Microeconomics

Recap of Chapter 7: Concepts that we need for Chapter 8

Fall 2026

Chapter 7 in one slide

1. Preferences. Complete, transitive, continuous $\Rightarrow$ a continuous utility function $u(\mathbf{x})$ represents them. Monotonic and convex $\Rightarrow$ SOCs of the maximization problem satisfied.

2. Utility $u(\mathbf{x})$: a convenient mathematical representation.

- Ordinal: any positive monotonic transformation $g(u(\mathbf{x}))$ represents the same preferences
- Indifference curve = level set $\{\mathbf{x} : u(\mathbf{x}) = \bar{u}\}$
- $\mathrm{MRS} = -MU_1/MU_2$ = slope of the indifference curve

3. Constrained optimization:

- Lagrangian $\rightarrow$ FOCs $\rightarrow$ optimality condition $p_1/p_2 = (\partial u/\partial x_1)/(\partial u/\partial x_2)$ (ERS = MRS)
- SOCs: u concave (convex preferences), tangency is a maximum
- Lagrange multiplier λ has an economic meaning

4. Optimal choices: Marshallian demand $\mathbf{x}(\mathbf{p}, m)$ and indirect utility $v(\mathbf{p}, m) = u(\mathbf{x}(\mathbf{p}, m))$ and Hicksian demand $\mathbf{h}(\mathbf{p}, u)$ and expenditure function $e(\mathbf{p}, u)$.

Two ways to describe the consumer's choice

UMP (utility maximization)

$$\max_{\mathbf{x}} \; u(\mathbf{x}) \quad \text{s.t. } \mathbf{px} = m$$

- Marshallian demand: $x_i(\mathbf{p}, m)$
- Indirect utility: $v(\mathbf{p}, m) = u(\mathbf{x}(\mathbf{p}, m))$
- Demand as p_i changes, **income fixed**

EMP (expenditure minimization)

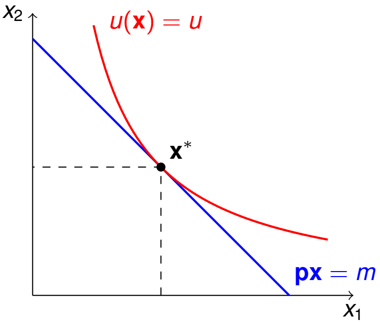
$$\min_{\mathbf{x}} \; \mathbf{px} \quad \text{s.t. } u(\mathbf{x}) = u$$

- Hicksian demand: $h_i(\mathbf{p}, u)$
- Expenditure function: $e(\mathbf{p}, u) = \mathbf{p}\,\mathbf{h}(\mathbf{p}, u)$
- Demand as p_i changes, **utility fixed** ("compensated")

Both problems give the **same optimality condition**:

$$\frac{p_1}{p_2} = \frac{\partial u(\mathbf{x})/\partial x_1}{\partial u(\mathbf{x})/\partial x_2} \qquad \text{(economic rate of substitution = MRS)}$$

Duality: UMP and EMP pick the same bundle



UMP: slide the indifference curve NE to the budget line (fixed).
EMP: slide the budget line SW to the indifference curve (fixed).

Under our assumptions both reach the same $\mathbf{x}^*$, so:

$$(1) \quad x_i(\mathbf{p}, m) = h_i\big(\mathbf{p}, v(\mathbf{p}, m)\big)$$
$$(2) \quad h_i(\mathbf{p}, u) = x_i\big(\mathbf{p}, e(\mathbf{p}, u)\big)$$

Identity (2) is the starting point of the Slutsky equation.

Shephard's lemma

Shephard's lemma (from the EMP):

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_i} = h_i(\mathbf{p}, u)$$

If p_i rises by one euro, the minimum spending needed to reach u rises by the quantity you buy of good i.

Shephard's lemma is needed in the Slutsky equation.

Where Chapter 7 shows up in Chapter 8

Chapter 7 concept	**Used in Chapter 8 for**
Tangency: MRS $= p_1/p_2$	Price offer curve and income expansion path
Marshallian demand $x_i(\mathbf{p}, m)$	Demand curves (p_i varies) and Engel curves (m varies)
Hicksian demand $h_i(\mathbf{p}, u)$	Substitution effect
Duality identity (2)	Deriving the Slutsky equation
Shephard's lemma	Rewriting $\partial e/\partial p_i$ as x_i
Convex preferences	Substitution effect is never positive

Chapter 8 question: what happens to the optimal bundle when $\mathbf{p}$ or m changes?