STATISTICAL LABORATORY

Lecture 8: Index Numbers (Theory and Practice)

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Applied Mathematics for Economics and Management
1st Year / 1st Semester – 2026/2027

Presentation Guidelines

- **Time Allocation:**
  - 5-student groups: 15 mins presentation + 2 mins Q&A
  - 6-student groups: 18 mins presentation + 3 mins Q&A
- **Strict Time Limit:** Presentations **cannot overrun** the allotted time.
- **Presentation Date:** November 24th.
- **Schedule:** A random draw will be conducted one week prior to determine the presentation sequence.
- **Submission:** Send slides to my email by **November 23rd at 8:00 PM**.
- **Grading Weight:** Group Project Grade = 70% Written Report + 30% Presentation.

Presentation Guideline

Criterion	Weight	Description
1. Time Management	20%	Stays strictly within the allotted time window without going over.
2. Group Participation & Cohesion	20%	Balanced speaking time and equal collaboration across all members.
3. Individual Delivery & Skills	15%	Clear voice and presentation with confidence.
4. Content & Quality of Analysis	25%	Accurate, rigorous analysis that addresses core objectives.
5. Visual Aids & Slide Design	20%	Well-structured, legible slides that enhance comprehension.

Program

1. Fundamental Concepts of Statistics
2. Exploratory Data Analysis
3. Organizing and Summarizing Data
4. Association and Relationships Between Variables
5. **Index Numbers**
6. Time Series Analysis

What Are Index Numbers?

Definition

- Index numbers are statistical measures that show changes in a variable or group of related variables over time or across different locations relative to a base value.

Key Points

- Expresses relative change rather than absolute values.
- Commonly used for prices, production, and economic indicators.
- Base period or base value is usually set to 100.

What Are Index Numbers? (Continued)

- Example 1
  - A price index of 120 means that prices have increased by 20% compared to the base period.
- Example 2 (source: A Practical Introduction to Index Numbers (Ralph, O'Neill and Winton 2015)

Table: Value of retail sales 2005–2008 for Great Britain.

	Value of retail sales (£bn)	Value relative to sales in 2005 (2005 = 100)
2005	281.450	100.00
2006	292.110	103.79
2007	303.621	107.88
2008	321.178	114.12

Why Are Index Numbers Important?

- A key application of index numbers is measuring inflation, which plays a crucial role in business and government policy decisions:
  1. Setting central bank monetary policy
  2. Adjusting private pension provisions
  3. Adjustment of amounts paid over long-term contracts
  4. Adjusting public transport fares and regulated prices
  5. Evaluating corporate and financial investment decisions
  6. Adjustment of tax allowances

Simple Index Numbers

Definition & Notation

A simple index number measures the relative change of a variable over time compared to a base period (x_0 = value in the base period, x_t = value in the current period).

1. Ratio Form (without $\times 100$):

- $I_{t|0} = \frac{x_t}{x_0}$
- $I_t = 1 \rightarrow$ no change
- $I_t > 1 \rightarrow$ increase
- $I_t < 1 \rightarrow$ decrease

2. Percentage Form (with $\times 100$):

- $I_{t|0} = \frac{x_t}{x_0} \times 100$
- $I_t = 100 \rightarrow$ no change
- $I_t > 100 \rightarrow$ increase
- $I_t < 100 \rightarrow$ decrease

Example: Base period: $x_0 = 50$, Current period: $x_t = 60$
$\Longrightarrow$ $I_t = \frac{60}{50} \times 100 = 120$ (Prices increased by 20%).

Calculating Simple Index Numbers: Example

Problem:

- The prices of a commodity over 5 years are shown in this table.
- Calculate the simple index numbers using 2019 as the base year, multiplying by 100.

Year	**Price ($)**
2019	50
2020	55
2021	53
2022	58
2023	60

Calculating Simple Index Numbers: Example

Year	Price ($)	Index Number ($I_{t|0}$)
2019	50	$(50/50) \times 100 = 100$ (Base year)
2020	55	$(55/50) \times 100 = 110$ (10% increase)
2021	53	$(53/50) \times 100 = 106$ (6% increase)
2022	58	$(58/50) \times 100 = 116$ (16% increase)
2023	60	$(60/50) \times 100 = 120$ (20% increase)

Conclusion: Prices increased every year relative to 2019, with the largest increase in 2023 (20%).

Usefulness

- Index numbers can be used to make effective comparisons between different series.
- It is often the case that data series have quite different values, which can make comparison difficult. By creating index number versions with a common base period, comparisons can be made more easily.

Example: Population

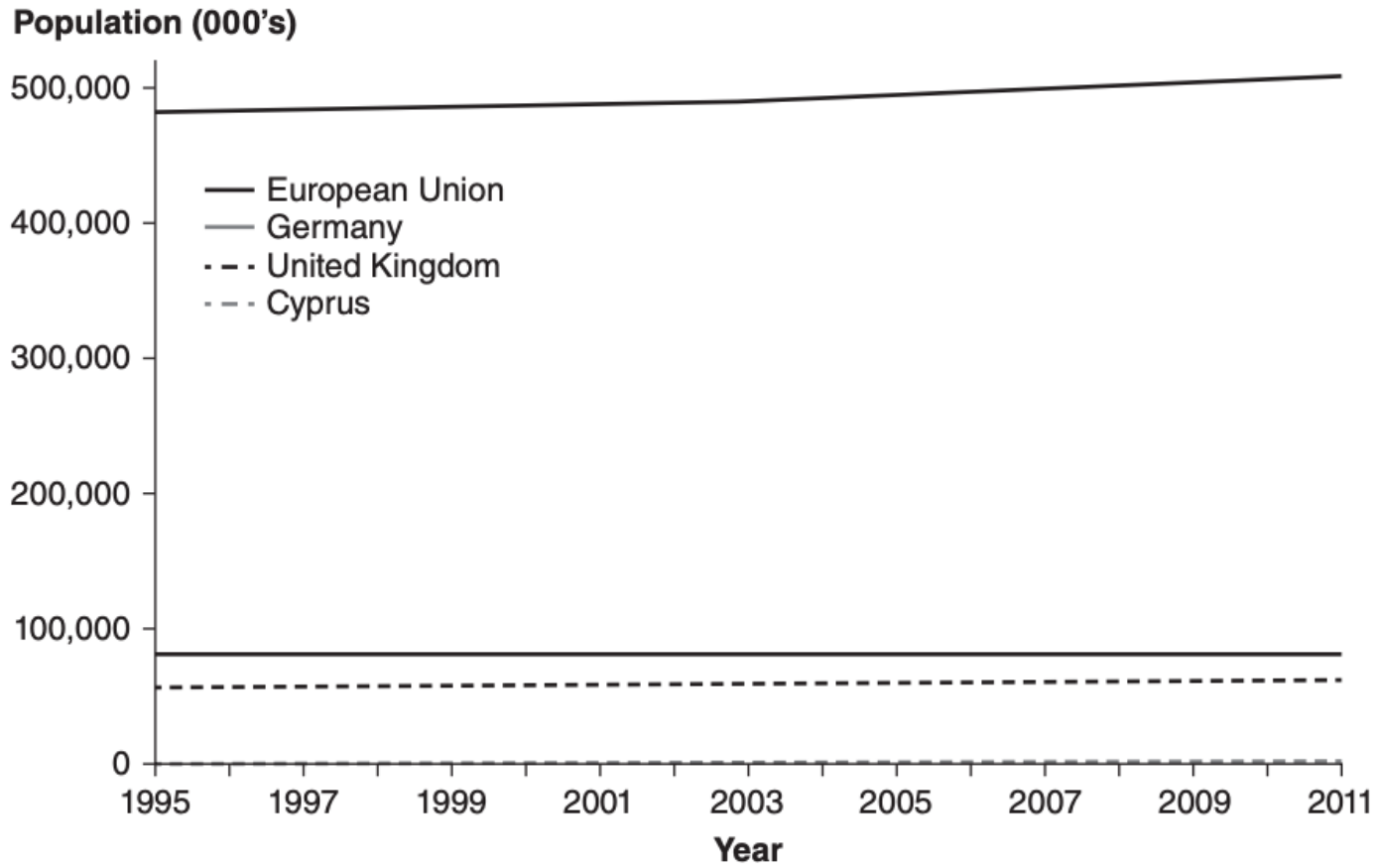


Figure 2.1 Population on 1st January, European Union, 1995–2011. Source: Eurostat. These graphs have been created using data from the Eurostat Statistics Database: http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database. Eurostat is the statistical office of the European Union and it is a Web site that provides access to all Eurostat's statistical databases and the associated electronic publications.

Example: Population

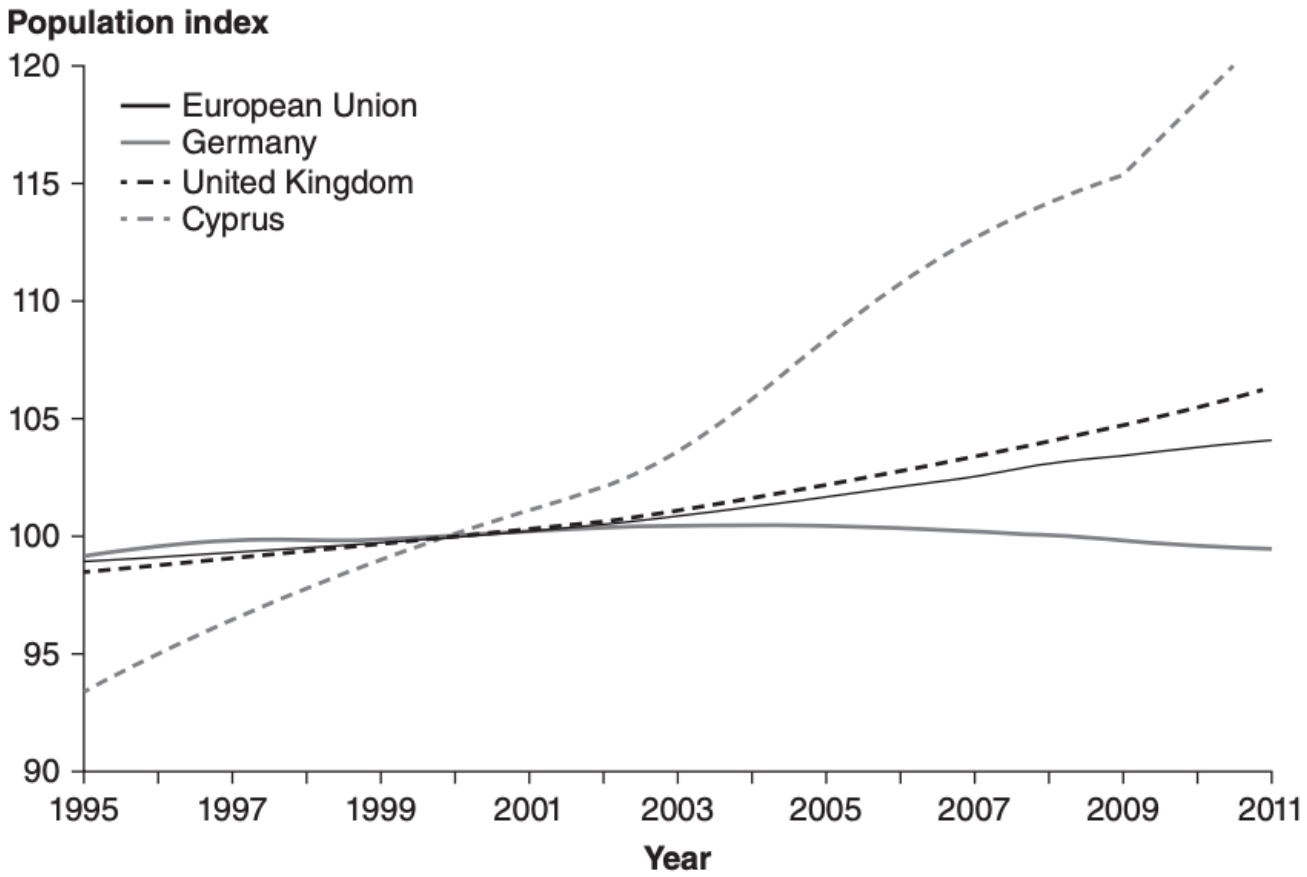


Figure 2.2 Index of population on 1 January (2000 = 100), European Union, 1995–2011. Source: Eurostat. These graphs have been created using data from the Eurostat Statistics Database: http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database. Eurostat is the statistical office of the European Union and it is a Web site that provides access to all Eurostat's statistical databases and the associated electronic publications.

Synthetic (or Aggregate) Indices

Definition

Synthetic indices are general terms for indices that combine multiple variables into one measure.

- They can be either composite or complex, depending on the units of the variables:
  - **Composite synthetic index:** all variables in the same units.
  - **Complex synthetic index:** variables in different units.

Summary

- Composite $\rightarrow$ same units
- Complex $\rightarrow$ different units
- Synthetic $=$ general term ($\rightarrow$ can be composite or complex)

Synthetic (or Aggregate) Indices

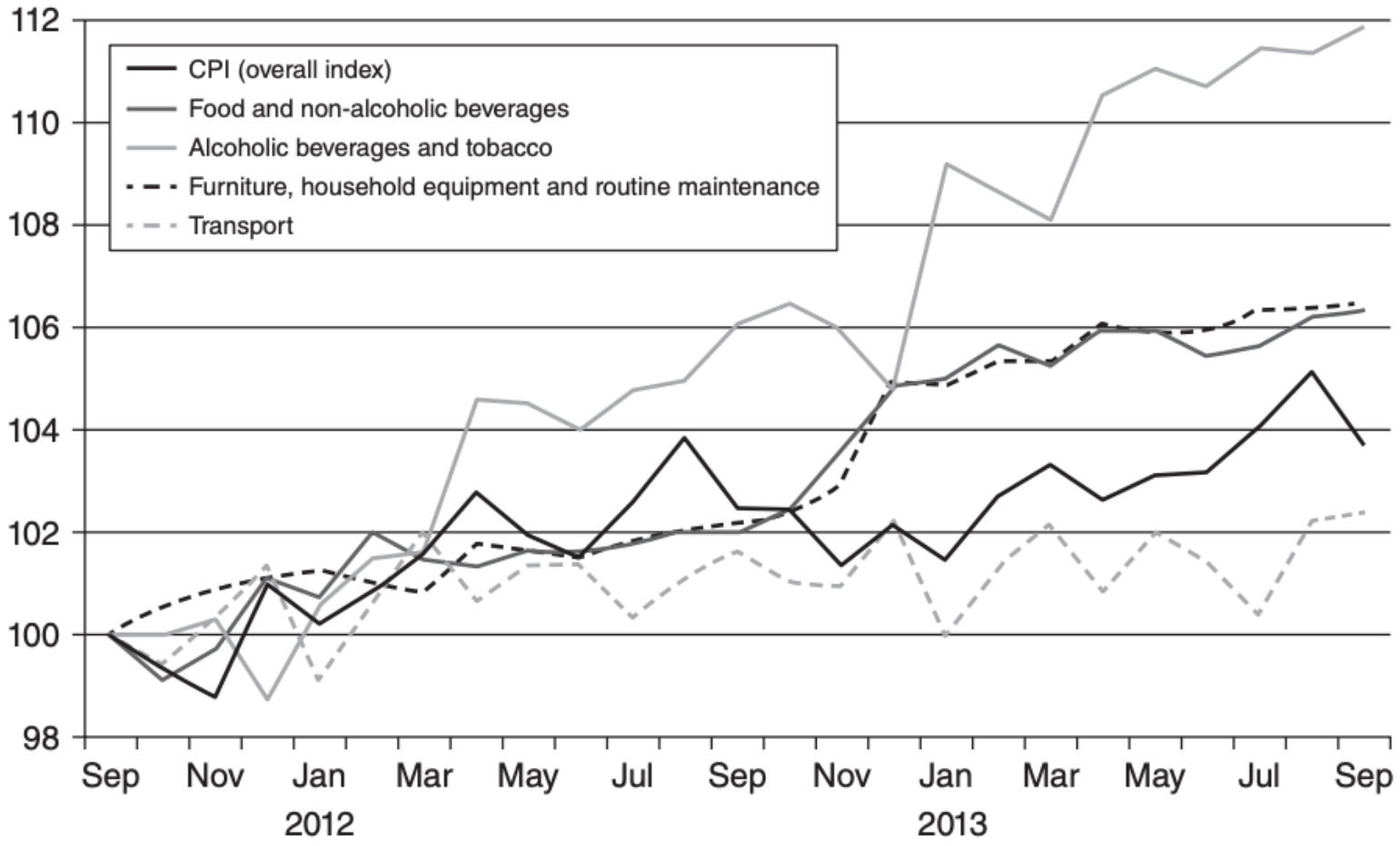


Figure 1.1 UK Consumer Prices Index and selected sectors, September 2011–2013. Source: Office for National Statistics (http://www.ons.gov.uk/ons/rel/rsi/retail-sales/july-2013/tsd-retail-sales--july-2013.html).

Synthetic (or Aggregate) Indices)

The overall CPI is a weighted average of its 12 sub-categories rather than a simple arithmetic mean. Each category is assigned a weight reflecting its proportion of total UK household consumption expenditure.

Table: Weights for categories of goods and services in the CPI 2013, UK.

	Parts per 1000
01 Food and non-alcoholic beverages	95
02 Alcoholic beverages and tobacco	38
03 Clothing and footwear	60
04 Housing, water, electricity, gas and other fuels	244
05 Furniture, household equipment and maintenance	52
06 Health	23
07 Transport	128
08 Communication	26
09 Recreation and culture	123
10 Education	18
11 Restaurants and hotels	103
12 Miscellaneous goods and services	90
	1000

Synthetic Index Calculation

The synthetic index $I_{t|0}$ can be calculated as an average of simple indices:

1. **Arithmetic mean:** $I_{t|0} = \frac{1}{m} \sum_{k=1}^{m} i_k(t|0)$
2. **Geometric mean:** $I_{t|0} = \left(\prod_{k=1}^{m} i_k(t|0) \right)^{\frac{1}{m}}$
3. **Harmonic mean:** $I_{t|0} = \frac{m}{\sum_{k=1}^{m} \frac{1}{i_k(t|0)}}$

Notation

- m variables: $x_k, \ k = 1, 2, \ldots, m$
- Base period values: $x_{10}, x_{20}, \ldots, x_{m0}$
- Current period values: $x_{1t}, x_{2t}, \ldots, x_{mt}$
- Simple index for each variable:

$$i_k(t|0) = \frac{x_{kt}}{x_{k0}} \times 100$$

Note: Choice of mean depends on the nature of variables and desired sensitivity. These formulas assume the variables are in the same units (composite synthetic index).

Why Multiple Synthetic Index Formulas Exist?

- **Time-Reversal Test:**
- If $I_{1|0}$ measures the price change from period 0 to period 1,
- and $I_{0|1}$ measures the price change from period 1 to period 0 (with the base year swapped),
- then the index should satisfy the property:

$$\frac{I_{1|0} \times I_{0|1}}{100} = 100 \quad (\text{or } I_{1|0} \times I_{0|1} = 1 \text{ (in ratio form)}) \qquad (1)$$

Why Multiple Synthetic Index Formulas Exist? (Continued)

- Suppose I have a market data with two items over two periods

	Period	
	0	1
Item 1	€10	€20
Item 2	€20	€10

Calculation in Ratio Form

- **1. Forward Price Relatives ($0 \to 1$):**

$$i_1(1|0) = \frac{20}{10} = 2.0, \quad i_2(1|0) = \frac{10}{20} = 0.5$$

- **2. Reverse Price Relatives ($1 \to 0$):**

$$i_1(0|1) = \frac{10}{20} = 0.5, \quad i_2(0|1) = \frac{20}{10} = 2.0$$

Calculation in Ratio Form

1. **Arithmetic Mean:**

$$I_{1|0} = \frac{2.0 + 0.5}{2} = 1.25, \quad I_{0|1} = \frac{0.5 + 2.0}{2} = 1.25$$
$$\Longrightarrow \ 1.25 \times 1.25 = 1.5625 \neq 1$$

Upward bias (>1), it fails the time reversal test

2. **Geometric Mean:**

$$I_{1|0} = \sqrt{2.0 \times 0.5} = 1.00, \quad I_{0|1} = \sqrt{0.5 \times 2.0} = 1.00$$
$$\Longrightarrow \ 1.00 \times 1.00 = 1.00 \quad (\textbf{Passes})$$

No bias, it passes the time reversal test

3. **Harmonic Mean:**

$$I_{1|0} = \frac{2}{\frac{1}{2.0} + \frac{1}{0.5}} = 0.80, \quad I_{0|1} = \frac{2}{\frac{1}{0.5} + \frac{1}{2.0}} = 0.80$$
$$\Longrightarrow \ 0.80 \times 0.80 = 0.64 \neq 1 \quad (\textbf{Fails})$$

Downward bias (<1), it fails the time reversal test

*To fix this, the Carruthers, Selwood, Ward, and Dalen (CSWD) index is defined specifically as the geometric mean of the arithmetic and harmonic price indices.

Synthetic Index Numbers - Using Averages of Variables

Alternative Method

Treat the synthetic index as a simple index of the average values of the variables at the current period (t) relative to the base (0).

$$I_{t|0} = \frac{\frac{1}{m}\sum_{k=1}^{m} x_{kt}}{\frac{1}{m}\sum_{k=1}^{m} x_{k0}} \times 100 = \frac{\sum_{k=1}^{m} x_{kt}}{\sum_{k=1}^{m} x_{k0}} \times 100$$

- x_{kt}: value of variable k in period t
- x_{k0}: value of variable k in the base period 0
- **Note:** This approach treats all variables equally via their arithmetic mean, producing a composite synthetic index if variables share the same units.

Weighted Synthetic Index

Formula

Another alternative is to define the synthetic index as a weighted index of the current values x_{kt} in relation to the base values x_{k0}, using weights w_k $(k = 1, 2, ..., m)$.

$$I_{t|0} = \frac{\sum_{k=1}^{m} w_k x_{kt}}{\sum_{k=1}^{m} w_k x_{k0}} \times 100$$

If all weights are equal $(w_k = 1/m)$, the formula becomes the simple unweighted index:

$$I_{t|0} = \frac{\frac{1}{m}\sum x_{kt}}{\frac{1}{m}\sum x_{k0}} \times 100$$

- w_k weights reflect the relative importance of each variable.
- Typically used when variables are in the same units (composite indices).

Calculation of Synthetic Indices: Example

Problem

The table presents prices (in euros) of three products in year 0 (base) and year t.

Product	x_{k0} **(Year 0)**	x_{kt} **(Year t)**
A	10	12
B	8	9
C	6	7

Compute the synthetic index using different methods.

Calculation of Synthetic Indices: Example (Continued)

Product	x_{k0} **(Year 0)**	x_{kt} **(Year t)**	**Simple Index** $i_{k(t\|0)}$
A	10	12	$(12/10) \times 100 = 120$
B	8	9	$(9/8) \times 100 = 112.5$
C	6	7	$(7/6) \times 100 = 116.7$

Calculation of Synthetic Indices: Example (Step 2)

Step 2: Indices Based on Simple Indices

1. **Arithmetic Mean Index**

$$I_{t|0} = \frac{120 + 112.5 + 116.7}{3} = 116.4$$

2. **Geometric Mean Index**

$$I_{t|0} = (120 \times 112.5 \times 116.7)^{1/3} \approx 116.6$$

3. **Harmonic Mean Index**

$$I_{t|0} = \frac{3}{\frac{1}{120} + \frac{1}{112.5} + \frac{1}{116.7}} \approx 116.0$$

Calculation of Synthetic Indices: Example (Step 3)

Step 3: Indices Based on Averages of Variables

1. **Arithmetic mean of variables**

$$I_{t|0} = \frac{\frac{1}{3}(12+9+7)}{\frac{1}{3}(10+8+6)} \times 100 = \frac{9.33}{8} \times 100 \approx 116.6$$

2. **Geometric mean of variables**

$$I_{t|0} = \frac{(12 \times 9 \times 7)^{1/3}}{(10 \times 8 \times 6)^{1/3}} \times 100 \approx 116.6$$

Calculation of Synthetic Indices: Example (Step 4)

Step 4: Weighted Synthetic Index (Values of Variables)
Assume weights: $w_A = 0.5, w_B = 0.3, w_C = 0.2$

$$I_{t|0} = \frac{0.5 \cdot 12 + 0.3 \cdot 9 + 0.2 \cdot 7}{0.5 \cdot 10 + 0.3 \cdot 8 + 0.2 \cdot 6} \times 100 = \frac{10.1}{8.6} \times 100 \approx 117.4$$

Note

- Overall increase from year 0 to t is $\approx 16 - 17\%$.
- Arithmetic, geometric, and harmonic mean indices rely on simple indices.
- Averages of variables and weighted indices use raw values and weights to reflect importance.

Choosing the Right Synthetic Index

1. Arithmetic mean of simple indices
   - Easy to calculate and interpret.
   - Good for similar importance/scales.
2. Geometric mean of simple indices
   - Better for relative changes (percentages).
   - Less affected by extreme values.
3. Harmonic mean of simple indices
   - Emphasizes smaller values; rarely used.
4. Indices based on averages
   - Reflects average behavior (same units).
5. Weighted synthetic index
   - Incorporates variable importance.

Conclusion:

- No universally best index.
- Choose based on variable type, importance, and extreme value sensitivity.

Laspeyres and Paasche Index Numbers

- Arithmetic, geometric, and harmonic means are sometimes uninformative when multiple items have unequal importance.
- Consider this examele:

	2012 price (base year)	2013 price	$I_{1\|0}$ (Ratio form)
Apples	0.40	0.42	1.050
Bananas	0.50	0.55	1.100
Pineapple	1.25	2.00	1.600

Laspeyres and Paasche Index Numbers

(Base = 2012)	
	Price index for 2013
Arithmetic	125.0
Geometric	122.7
Harmonic	120.7

	2012 price	2013 price	2012 quantity
Apples	0.40	0.42	220
Bananas	0.50	0.55	230
Pineapple	1.25	2.00	50

Laspeyres and Paasche Index Numbers

- Conclusion: Although consumed in smaller quantities, pineapple's high price increase pulls up the arithmetic, geometric, and harmonic means.
- Not all items in our basket are equally important.
- When this is the case, we would like to apply a weighting to take this into account.

Laspeyres and Paasche Index Numbers

What are they?

Laspeyres and Paasche indices are **composite (synthetic) index numbers** used to measure how **prices** or **quantities** change between a **base period** (0) and a **current period** (t).

Why are they useful?

- To measure **inflation** and changes in the **cost of living**.
- To evaluate **production** or **consumption** volume changes.
- To summarize complex multi-product changes into **one single number**.
- Widely used in **national accounts**, **macroeconomic analysis**, and **indexing salaries or contracts**.

How do they differ?

The difference lies **only in the weights**: Laspeyres uses **base-period** weights, while Paasche uses **current-period** weights.

How do they differ?

Index	Weights Used	Interpretation	Tendency
Laspeyres Index	Base-period weights (quantities or prices from period 0)	Keeps the base-period consumption structure fixed	Tends to **overstate** price increases
Paasche Index	Current-period weights (quantities or prices from period t)	Reflects current consumption patterns	Tends to **understate** price increases

- The Laspeyres index assumes people keep buying the exact same base-period basket, ignoring consumer substitution toward cheaper alternatives.
- The Paasche index understates price increases because it evaluates prices using the current basket–a basket consumers already modified to avoid expensive items.

Laspeyres Index: Formula and Interpretation

1. Laspeyres Indices

Price Index (Fixed base quantities):

$$L^P_{t|0} = \frac{\sum_{k=1}^m q_{k0} p_{kt}}{\sum_{k=1}^m q_{k0} p_{k0}} \times 100 = \frac{\mathbf{q}_0' \mathbf{p}_t}{\mathbf{q}_0' \mathbf{p}_0} \times 100$$

Quantity Index (Fixed base prices):

$$L^Q_{t|0} = \frac{\sum_{k=1}^m p_{k0} q_{kt}}{\sum_{k=1}^m p_{k0} q_{k0}} \times 100 = \frac{\mathbf{p}_0' \mathbf{q}_t}{\mathbf{p}_0' \mathbf{q}_0} \times 100$$

where $\mathbf{p}_0$, $\mathbf{p}_t$, $\mathbf{q}_0$, and $\mathbf{q}_t$ are all column vectors (where $'$ denotes the transpose):

$$\mathbf{p}_0 = (p_{10}, p_{20}, \ldots, p_{m0})', \qquad \mathbf{p}_t = (p_{1t}, p_{2t}, \ldots, p_{mt})'$$
$$\mathbf{q}_0 = (q_{10}, q_{20}, \ldots, q_{m0})', \qquad \mathbf{q}_t = (q_{1t}, q_{2t}, \ldots, q_{mt})'$$

Notation: m products, base period price p_{k0} and quantity q_{k0}, current period price p_{kt} and quantity q_{kt}.

Paasche Index: Formula and Interpretation

2. Paasche Indices

Price Index (Fixed current quantities):

$$P^P_{t|0} = \frac{\sum_{k=1}^m q_{kt} p_{kt}}{\sum_{k=1}^m q_{kt} p_{k0}} \times 100 = \frac{\mathbf{q}_t' \mathbf{p}_t}{\mathbf{q}_t' \mathbf{p}_0} \times 100$$

Quantity Index (Fixed current prices):

$$P^Q_{t|0} = \frac{\sum_{k=1}^m p_{kt} q_{kt}}{\sum_{k=1}^m p_{kt} q_{k0}} \times 100 = \frac{\mathbf{p}_t' \mathbf{q}_t}{\mathbf{p}_t' \mathbf{q}_0} \times 100$$

Notation: Uses current period weights (q_{kt} for price index, p_{kt} for quantity index).

Laspeyres VS Paasche Index

- **Laspeyres price index:** How much more expensive is my **old shopping basket** if I buy it at today's prices?
- **Paasche price index:** How much more expensive is my **today's shopping basket** compared to what it would have cost at past prices?
- **Laspeyres quantity index:** How much has my overall consumption grown, if we value both old and new quantities using **old prices**?
- **Paasche quantity index:** How much has my overall consumption grown, if we value both old and new quantities using **today's prices**?

Worked Example: Laspeyres and Paasche Price Indices

Product k	p_{k0}	q_{k0}	p_{kt}	q_{kt}
1	4	10	6	12
2	5	6	8	5
3	2	20	3	18

Compute Laspeyres Price Index

$$\sum q_{k0}p_{kt} = 10 \cdot 6 + 6 \cdot 8 + 20 \cdot 3 = 60 + 48 + 60 = 168$$

$$\sum q_{k0}p_{k0} = 10 \cdot 4 + 6 \cdot 5 + 20 \cdot 2 = 40 + 30 + 40 = 110$$

$$L^P_{t|0} = \frac{168}{110} \times 100 \approx \mathbf{152.73}$$

$\rightarrow$ Prices increased by about **52.7%** using base-period quantities.

Worked Example: Laspeyres and Paasche Price Indices

Compute Paasche Price Index

$$\sum q_{kt} p_{kt} = 12 \cdot 6 + 5 \cdot 8 + 18 \cdot 3 = 72 + 40 + 54 = 166$$

$$\sum q_{kt} p_{k0} = 12 \cdot 4 + 5 \cdot 5 + 18 \cdot 2 = 48 + 25 + 36 = 109$$

$$P^P_{t|0} = \frac{166}{109} \times 100 \approx \mathbf{152.29}$$

$\rightarrow$ Prices increased by about **52.3%** using current-period quantities.

Laspeyres Quantity Index

$$L_{t|0}^{Q} = \frac{\sum_{k=1}^{m} p_{k0} q_{kt}}{\sum_{k=1}^{m} p_{k0} q_{k0}} \times 100$$

Compute numerator:

$$\sum p_{k0} q_{kt} = 4 \cdot 12 + 5 \cdot 5 + 2 \cdot 18 = 48 + 25 + 36 = 109$$

Denominator (already computed):

$$\sum_{k} p_{k0} q_{k0} = 4 \cdot 10 + 5 \cdot 6 + 2 \cdot 20 = 110$$

$$L_{t|0}^{Q} = \frac{109}{110} \times 100 \approx \mathbf{99.09}$$

Quantities (valued at based prices) fell by 0.91%.

Paasche Quantity Index

$$P_{t|0}^{Q} = \frac{\sum_{k=1}^{m} p_{kt} q_{kt}}{\sum_{k=1}^{m} p_{kt} q_{k0}} \times 100$$

Compute numerator:

$$\sum p_{kt} q_{kt} = 6 \cdot 12 + 8 \cdot 5 + 3 \cdot 18 = 72 + 40 + 54 = 166$$

Denominator:

$$\sum_{k} p_{kt} q_{k0} = 6 \cdot 10 + 8 \cdot 6 + 3 \cdot 20 = 168$$

$$P_{t|0}^{Q} = \frac{166}{168} \times 100 \approx \mathbf{98.81}$$

Quantities (valued at current prices) fell by 1.19%.

Comparing Laspeyres and Paasche Price Indices

Comparison Details

- The two quantity indices differ because they use different price weights: Laspeyres values current quantities at base prices, while Paasche values them at current prices.
- When prices change (here prices rose), using current prices as weights (Paasche) gives a slightly different view of quantity change compared to base prices (Laspeyres).

Fisher Ideal Index

If you want a single "neutral" measure, use the **Fisher Ideal index** (geometric mean of Laspeyres and Paasche).

Combined (Composite) Price Indices

According to Silvestre (2007), composite price indices combine Laspeyres and Paasche to reduce bias:

1. **Fisher Ideal Price Index (geometric mean):**

$$F_{t|0}^{P} = \sqrt{L_{t|0}^{P} \cdot P_{t|0}^{P}}$$

2. **Marshall-Edgeworth Price Index (average quantities):**

$$ME_{t|0}^{P} = \frac{\sum_{k=1}^{m} p_{kt}\left(\frac{q_{k\mathbf{0}}+q_{kt}}{2}\right)}{\sum_{k=1}^{m} p_{k0}\left(\frac{q_{k\mathbf{0}}+q_{kt}}{2}\right)} \times 100 = \frac{\sum_{k=1}^{m} p_{kt}(q_{k0}+q_{kt})}{\sum_{k=1}^{m} p_{k0}(q_{k0}+q_{kt})} \times 100$$

3. **Bowley Price Index (arithmetic mean):**

$$B_{t|0}^{P} = \frac{L_{t|0}^{P} + P_{t|0}^{P}}{2}$$

Why Use Combined (Composite) Price Indices?

- There is no general agreement on whether the **Laspeyres** or **Paasche** index provides the best measure of price variation.
  - **Laspeyres** tends to **overestimate**, because it uses **base-period quantities** (no substitution).
  - **Paasche** tends to **underestimate**, because it uses **current-period quantities** (already adjusted).
  - Therefore, combined indices aim to produce a **more balanced measure**.

Purpose of Each Composite Index

Composite Index	**Construction Logic**	**Purpose / Interpretation**
Fisher Ideal Index	Geometric mean of Laspeyres and Paasche	Provides a balanced and less biased estimate (considered "ideal").
Marshall–Edgeworth Index	Uses the average of base and current quantities	Reduces the impact of quantity variation and smooths substitution effects.
Bowley Index	Arithmetic mean of Laspeyres and Paasche	Simple compromise index between the two measures.

Purpose of Each Composite Index

Note

- **Fisher Index:** Balances the upward bias of Laspeyres and the downward bias of Paasche, resulting in a more balanced and less biased estimate of price variation.
- **Marshall–Edgeworth Index:** Uses the average of base-period and current-period quantities, which reduces the impact of quantity variation and provides a smoother measure of price change.
- **Bowley Index:** Does not correct the bias; it simply takes the midpoint between Laspeyres and Paasche, providing a straightforward compromise between the two measures.

Worked Example: Combined (Composite) Price Indices

Product k	p_{k0}	q_{k0}	p_{kt}	q_{kt}
1	4	10	6	12
2	5	6	8	5
3	2	20	3	18

Fisher Price Index $F^P_{t|0}$

$$F^P_{t|0} = \sqrt{L^P_{t|0} \cdot P^P_{t|0}} \approx \sqrt{152.73 \cdot 147.9} \approx 150.2$$

Marshall–Edgeworth Price Index $ME^P_{t|0}$

$$ME^P_{t|0} = \frac{\sum_k p_{kt} \frac{q_{k0}+q_{kt}}{2}}{\sum_k p_{k0} \frac{q_{k0}+q_{kt}}{2}} \cdot 100$$

$$ME^P_{t|0} = \frac{6 \cdot 11 + 8 \cdot 5.5 + 3 \cdot 19}{4 \cdot 11 + 5 \cdot 5.5 + 2 \cdot 19} \cdot 100 = \frac{167}{109.5} \cdot 100 \approx 152.5$$

Worked Example: Combined (Composite) Price Indices

Bowley Price Index $B^P_{t|0}$

$$B^P_{t|0} = \frac{L^P_{t|0} + P^P_{t|0}}{2} = \frac{152.73 + 147.9}{2} \approx 150.3$$

Interpretation

- The three indices indicate a price increase of around **50%** from period 0 to period t.
- **Fisher** (≈ 150.2) and **Bowley** (≈ 150.3) give very similar results, reflecting a balanced measure of price change.
- **Marshall–Edgeworth** (≈ 152.5) is slightly higher, due to giving more weight to the quantities of the base and current periods combined.

Exercise 5.1

The prices (in monetary units) and quantities (in kg) consumed of two raw materials, A and B, in a factory are presented below:

Year	**1**	**2**	**3**	**4**	**5**
P_A	3	4	4.5	4	5.25
Q_A	12	14	8	6	10
P_B	5	6	6.5	5.5	3.9
Q_B	9	11	8	10	12

a) Analyse the evolution of prices of raw material A using index numbers.

b) For year 2, relative to year 1, compute the Laspeyres quantity index and comment on the result.

Exercise 5.1a (Suggested Solution)

$$I_{t|1} = \frac{p_{A,t}}{p_{A,1}} \times 100$$

Year (t)	**P_A (Price)**	**Simple Index ($I_{t\|1}$)**	**Interpretation / Change**
1	3.00	$\frac{3.00}{3.00} \times 100 = 100.00$	Base Year (0%)
2	4.00	$\frac{4.00}{3.00} \times 100 = 133.33$	Increase of 33.33%
3	4.50	$\frac{4.50}{3.00} \times 100 = 150.00$	Increase of 50.00%
4	4.00	$\frac{4.00}{3.00} \times 100 = 133.33$	Increase of 33.33%
5	5.25	$\frac{5.25}{3.00} \times 100 = 175.00$	Increase of 75.00%

Comment: The price of raw material A shows an overall upward trend over the 5-year period, reaching a peak in Year 5 with a 75% price increase relative to Year 1.

Exercise 5.1b (Suggested Solution)

$$L^{Q}_{t|0} = \frac{\sum_{k=1}^{m} p_{k0} q_{kt}}{\sum_{k=1}^{m} p_{k0} q_{k0}} \times 100$$

Here, base period 0 = Year 1, current period t = Year 2, and items $k \in \{A, B\}$.

Data:

- Year 1 (0): $p_{A,1} = 3$, $q_{A,1} = 12$, $p_{B,1} = 5$, $q_{B,1} = 9$
- Year 2 (t): $p_{A,2} = 4$, $q_{A,2} = 14$, $p_{B,2} = 6$, $q_{B,2} = 11$

Calculations:

- **Numerator** ($\sum p_{k1} q_{k2}$): $p_{A,1} q_{A,2} + p_{B,1} q_{B,2} = (3 \times 14) + (5 \times 11) = 42 + 55 = 97$
- **Denominator** ($\sum p_{k1} q_{k1}$): $p_{A,1} q_{A,1} + p_{B,1} q_{B,1} = (3 \times 12) + (5 \times 9) = 36 + 45 = 81$

$$L^{Q}_{2|1} = \frac{97}{81} \times 100 \approx 119.75$$

Conclusion: Valued at base-period (Year 1) prices, the overall quantity consumed of raw materials increased by 19.75% from Year 1 to Year 2.

Exercise 5.2

The following data refer to the evolution of prices and quantities sold of three products:

	Year 1		Year 2		Year 3	
	Price	**Quantity**	**Price**	**Quantity**	**Price**	**Quantity**
Product 1	10	100	12	100	14	110
Product 2	12	150	16	120	15	120
Product 3	8	30	10	35	10	45

a) Assess the evolution of quantities sold using the Paasche Quantity Index, relative to year 1.

Exercise 5.2 (Continued)

b) Suppose the price index (PI) of these products is given by:

	Year 1	Year 2	Year 3
PI	100	110	115

The values for years 4 and 5 were calculated relative to year 3:

	Year 4	Year 5
PI	120	135

From the data provided, compute and interpret the reconciled index, using year 3 as the base year.

Exercise 5.2a (Suggested Solution)

$$P_{t|0}^{Q} = \frac{\sum_{k=1}^{m} p_{kt} q_{kt}}{\sum_{k=1}^{m} p_{kt} q_{k0}} \times 100$$

1 Year 2 relative to Year 1 ($t = 2$):

$$\sum p_{k2} q_{k2} = (12 \times 100) + (16 \times 120) + (10 \times 35) = 3470$$

$$\sum p_{k2} q_{k1} = (12 \times 100) + (16 \times 150) + (10 \times 30) = 3900$$

$$P_{2|1}^{Q} = \frac{3470}{3900} \times 100 \approx 88.97$$

Quantities sold (at Year 2 prices) decreased by 11.03% *relative to Year 1.*

Exercise 5.2a (Suggested Solution)

2 Year 3 relative to Year 1 ($t = 3$):

$$\sum p_{k3} q_{k3} = (14 \times 110) + (15 \times 120) + (10 \times 45) = 3790$$

$$\sum p_{k3} q_{k1} = (14 \times 100) + (15 \times 150) + (10 \times 30) = 3950$$

$$P^{Q}_{3|1} = \frac{3790}{3950} \times 100 \approx 95.95$$

Quantities sold (at Year 3 prices) decreased by 4.05% *relative to Year 1.*

Exercise 5.2b (Suggested Solution)

Given Data:

- Series 1 (Base Year 1): $\text{PI}_1 = 100$, $\text{PI}_2 = 110$, $\text{PI}_3 = 115$
- Series 2 (Base Year 3): $\text{PI}_3 = 100$, $\text{PI}_4 = 120$, $\text{PI}_5 = 135$

Reconciliation Procedure (Re-basing to Year 3): To shift Series 1 so that Year 3 becomes 100, multiply each value by $\frac{100}{\text{PI}_3^{(1)}} = \frac{100}{115}$:

- **Year 1:** $100 \times \frac{100}{115} \approx \mathbf{86.96}$
- **Year 2:** $110 \times \frac{100}{115} \approx \mathbf{95.65}$
- **Year 3:** $115 \times \frac{100}{115} = \mathbf{100.00}$
- **Year 4:** Given directly as $\mathbf{120.00}$
- **Year 5:** Given directly as $\mathbf{135.00}$

Interpretation: Prices increased steadily from Year 1 through Year 5. By Year 5, prices were 35% higher than in Year 3, and prices in Year 1 were 13.04% lower than in Year 3.

Exercise 5.4

The following table contains information on the prices and quantities of products A and B traded in years 1 and 2:

	Product A		Product B	
Year	**Price**	**Quantity**	**Price**	**Quantity**
1	x	10	12	5
2	7	y	15	10

a) For product B, assess the evolution of the price in comparison with the evolution of the quantity traded using index numbers.

b) From the collected data, the Laspeyres Price Index for year 1 relative to year 2 is 1.2, and the Paasche Quantity Index for year 2 relative to year 1 is 1.3. Under these conditions, determine x and y.

Exercise 5.4a (Suggested Solution)

Data for Product B:

- Prices: $p_{B,1} = 12$, $p_{B,2} = 15$
- Quantities: $q_{B,1} = 5$, $q_{B,2} = 10$

Calculations (Percentage Form):

- **Price Index for Product B:**

$$I^P_{2|1}(B) = \frac{p_{B,2}}{p_{B,1}} \times 100 = \frac{15}{12} \times 100 = 125.00$$

The price of Product B increased by 25% from Year 1 to Year 2.

- **Quantity Index for Product B:**

$$I^Q_{2|1}(B) = \frac{q_{B,2}}{q_{B,1}} \times 100 = \frac{10}{5} \times 100 = 200.00$$

The quantity traded of Product B doubled (+100%) from Year 1 to Year 2.

Exercise 5.4b (Suggested Solution)

Given Information:

- Laspeyres Price Index for Year 1 relative to Year 2: $L^P_{1|2} = 1.2$ (Ratio form)
- Paasche Quantity Index for Year 2 relative to Year 1: $P^Q_{2|1} = 1.3$ (Ratio form)

1. Determining x via $L^P_{1|2} = 1.2$: Note that base period 0 = Year 2 and current period t = Year 1.

$$L^P_{1|2} = \frac{\sum q_{k2} p_{k1}}{\sum q_{k2} p_{k2}} = \frac{q_{A,2} p_{A,1} + q_{B,2} p_{B,1}}{q_{A,2} p_{A,2} + q_{B,2} p_{B,2}}$$

Substitute known values
($p_{A,1} = x, q_{A,2} = y, p_{B,1} = 12, q_{B,1} = 5, p_{A,2} = 7, p_{B,2} = 15, q_{B,2} = 10$):

$$L^P_{1|2} = \frac{y \cdot x + 10 \cdot 12}{y \cdot 7 + 10 \cdot 15} = \frac{xy + 120}{7y + 150} = 1.2$$

$$xy + 120 = 1.2(7y + 150) \implies xy + 120 = 8.4y + 180 \implies xy - 8.4y = 60 \text{ (Equation 1)}$$

Exercise 5.4b (Suggested Solution)

2. Determining y via $P^Q_{2|1} = 1.3$: Here base period 0 = Year 1 and current period t = Year 2.

$$P^Q_{2|1} = \frac{\sum p_{k2} q_{k2}}{\sum p_{k2} q_{k1}} = \frac{p_{A,2} q_{A,2} + p_{B,2} q_{B,2}}{p_{A,2} q_{A,1} + p_{B,2} q_{B,1}}$$

Substitute known values:

$$P^Q_{2|1} = \frac{7 \cdot y + 15 \cdot 10}{7 \cdot 10 + 15 \cdot 5} = \frac{7y + 150}{70 + 75} = \frac{7y + 150}{145} = 1.3$$

Solving for y:

$$7y + 150 = 1.3 \times 145 \implies 7y + 150 = 188.5 \implies 7y = 38.5 \implies \mathbf{y = 5.5}$$

3. Solving for x using Equation 1:

$$xy - 8.4y = 60 \implies 5.5x - 8.4(5.5) = 60 \implies 5.5x - 46.2 = 60$$

$$5.5x = 106.2 \implies \mathbf{x} = \frac{\mathbf{106.2}}{\mathbf{5.5}} \approx \mathbf{19.31}$$

Final Solution: $x \approx 19.31$ monetary units, and $y = 5.5$ kg.

THANKS!

Questions?