Master in Mathematical Financial

Interest Rates and Credit Risk Models

Exam – 10 January 2025

1. Assuming that you have the following information available about the 1-year rating transition matrix from the statistics of a recognized rating agency, aggregating all information regarding investment and speculative grade ratings respectively in a rating class *I* and *S* (corresponding *D* to default):

	I	S	D
I	0,85	0,13	0,02
S	0,25	0,65	0,1

1.1. Compute the 2-year probability of default of a company with a rating classification of “I”, considering the potential rating migrations and interpret the result obtained. (2,0)

- see spreadsheet

1.2. What would be the default intensity necessary to get the 1-year default rate for the company and the same moment considered in the previous question, as well as the corresponding expected time to default, assuming that this default intensity follows a Poisson process? (3,0)

- see spreadsheet

1.3. Which additional information would you need to calculate the cumulative probability of default of the company for *t* years assuming that default follows a Cox process and how would you calculate it. (2,5)

Probability of Survival (Probability of no jumps with a time-varying λ)

$\mathbf{P}[N(T)-N(t)=n]=\frac{1}{n!}\left(\int_t^T \lambda(s)ds\right)^n \exp\left\{-\int_t^T \lambda(s)ds\right\}$

- knowing time-varying λ’s is necessary

1.4. Compute the 2-year cumulative probability of default, assuming that the annual hazard rate for a company is 1% in the first year and increases to 3% or 6% in the second year, under two different scenarios, being 0,6 the probability of the former scenario. (2,5)

- see spreadsheet

2. Assuming that the 3- and the 6-month spot rates are 3,4% and 3,5%, respectively, while the forward rate for a 6-month maturity and a time to settlement of 3 months is 3,75%:

2.1. Compute the 9-month spot rate (2,0/20)

- see spreadsheet

2.2. Compute the 1-year spot rate, for a 1-year bond at par, paying quarterly coupons and being its annualized coupon rate equal to 3%, considering also the result obtained in the previous question. (2,0/20)

- see spreadsheet

2.3. Quantify the impact on the price of the bond mentioned in the previous question of an increase in the 1-year interest rate by 1 percentage point, explaining the measure used, both with discretely and continuously compounding interest rates and also how would that impact change with a coupon rate equal to 5%, as well as for a zero-coupon bond. (3,0/20)

- bond at par => yield = coupon rate
- with a higher coupon rate, the weight of the coupons increase => duration is lower.
- for a zero-coupon bond, duration = residual maturity
- see spreadsheet

2.4. Assuming that most shifts in this yield curve are parallel or just involve changes in the slope, while the volatility of interest rates is constant, present an adequate affine model to characterize the behavior and the shape of the curve along time, by specifying the main equations (e.g. the spot, the one-period forward, the volatility and the term premium curves). (3,0/20)

- Vasicek model with 2 factors:

Bond prices:

$$-p_{n,t} = A_n + B_{1,n} z_{1t} + B_{2,n} z_{2t}$$

Yield curve:

$$y_{n,t} = \frac{1}{n}\left(A_n + B_{1,n} z_{1t} + B_{2,n} z_{2t}\right)$$

Factor loadings:

$$A_n = A_{n-1} + \delta + \frac{1}{2}\sum_{i=1}^{2}\left[\lambda_i^2\sigma_i^2 - (\lambda_i\sigma_i + B_{i,n-1}\sigma_i)^2\right]$$

$$B_{i,n} = (1 + B_{i,n-1}\varphi_i)$$