

Special Period, June 30, 2026 - Duration: 2h00

Justify your answers

1. [1,0 pts] Verify whether the vector $u = (-4, -3, -2)$ can be obtained as a linear combination of the vectors $v = (2, 1, 0)$ and $w = (-1, -1, 1)$.

2. [1,0 pts] Determine for which values of $\alpha \in \mathbb{R}$ the vectors $u = (3, 2, -1)$ and $v = (-6, \alpha, 2)$ are linearly independent.

3. Consider the matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$.

a. [1,0 pts] Show that the matrix A satisfies the equation $A^2 - 2I = A$, where $I \in M_{2 \times 2}(\mathbb{R})$ is the identity matrix.

b. [1,5 pts] Using the result from the previous item, deduce that A is invertible and express A^{-1} .

4. Consider the system of linear equations
$$\begin{cases} x + z = 2 \\ 2x + \alpha y + 3z = 1 \\ y + z = 5 \end{cases}$$

where $\alpha \in \mathbb{R}$ is a real parameter.

a. [1,5 pts] Determine the values of α , if they exist, for which the system has infinitely many solutions.

b. [1,5 pts] Determine the solution set of the system when $\alpha = 0$.

5. Consider the series

$$\sum_{n=2}^{+\infty} \frac{3^{n-1}}{5^{2n-2}}.$$

a. [1,0 pts] Show that the series is convergent.

b. [1,5 pts] Determine the sum of the series, presenting the result as an irreducible fraction.

6. Consider the function $f : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ defined by the expression $f(x) = 2x^2e^{3x-6}$.

a. [1,0 pts] Show that the function f is strictly increasing and briefly justify that it is a bijective function.

b. [1,5 pts] Determine the equation of the tangent line to the graph of the inverse function f^{-1} at the point of coordinates $(8, 2)$.

7. [2,5 pts] Use the Taylor polynomial of order 2 centred at $a = 0$ of the function f defined by the expression $f(x) = \sin(x)e^{2x}$ to show that

$$\sin(0, 1)e^{0,2} \approx 0, 12.$$

8. Consider the antiderivatives

$$F(x) = \int \frac{dx}{\sqrt{x} + \sqrt[6]{x}}$$

a. [1,0 pts] Using the substitution $x = t^6$, show that

$$\int \frac{dx}{\sqrt{x} + \sqrt[6]{x}} = 6 \int \frac{t^4 dt}{t^2 + 1}.$$

b. [1,5 pts] Using the previous item, compute $F(x)$.

9. [2,5 pts] Compute

$$\int_0^1 xe^{x^2}(1 + e^{x^2})dx + \int_0^1 x \arcsin(x)dx.$$