this change to the labor first-order condition, $\beta^{t-1} v'\left(D\left(\frac{P_t(i)}{\bar{P}_t}\right)/Z_t\right)\frac{1}{\rho} = Z_t P_t(i)\mu_t$, we get that

$$\beta^{t-1} v'\left(L_t\right)\frac{1}{\rho} = Z_t P_t(i)\mu_t.$$

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- If we use our first-order conditions for money and consumption, this equation becomes

$$\begin{aligned}\beta^{t-1} v'(L_t)\frac{1}{\rho} &= Z_t P_t(i)\left[\mu_{t+1}+\lambda_{t+1}\right] \\ &= \frac{\bar{P}_t}{\bar{P}_{t+1}} Z_t \beta^t u'(C_{t+1}) = \frac{\bar{P}_t}{\bar{P}_{t+1}} Z_t \beta^t \frac{1}{Z_{t+1} L_{t+1}}\end{aligned}$$

  which is just as before (given that everyone sets the same price $\bar{P}$) and (using $\beta^{t-1} u'(C_t) = \bar{P}_t[\lambda_t + \mu_t]$ and $Z_{t+1} L_{t+1} = C_{t+1}$)
- However, when we substitute in for prices using our new condition, $\bar{P}_t = \frac{\bar{M}_t(1+\tau_t)}{Z_t L_t}$, we get

$$v'(L_t)\frac{1}{\rho} = \frac{\frac{\bar{M}_{t+1}}{Z_t L_t}}{\frac{\bar{M}_{t+1}(1+\tau_{t+1})}{Z_{t+1} L_{t+1}}} Z_t \beta \frac{1}{Z_{t+1} L_{t+1}} = \frac{1}{1+\tau_{t+1}} \frac{Z_{t+1} L_{t+1}}{L_t} \beta \frac{1}{Z_{t+1} L_{t+1}}$$

- Even if we bring back uncertainty, we are going to get $L_t^{1+\gamma} = \rho\beta\mathbb{E}\left\{\frac{1}{1+\tau_{t+1}}\right\}$

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- **OK, Take III:**
- The only difference is that it is the monetary injection tomorrow that affects the present
- The time t injection now scales up P_t and P_{t+1} by an equal amount and hence has no effect on the return you earn from holding money between today and tomorrow. Also no demand increase today.
- Now assume that the transfer occurs after they set the price and right before the shopper gets to the good market.
- Assume they do not know the transfer at the time they set the price.
- So they will forecast the anticipated transfer and factor that into setting their price. Assume no uncertainty in Z_t
- The level of demand will now depend explicitly on the realized money supply shock, and to emphasize this, we will write

$$D\left(\frac{P_t(i)}{\bar{P}_t}, \tau_t\right).$$

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- If the forecasted money supply injection is $\bar{\tau}_t$, then the forecasted level of demand and hence consumption is given by

$$D\left(1, \bar{\tau}_t\right) = \bar{C}_t = \frac{M_t(1+\bar{\tau}_t)}{\bar{P}_t},$$

  where the last equality follows from the c.i.a. constraint holding as an equality at the forecasted level of the money supply.

The *actual* level of consumption will be given by

$$C_t = \frac{M_t(1+\tau_t)}{\bar{P}_t} = \frac{M_t(1+\tau_t)}{M_t(1+\bar{\tau}_t)/\bar{C}_t} = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)}\bar{C}_t.$$

So consumption today will be higher than forecasted if $\tau_t > \bar{\tau}_t$ and lower if the reverse is true.

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- Since consumption is produced with labor, this implies that forecasted labor $Z_t \bar{L}_t = \bar{C}_t$, while actual labor is $Z_t L_t = C_t$.
- Since
$$C_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)} \bar{C}_t,$$
- Hence, the level of labor input is given by
$$L_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)} \bar{L}_t,$$
and labor will be higher in response to an inflation surprise.
- This last equation allows us to think in terms of a labor effort target $\bar{L}$, and the impact of the deviation in money growth relative to its forecast on actual labor.

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Putting the Pieces Together: Timing of Decisions

- The HH objective is given by

$$max_{P_1(i)}\mathbb{E}_1\left\{\begin{array}{c} max_{C_1,M_1,B_1}u(C_1)-v\left(D\left(\frac{P_1(i)}{\bar{P}_1},\tau_1\right)/Z_1\right) \\ +\beta max_{P_2(i)}\mathbb{E}_2\left\{max_{C_2,M_2,B_2}u(C_2)-v\left(D\left(\frac{P_2(i)}{\bar{P}_2},\tau_2\right)/Z_2\right)\right\}\end{array}\right.$$

- In a period, the HH first choses its price $P_t(i)$ knowing τ_{t-1} and being able to infer $\bar{P}_t$ hence $\mathbb{E}_t$
- Then it finds out τ_t and chooses $\{C_t, M_t, B_t\}$ knowing also q_t
- Then in period $t+1$ it will start over choosing $P_{t+1}(i)$ knowing no more than it did at the end of period t hence $\mathbb{E}_{t+1}$

Putting the Pieces Together: Timing of Decisions

- Focus on the parts that involve P_1 in the Lagrangian and ignore the others to get

$$\mathbb{L} = max_{P_1(i)} \mathbb{E}_1 \left\{ -v\left(D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)/Z_1 \right) + \mu_1(\tau_1) P_1(i) D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \right\}$$

- The first-order condition for the price will be

$$\sum_{\tau_1} \left\{ \begin{array}{c} -v'\left(D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)/Z_1 \right) \frac{1}{Z_1} \frac{\partial}{\partial P_1(i)} D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \\ +\mu_1(\tau_1) D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) + \mu_1(\tau_1) P_1 \frac{\partial}{\partial P_1(i)} D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \end{array} \right\} Pr\{\tau_1\} = 0$$

- But we know this partial and these are the same terms we saw before on slide 17.

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So substitute to get

$$\sum_{\tau_1} \left\{ \begin{array}{c} -v'\left(D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)/Z_1\right)\frac{1}{Z_1}D\left(\frac{P_1(i)}{\bar{P}_1}\right)\frac{1}{\rho-1}P_1(i)^{-1} \\ +\mu_1(\tau_1)D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) + \mu_1(\tau_1)P_1(i)D\left(\frac{P_1(i)}{\bar{P}_1}\right)\frac{1}{\rho-1}P_1(i)^{-1} \end{array} \right\} Pr\{\tau_1\}$$
$$= 0$$

Then, again making use of the fact that (i) $P_t(i) = \bar{P}_t$ since all prices are the same in equilibrium, and (ii) that $D\left(\frac{P_t(i)}{\bar{P}_t}, \tau_t\right) = Z_t L_t$, to get that

$$\sum_{\tau_t} \left\{ -\beta^{t-1} v'(L_t) L_t \frac{1}{\rho-1} \bar{P}_t^{-1} + \mu_t(\tau_t) Z_t L_t + \mu_t(\tau_t) Z_t L_t \frac{1}{\rho-1} \right\} Pr\{\tau_t\}$$
$$= 0$$

$$\sum_{\tau_t} \left\{ -\beta^{t-1} v'(L_t) + \mu_t(\tau_t) \bar{P}_t Z_t \rho \right\} L_t(\tau_t) Pr\{\tau_t\} = 0$$

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Certainty Equivalence

- Assume we have a random variable x and we want to know

$$\mathbb{E}\left\{F(x)\right\} = \sum_x F(x)Pr\{x\}$$

- We can approximate things as follows

$$\begin{aligned}\mathbb{E}\left\{F(x)\right\} &\approx \sum_x \left\{ \begin{array}{c} F(\mathbb{E}\{x\}) + F'(\mathbb{E}\{x\})(x-\mathbb{E}\{x\}) \\ +\frac{1}{2}F''(\mathbb{E}\{x\})(x-\mathbb{E}\{x\})^2 \end{array} \right\} Pr\{x\} \\ &= F(\mathbb{E}\{x\}) + F''(\mathbb{E}\{x\}) \times \frac{\sigma_x^2}{2}\end{aligned}$$

- Linear approximation methods lead to certainty equivalence.
- We are going to fudge in our first-order conditions (simplify) by saying that

$$\mathbb{E}\left\{F(x)\right\} = F(\mathbb{E}\{x\})$$

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$$\sum_{\tau_t}\left\{-\beta^{t-1}v'\left(L_t\right)+\mu_t(\tau_t)\bar{P}_t Z_t\rho\right\}L_t(\tau_t)Pr\{\tau_t\}=0$$

Assume that we use this condition for expected labor, $\bar{L}_t$, and ignore uncertainty.

$$-\beta^{t-1}v'(\bar{L}_t)+\mathbb{E}\{\mu_t\}\bar{P}_t Z_t\rho=0$$

Now, we know $\bar{P}_t = \frac{M_t(1+\bar{\tau}_t)}{Z_t L_t}$ and also

$$\mu_t=\mathbb{E}\{\mu_{t+1}+\lambda_{t+1}\} \implies \mathbb{E}\{\mu_t\}=\mathbb{E}\left\{\beta^t u'\left(C_{t+1}\right)\right\}/\bar{P}_{t+1} \tag{1}$$

so plugging in yields

$$-\beta^{t-1}v'(\bar{L}_t)+\left\{\frac{\beta^t Z_t\bar{P}_t\rho}{Z_{t+1}\bar{L}_{t+1}\bar{P}_{t+1}}\right\}=0$$

$$-v'(\bar{L}_t)\bar{L}_t+\beta\frac{1}{1+\bar{\tau}_{t+1}}\rho=0 \implies \bar{L}_t=\left\{\frac{\beta\rho}{1+\bar{\tau}_{t+1}}\right\}^{1/(1+\gamma)}$$

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$$\bar{L}_t = \left\{ \frac{\beta\rho}{1 + \bar{\tau}_{t+1}} \right\}^{1/(1+\gamma)}$$

$$L_t = \frac{(1 + \tau_t)}{(1 + \bar{\tau}_t)} \bar{L}_t,$$

$$\tau_t = \rho_\tau \tau_{t-1} + B_\tau + \sigma_\tau \varepsilon_{\tau,t}.$$

So, the expectations as of time t are

$$\bar{\tau}_t = \rho_\tau \tau_{t-1} + B_\tau, \quad \bar{\tau}_{t+1} = \rho_\tau(\rho_\tau \tau_{t-1} + B_\tau) + B_\tau$$

Our Model Meets Mr. Keynes

Building up all of our variables:
Determine our forecasts for τ :

$$\bar{\tau}_t = \rho_\tau \tau_{t-1} + B_\tau, \quad \bar{\tau}_{t+1} = \rho_\tau(\rho_\tau \tau_{t-1} + B_\tau) + B_\tau$$

Determine our target labor level

$$\bar{L}_t = \left\{ \frac{\beta\rho}{1 + \bar{\tau}_{t+1}} \right\}^{1/(1+\gamma)}$$

Determine the price index

$$\bar{P}_t = \frac{M_t(1 + \bar{\tau}_t)}{Z_t \bar{L}_t}$$

Determine realized labor

$$L_t = \frac{(1 + \tau_t)}{(1 + \bar{\tau}_t)} \bar{L}_t$$

Our Model Meets Mr. Keynes

Impulse response functions are a nice way to understand the dynamic implications of a model. They are constructed as follows

- The prior value of the random variable is set equal to its mean, so

$$\tau_0 = B_\tau/(1-\rho_\tau)$$

  - In period 1, $\varepsilon_{\tau,1} = 1$, so

$$\tau_1 = B_\tau/(1-\rho_\tau) + C$$

  - After period 1 the shock is set to zero, so

$$\tau_t = B_\tau/(1-\rho_\tau) + \rho_\tau^{t-1} C \text{ for all } t \geq 1$$

We then push these shocks through our model, and labor in particular to trace what happens period $t \geq 1$

$$\bar{L}_t = \left[\frac{\beta\rho}{1+\bar{\tau}_{t+1}}\right]^{1/(1+\gamma)} \qquad L_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)}\bar{L}_t,$$

Our Model Meets Mr. Keynes

When you trace out the impulse response, you will find that the positive effect is 1 period, while the negative effect can last a long time after that.

One reason for this very short-term gain is that we are only assuming that prices are sticky for one period. But in lining our model up with the data, we might want to think carefully about how long a period is.

We can also use our model to see what happens when people are fooled by a change in policy or an announcement. For example the government's policy rule is

$$\tau_t = \rho_\tau \tau_{t-1} + B_\tau + \sigma_\tau \varepsilon_{\tau,t}.$$

But the public thinks that its

$$\tau_t = \rho_\tau \tau_{t-1} + \tilde{B}_\tau + \sigma_\tau \varepsilon_{\tau,t}.$$

So, the expectations as of time t are

$$\bar{\tau}_t = \rho_\tau \tau_{t-1} + \tilde{B}_\tau, \quad \bar{\tau}_{t+1} = \rho_\tau (\rho_\tau \tau_{t-1} + \tilde{B}_\tau) + \tilde{B}_\tau$$

Our Model Meets Mr. Keynes

- The Federal Reserve has sought to become more transparent over time. By this I mean that it ties to more clearly signal what it is going to be doing and why in the coming weeks and months.
- One way to think about modeling this change is to think that the Fed is actually telling households what the upcoming monetary shock is likely to be.
- We can think of the shock ε_t as consisting of a preannounced component, ε_t^P, and an unannounced surprise component, ε_t^U. In this interpretation, the realized shock would be given by

$$\varepsilon_t = \alpha * \varepsilon_t^P + (1 - \alpha) * \varepsilon_t^U,$$

  while α would govern the importance of the preannounced and the unannounced components given that both shocks are standard normals.
- How would the impulse responses differ for these two kinds of shocks?