Chapter 10, slide 23

Solutions

November 25, 2025

Consider a consumer with utility function $u = 2\sqrt{x_1} + x_2$ and budget constraint $10 = x_1 + 2x_2$.

1. Calculate the CV for an increase in p_1 from 1 to 2.
2. Calculate the EV for an increase in p_1 from 1 to 2.
3. Why is it that $CV = EV$?
4. Calculate ΔCS for an increase in p_1 from 1 to 2.

Solution

Consider the utility function

$$u(x_1, x_2) = 2\sqrt{x_1} + x_2$$

with general prices p_1, p_2 and income m.

The consumer solves

$$\max_{x_1,x_2} 2\sqrt{x_1} + x_2 \quad \text{s.t.} \quad p_1x_1 + p_2x_2 = m.$$

The Lagrangian is

$$\mathcal{L} = 2\sqrt{x_1} + x_2 - \lambda(p_1x_1 + p_2x_2 - m).$$

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial x_1} : \frac{1}{\sqrt{x_1}} - \lambda p_1 = 0,$$
$$\frac{\partial \mathcal{L}}{\partial x_2} : 1 - \lambda p_2 = 0$$

From the x_2 condition,

$$\lambda = \frac{1}{p_2}.$$

Substituting into the x_1 condition,

$$\frac{1}{\sqrt{x_1}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \sqrt{x_1} = \frac{p_2}{p_1} \quad \Rightarrow \quad x_1(\mathbf{p}, m) = \left(\frac{p_2}{p_1}\right)^2.$$

Plugging x_1 into the budget constraint to find x_2,

$$
\begin{aligned}
p_1\left(\frac{p_2}{p_1}\right)^2 + p_2x_2 = m& \\
x_2(\mathbf{p}, m) &= \frac{m}{p_2} - \frac{p_2}{p_1}.
\end{aligned}
$$

So the Marshallian demand is

$$x_1(\mathbf{p}, m) = \left(\frac{p_2}{p_1}\right)^2, \qquad x_2(\mathbf{p}, m) = \frac{m}{p_2} - \frac{p_2}{p_1}.$$

The indirect utility is

$$
\begin{aligned}
v(\mathbf{p}, , m) &= u\big(x_1(\mathbf{p}, m), x_2(\mathbf{p}, m)\big) \\
&= 2\sqrt{\left(\frac{p_2}{p_1}\right)^2} + \left(\frac{m}{p_2} - \frac{p_2}{p_1}\right) \\
&= \frac{2p_2}{p_1} + \frac{m}{p_2} - \frac{p_2}{p_1} = \frac{m}{p_2} + \frac{p_2}{p_1}.
\end{aligned}
$$

To get the expenditure function, let $e(\mathbf{p}, u)$ be the minimum expenditure needed to reach utility u at prices (p_1, p_2). By definition,

$$u = v\big(\mathbf{p}, e(\mathbf{p}, u)\big) = \frac{e(\mathbf{p}, u)}{p_2} + \frac{p_2}{p_1}.$$

Solve for e:

$$\frac{e(p_1, p_2, u)}{p_2} = u - \frac{p_2}{p_1} \quad \Rightarrow \quad e(p_1, p_2, u) = p_2u - \frac{p_2^2}{p_1}.$$

1. Numerical values for compensating variation (CV)

Take

$$m = 10, \quad p_2 = 2, \quad p_1^0 = 1, \quad p_1^1 = 2.$$

First compute initial utility:

$$v^0 = v(p_1^0, p_2, m) = \frac{10}{2} + \frac{2}{1} = 5 + 2 = 7.$$

The expenditure needed to reach utility v^0 at the new prices (p_1^1, p_2) is

$$
\begin{aligned}
e(p_1^1, p_2, v^0) &= p_2v^0 - \frac{p_2^2}{p_1^1} \\
&= 2 \cdot 7 - \frac{4}{2} = 14 - 2 = 12.
\end{aligned}
$$

The compensating variation is

$$CV = e(p_1^1, p_2, v^0) - m = 12 - 10 = 2.$$

2. Numerical values for equivalent variation (EV)

Utility after the price change at the original income is

$$v^1 = v(p_1^1, p_2, m) = \frac{10}{2} + \frac{2}{2} = 5 + 1 = 6.$$

The expenditure needed to reach v^1 at the old prices (p_1^0, p_2) is

$$\begin{aligned} e(p_1^0, p_2, v^1) &= p_2 v^1 - \frac{p_2^2}{p_1^0} \\ &= 2 \cdot 6 - \frac{4}{1} = 12 - 4 = 8. \end{aligned}$$

The equivalent variation is

$$EV = m - e(p_1^0, p_2, v^1) = 10 - 8 = 2.$$

Hence $CV = EV = 2$.

3. Why $CV = EV$?

The Marshallian demand for good 1 is

$$x_1(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2,$$

which does not depend on income m. So good 1 has no income effect and the Hicksian and Marshallian demand for good 1 coincide. In this case the three money measures of the price change coincide:

$$CV = EV = \Delta CS.$$

4. Change in consumer surplus

The change in consumer surplus for a price increase from p_1^0 to p_1^1 is

$$\Delta CS = \int_{p_1^0}^{p_1^1} x_1(p_1, p_2, m)\, dp_1.$$

Here

$$x_1(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2 \quad \text{and} \quad p_2 = 2,$$

so it follows $x_1(p_1, p_2, m) = \dfrac{4}{p_1^2}$ and

$$\Delta CS = \int_1^2 \frac{4}{p_1^2}\, dp_1 = 4 \int_1^2 p_1^{-2}\, dp_1.$$

Compute the integral:

$$4\int_1^2 p_1^{-2}\,dp_1 = -4\frac{1}{p_1}\Big|_1^2 = -\frac{4}{2} - \left(-\frac{4}{1}\right) = 2.$$

Thus

$$\Delta CS = 2 = CV = EV.$$

ΔCS is the area left of the Marshallian demand. Here, because utility is quasi linear in x_2, there is no income effect and the Hicksian demand for good 1 equals the Marshallian demand. The area to the left of either demand curve between p_1^0 and p_1^1 is the same. Hence the loss in consumer surplus, the compensating variation and the equivalent variation all have the same numerical value.