Lisbon School of Economics & Management
Universidade de Lisboa

# Microeconomics

Fall 2026-2027

Midterm – **SOLUTIONS**

October 2026

**Duration:** 1 hour (60 minutes)

**Total:** 20 points (Part A: 10 points, Part B: 10 points)

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## True / False questions

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Each statement is worth 1 point (10 points in total): 1 point for the correct answer with a correct justification, partial points for the correct answer with an incomplete or partly incorrect justification, and 0 otherwise. An answer without justification does not receive any marks. Each statement is followed by the correct answer and a short justification.

**1.** Convexity of preferences is a necessary condition for indifference curves to be downward sloping.

**False.** The downward slope comes from monotonicity. Convexity only determines the curvature of the indifference curves, not the sign of their slope.

**2.** Monotonicity of preferences means that a consumer strictly prefers bundle $\mathbf{x}$ to bundle $\mathbf{y}$ whenever $\mathbf{x}$ contains more of at least one good, even if it contains less of the other goods.

**False.** Monotonicity requires $\mathbf{x} \geq \mathbf{y}$ (at least as much of *every* good) and $\mathbf{x} \neq \mathbf{y}$ (strictly more of some good). If $\mathbf{x}$ has less of some good, the consumer need not prefer it.

**3.** If the income effect of good 1 is zero, then $CV = EV = \Delta CS$ for a change in the price of good 1.

**True.** With $\partial x_1/\partial m = 0$ the Marshallian demand and the two Hicksian demands coincide, $x_1 = h_1(\cdot, v^0) = h_1(\cdot, v^1)$, so the areas to the left of these curves (CV, EV, $\Delta CS$) are equal.

**4.** The Hicksian demand $h(p, u)$ is equal to the Marshallian demand $x(p, m)$ whenever $u = v(p, m)$, that is, whenever $m = e(p, u)$.

**True.** By duality, the UMP and the EMP select the same bundle when $u = v(p, m)$, equivalently $m = e(p, u)$: $h(p, u) = x(p, m)$.

**5.** Shephard's lemma states that the partial derivative of the indirect utility function with respect to the price $p_i$ is equal to the Marshallian demand for good $i$.

**False.** Shephard's lemma applies to the *expenditure function*: $\partial e(\mathbf{p}, u)/\partial p_i = h_i(\mathbf{p}, u)$, the Hicksian demand. For the indirect utility function, Roy's identity gives $x_i = -\dfrac{\partial v/\partial p_i}{\partial v/\partial m}$.

**6.** A Giffen good must be an inferior good, but an inferior good is not necessarily a Giffen good.

**True.** Both $IE < 0$ Giffen has $IE < SE < 0$ and $TE > 0$. An inferior has $SE < IE < 0$ and $TE < 0$.

**7.** The substitution effect of an own-price increase is non-positive only for normal goods; for inferior goods it can be positive.

**False.** The substitution effect is non-positive for *any* good. Only the income effect depends on whether the good is normal or inferior.

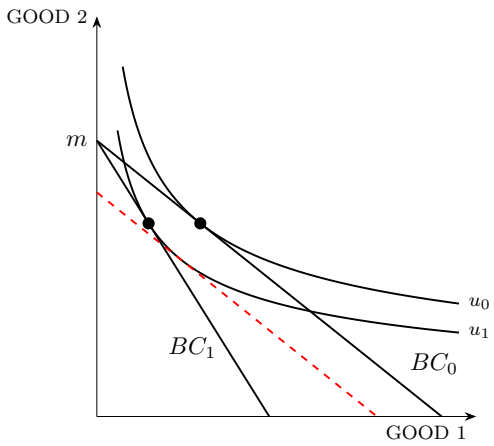
**8.** In the utility maximization problem, the Lagrange multiplier $\lambda$ measures how much maximal utility changes when income $m$ increases marginally.

**True.** $\lambda$ is the marginal utility of income, $\lambda = \partial v(\mathbf{p}, m)/\partial m$. By the envelope theorem, only the direct effect of $m$ on the Lagrangian matters.

**9.** A consumer's utility function satisfies $u(\mathbf{x}) = 4$ and $u(\mathbf{y}) = 2$. Therefore the consumer prefers $\mathbf{x}$ to $\mathbf{y}$, and also gets twice as much satisfaction from $\mathbf{x}$.

**False.** The first part is correct: $u(\mathbf{x}) > u(\mathbf{y})$ implies $\mathbf{x} \succ \mathbf{y}$. The second part is not: utility is ordinal, so only the ranking matters, not the size of the numbers.

**10.** The figure shows the budget line rotating inward from $BC_0$ (old price $p^0$) to $BC_1$ (new, higher price $p^1$) after an increase in the price of good 1, with income $m$ fixed. The dashed line is parallel to $BC_0$ and tangent to $u_1$. The distance between $m$ and the point where the dashed line crosses the Good 2 axis is the compensating variation (CV) of this price change.




**False.** The intercept is $e(\mathbf{p}^0, u_1)$, so the distance to $m$ is $m - e(\mathbf{p}^0, u_1)$, which is the *equivalent variation* (EV). The CV would use the new prices and the old utility level.

## Calculation questions

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Worked solutions (questions 11–20: 1 point per question, 10 points in total). The splits in square brackets are suggested partial-credit allocations.

**11.** Find the partial derivatives $f_x$ and $f_y$ of $f(x,y) = \dfrac{x^2 y}{x+y}$.

Quotient rule:
$f_x = \dfrac{2xy(x+y) - x^2 y}{(x+y)^2} = \dfrac{x^2 y + 2xy^2}{(x+y)^2} = \dfrac{xy(x+2y)}{(x+y)^2}$
$f_y = \dfrac{x^2(x+y) - x^2 y}{(x+y)^2} = \dfrac{x^3}{(x+y)^2}$

**12.** Let $h(x,y) = 14x + 15y - 2x^2 - xy - 3y^2$. Find the critical point by solving the first-order conditions, and use the second-order conditions to classify it.

FOCs: $h_x = 14 - 4x - y = 0$ and $h_y = 15 - x - 6y = 0$.
From the first, $y = 14 - 4x$. Substituting: $15 - x - 6(14 - 4x) = 23x - 69 = 0$, so $x = 3$ and $y = 2$.
Critical point: $(x,y) = (3,2)$, with $h = 36$.

SOCs: $h_{xx} = -4$, $h_{yy} = -6$, $h_{xy} = -1$.
$h_{xx} < 0$ and $h_{xx}h_{yy} - h_{xy}^2 = 24 - 1 = 23 > 0$, so the Hessian is negative definite and the point is a (local) maximum.

**13.** A consumer has utility $u(x_1,x_2) = x_1 x_2^2$, income $m = 90$ and faces prices $p_1 = 3$ and $p_2 = 4$. Find the optimal bundle. Show intermediate steps.

$\mathcal{L} = x_1 x_2^2 - \lambda\,(3x_1 + 4x_2 - 90)$.
FOCs: $\quad x_2^2 = 3\lambda, \qquad 2x_1 x_2 = 4\lambda, \qquad 3x_1 + 4x_2 = 90.$

Dividing the first two FOCs: $\dfrac{x_2^2}{2x_1 x_2} = \dfrac{3}{4} \;\Rightarrow\; x_2 = \dfrac{3}{2}x_1$ .
Budget: $3x_1 + 4 \cdot \frac{3}{2}x_1 = 9x_1 = 90$, so $x_1 = 10$ and $x_2 = 15$.
Optimal bundle: $(x_1,x_2) = (10,15)$.

**14.** A consumer has utility $u(x_1,x_2) = \sqrt{x_1 x_2}$. Write the indifference curve for utility level $\bar{u}$ as $x_2$ as a function of $x_1$.

$\sqrt{x_1 x_2} = \bar{u} \;\Rightarrow\; x_1 x_2 = \bar{u}^2 \;\Rightarrow\; x_2 = \dfrac{\bar{u}^2}{x_1}$.

**15.** A consumer wants to reach utility $\bar{u} = 25$ with $u(x_1,x_2) = x_1 x_2$ at prices $p_1 = 2$ and $p_2 = 8$. Set up the expenditure minimization problem and find the Hicksian demands $h_1$ and $h_2$.

EMP: min $2x_1 + 8x_2$ s.t. $x_1 x_2 = 25$.
Lagrangian: $\mathcal{L} = 2x_1 + 8x_2 - \lambda\,(x_1 x_2 - 25)$.
FOCs: $2 = \lambda x_2, \quad 8 = \lambda x_1, \quad x_1 x_2 = 25.$
Dividing the first two: $\dfrac{2}{8} = \dfrac{x_2}{x_1}$, so $x_2 = \dfrac{x_1}{4}$. Constraint: $x_1 \cdot \dfrac{x_1}{4} = 25 \Rightarrow x_1^2 = 100$.
$h_1 = 10, \quad h_2 = 2.5$.

**16.** A consumer has the indirect utility function $v(p_1, p_2, m) = \dfrac{m^2}{4p_1p_2}$. Find the Lagrange multiplier $\lambda$ of the utility maximization problem at $p_1 = 1$, $p_2 = 4$, $m = 16$.

The multiplier is the marginal utility of income (envelope theorem), so we differentiate $v$ with respect to income:
$\lambda = \dfrac{\partial v}{\partial m} = \dfrac{m}{2p_1p_2} = \dfrac{16}{8} = 2.$

**17.** A consumer has the indirect utility function $v(p_1, p_2, m) = m\, p_1^{-1/4} p_2^{-3/4}$. Use Roy's identity to derive the demand for good 1. Have you found a Marshallian or Hicksian demand?

$\dfrac{\partial v}{\partial m} = p_1^{-1/4} p_2^{-3/4}, \quad \dfrac{\partial v}{\partial p_1} = -\dfrac{1}{4}\, m\, p_1^{-5/4} p_2^{-3/4}.$

Roy's identity: $x_1 = -\dfrac{\partial v/\partial p_1}{\partial v/\partial m} = \dfrac{\frac{1}{4}\, m\, p_1^{-5/4} p_2^{-3/4}}{p_1^{-1/4} p_2^{-3/4}} = \dfrac{m}{4p_1}.$

This is a Marshallian demand: it is a function of prices and income, $x_1(p_1, p_2, m)$, not of a target utility level.

**18.** A consumer has the Marshallian demand $x_1(p_1, p_2, m) = \dfrac{m}{2p_1}$ and the Hicksian demand $h_1(p_1, p_2, u) = u\sqrt{\dfrac{p_2}{p_1}}$. At $p_1 = 1$, $p_2 = 1$, $m = 4$ (where $u = 2$), compute the total effect, the substitution effect and the income effect of a marginal increase in $p_1$, and verify that they satisfy the Slutsky equation.

At the point, $x_1 = \dfrac{4}{2} = 2$ and $\dfrac{\partial x_1}{\partial m} = \dfrac{1}{2p_1} = \dfrac{1}{2}$.

Total effect: $\dfrac{\partial x_1}{\partial p_1} = -\dfrac{m}{2p_1^2} = -2.$

Substitution effect: $\dfrac{\partial h_1}{\partial p_1} = -\dfrac{1}{2}\, u\, p_2^{1/2} p_1^{-3/2} = -\dfrac{1}{2} \cdot 2 = -1.$

Income effect: $x_1 \dfrac{\partial x_1}{\partial m} = -2 \cdot \dfrac{1}{2} = -1.$

Slutsky: $\dfrac{\partial x_1}{\partial p_1} = \dfrac{\partial h_1}{\partial p_1} - x_1 \dfrac{\partial x_1}{\partial m}$, i.e. $-2 = -1 + (-1)$.

**19.** A consumer has utility $u(x_1, x_2) = \sqrt{x_1} + x_2$, income $m$ and prices $p_1$ and $p_2$. Show that $x_1 = \left(\dfrac{p_2}{2p_1}\right)^2$, is the Marshallian demand for good 1.

Marshallian demand for good 1. From FOCs of Lagrangian: $\frac{1}{2\sqrt{x_1}} = \frac{p_1}{p_2} \iff x_1^* = \left(\frac{p_2}{2p_1}\right)^2$. This does not depend on income because utility is quasilinear.

**20.** Consider the same consumer with utility $u(x_1, x_2) = \sqrt{x_1} + x_2$ and income $m = 100$. The price $p_2 = 1$ is fixed and $p_1$ rises from 1 to 4. Find the compensating variation (CV). Some results will be non-integers.

$CV = e(p^1, v^0) - e(p^0, v^0) = e(p^1, v^0) - m$, with $p^0 = (1, 1)$ and $p^1 = (4, 1)$.
Initial utility: $v^0 = v(p^0, m) = \frac{1}{4} + 100 = 100.25$.
Expenditure function (derived from $v(p, m)$ using $e(p, u) = m$ and $v(p, m) = u$: $e(p, u) = p_2 u - \frac{p_2^2}{4p_1}$.
At the new prices: $e(p^1, v^0) = 100.25 - \frac{1}{16} = 100.19$.
Therefore $CV = 100.19 - 100 = \frac{3}{16} = 0.19$.