Microeconomics

Chapter 10

Consumers' surplus

Fall 2023

Measures of welfare change

We often want to measure how **consumers' welfare** is affected by **changes in the economic environment**. For instance, a change in the price, or more generally, a change in governmental policy. There are several methods to do so.

The classical measure of welfare change is consumers' surplus. However, only in special circumstances consumers' surplus is an exact measure of welfare change. This chapter describes **three methods for welfare change**, which includes consumers' surplus as a special case.

Measures of welfare change

Let $\mathbf{p}^t = (p_1^t, p_2^t)$ be the prices of $\mathbf{x} = (x_1, x_2)$ at time period t. Consider the following change in the economic environment between $t = 0$ and $t = 1$:

In period $t = 0$ we have $(\mathbf{p}^0, m)$, which are original prices and income. Denote $v^0 = v(\mathbf{p}^0, m)$ as the utility one can reach in $t = 0$.

In period $t = 1$ we have $(\mathbf{p}^1, m)$, which are new prices and original income. New prices are such that $p_1^1 > p_1^0$ and $p_2^1 = p_2^0$. Let $p_2^1 = p_2^0 = 1$, which is a normalization without loss of generality. Denote $v^1 = v(\mathbf{p}^1, m)$ as the utility one can reach in $t = 1$.

We will **use the change in the economic environment above to analyze the three welfare measures** throughout the slides.

Measures of welfare change

An obvious measure of welfare change may be:

$$v^1 - v^0 \lesseqgtr 0.$$

If this difference is positive, then the change increases consumers' utility, whereas if its negative, the change decreases consumers' utility.

When using utility, this is about the best we can do: there is **no unambiguously right way to quantify utility changes** since the only relevant feature of the utility function is its ordinal character. Recall that the utility function is invariant to positive monotonic transformations.

Three measures of welfare change

However, policy makers may want to have **quantifiable monetary measures** of changes in welfare. Such measures would allow them to, for instance, rank-order different policy changes or to compare benefits and costs to different groups of consumers.

Three measures of changes in welfare:

(1) Compensating variation (CV)
(2) Equivalent variation (EV)
(3) Changes in consumers' surplus (ΔCS)

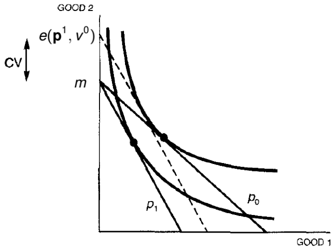
Compensating variation

Let $e(\mathbf{p}^0, v^0)$ be the income you need to reach utility v^0 at prices $\mathbf{p}^0$.
Let $e(\mathbf{p}^1, v^0)$ be the income you need to reach utility v^0 at prices $\mathbf{p}^1$.

The **compensating variation** (CV) is defined as:

$$\begin{aligned} CV &= e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0) \\ &= e(\mathbf{p}^1, v^0) - m. \end{aligned}$$

The CV measures how much the consumer's income must change (in this situation increase) under the new price $\mathbf{p}^1$ to make her as well off as she would be facing $\mathbf{p}^0$. The CV takes the new price $\mathbf{p}^1$ as base price.



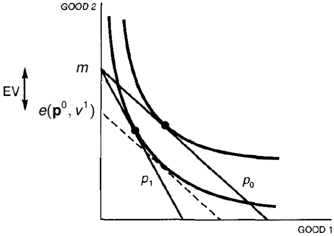
Equivalent variation

Let $e(\mathbf{p}^1, v^1)$ be the income you need to reach utility v^1 at prices $\mathbf{p}^1$.
Let $e(\mathbf{p}^0, v^1)$ be the income you need to reach utility v^1 at prices $\mathbf{p}^0$.

The **equivalent variation** (EV) is defined as:

$$\begin{aligned} EV &= e(\mathbf{p}^1, v^1) - e(\mathbf{p}^0, v^1) \\ &= m - e(\mathbf{p}^0, v^1). \end{aligned}$$

The EV measures how much the consumer's income must change (in this situation decrease) under the original price $\mathbf{p}^0$ to make her as well off as she would be facing $\mathbf{p}^1$. The EV takes the original price $\mathbf{p}^0$ as base price.



Calculate CV and EV

Three steps to calculate the **compensating variation**:

Step 1. Solve the UMP at original prices $\mathbf{p}^0$ and m to find $v^0 = v(\mathbf{p}^0, m)$.

Step 2. Solve the EMP at new prices $\mathbf{p}^1$ and v^0 to find $e(\mathbf{p}^1, v^0)$.

Step 3. Calculate $CV = e(\mathbf{p}^1, v^0) - m$.

Three steps to calculate the **equivalent variation**:

Step 1. Solve the UMP at new prices $\mathbf{p}^1$ and m to find $v^1 = v(\mathbf{p}^1, m)$.

Step 2. Solve the EMP at original prices $\mathbf{p}^0$ and v^1 to find $e(\mathbf{p}^0, v^1)$.

Step 3. Calculate $CV = m - e(\mathbf{p}^0, v^1)$.

Exercise

Consider a consumer with utility function $u = x_1^{1/2} x_2^{1/2}$ and budget constraint $100 = x_1 + x_2$.

1. Calculate the CV for an increase in p_1 from 1 to 2.
2. Calculate the EV for an increase in p_1 from 1 to 2.
3. Why is the CV bigger than the EV?

(not all solutions will be integers)

Link between Hicksian demand and CV and EV

From Shephard's lemma we know that:

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_1} = h_1(\mathbf{p}, u).$$

In words, the Hicksian demand h_1 is the derivative of the expenditure function towards p_1. Note that u can be replaced by any utility, such as v^0 and v^1.

Recall that the CV and EV are equal to:

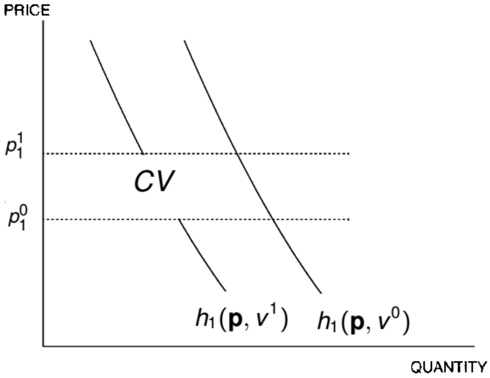
$$\begin{aligned} CV =& e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0). \\ EV =& e(\mathbf{p}^1, v^1) - e(\mathbf{p}^0, v^1). \end{aligned}$$

Hence, using the Fundamental Theorem of Calculus we write CV and EV as:

$$CV = \int_{p_1^0}^{p_1^1} h_1(\mathbf{p}, v^0) dp_1 = e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0).$$

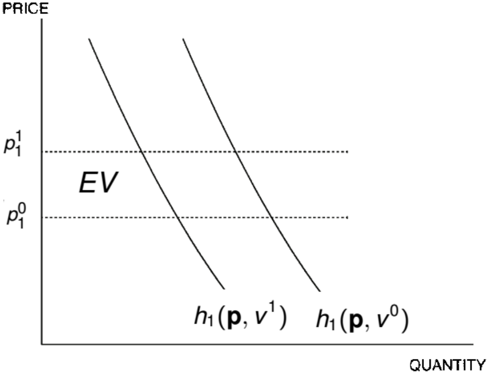
$$EV = \int_{p_1^0}^{p_1^1} h_1(\mathbf{p}, v^1) dp_1 = e(\mathbf{p}^1, v^1) - e(\mathbf{p}^0, v^1).$$

Link between Hicksian demand and CV and EV



The CV and EV can be interpreted as **the area to the left of the Hicksian demand curve**. CV uses the Hicksian demand curve related to v^0.

Link between Hicksian demand and CV and EV



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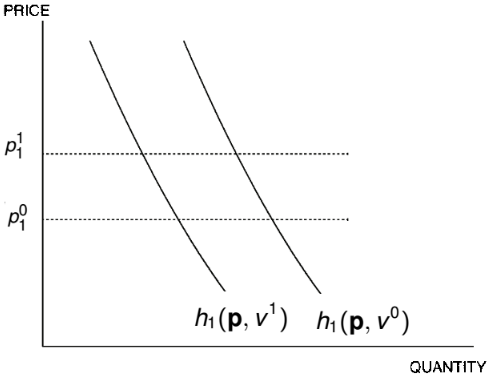
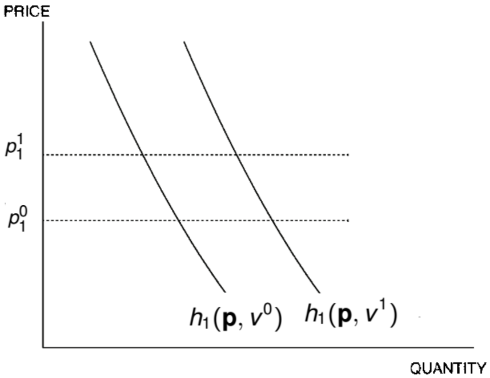
Link between Hicksian demand and CV and EV



We assumed that x_1 is a **normal good**, so that $h_1(\mathbf{p}, v^0) > h_1(\mathbf{p}, v^1)$ since income required for v^0 is higher than for v^1. This implies $CV > EV$.

Link between Hicksian demand and CV and EV



However, if we had assumed that x_1 is an **inferior good**, then $h_1(\mathbf{p}, v^1) > h_1(\mathbf{p}, v^0)$ since income required for v^0 is higher than for v^1. This would imply that $EV > CV$.

Consumers' surplus

What if we calculate the area to the left of the Marshallian demand curve instead of the Hicksian demand curve?

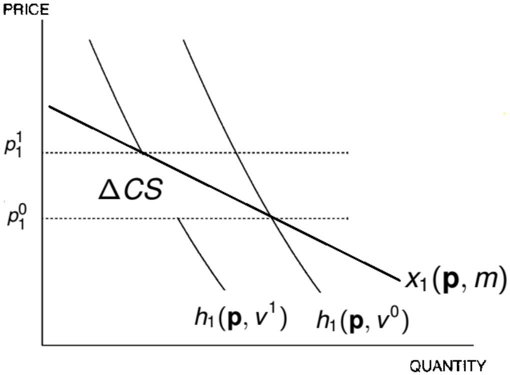
Let $x_1(\mathbf{p}, m)$ be the Marshallian demand function for good 1. We can define the **change in consumer surplus** as:

$$\Delta CS = \int_{p_1^0}^{p_1^1} x_1(\mathbf{p}, m) dp_1 .$$

In practice, we often calculate ΔCS instead of CV and EV. The reason is that it is possible to estimate the Marshallian demand function $x_1(\mathbf{p}, m)$ with data on quantities, prices, and income. In contrast, it is often impossible to estimate the Hicksian demand function $h_1(\mathbf{p}, v)$ as we cannot observe utility.

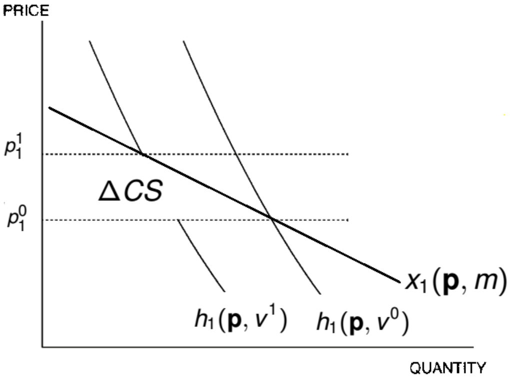
Unfortunately, in contrast to CV and EV, there is **no good theoretical foundation** for why ΔCS should be a measure of welfare change

Consumers' surplus



ΔCS can be interpreted as **the area to the left of the Marshallian demand curve**.

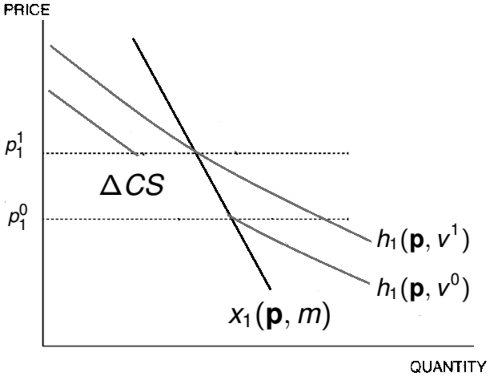
Consumers' surplus



Since we assumed that x_1 is a **normal good**, we have that $x_1(\mathbf{p}, m)$ is less steep than $h_1(\mathbf{p}, v)$. Notice that in the endpoints duality ensures that:

$$x_1(\mathbf{p}^1, m) = h_1(\mathbf{p}^1, v^1).$$
$$x_1(\mathbf{p}^0, m) = h_1(\mathbf{p}^0, v^0).$$

Consumers' surplus



However, if we had assumed that x_1 is an **inferior good**, then $x_1(\mathbf{p}, m)$ is steeper than $h_1(\mathbf{p}, v)$. Notice that in the endpoints duality ensures that:

$$x_1(\mathbf{p}^1, m) = h_1(\mathbf{p}^1, v^1).$$
$$x_1(\mathbf{p}^0, m) = h_1(\mathbf{p}^0, v^0).$$

Luckily ΔCS is in between CV and EV

For a **normal good** (with $\frac{\partial x_1(\mathbf{p},m)}{\partial m} > 0$) we have that CV>EV and ΔCS is in between,

$$CV > \Delta CS > EV.$$

For an **inferior good** (with $\frac{\partial x_1(\mathbf{p},m)}{\partial m} < 0$) we have that EV>CV and ΔCS is in between,

$$EV > \Delta CS > CV.$$

Note that for a price decrease (i.e., $p_1^1 < p_1^0$) the bounds are reversed.

Sometimes ΔCS is equal to CV and EV

Recall that for a good with an **income effect equal to zero** (with $\frac{\partial x_1(\mathbf{p},m)}{\partial m} = 0$) the Marshallian demand curve ($x_1(\mathbf{p})$) is equal to the Hicksian demand curve and both Hicksian demand curves are equal ($h_1(\mathbf{p}) = h_1(\mathbf{p}, v^0) = h_1(\mathbf{p}, v^1)$).

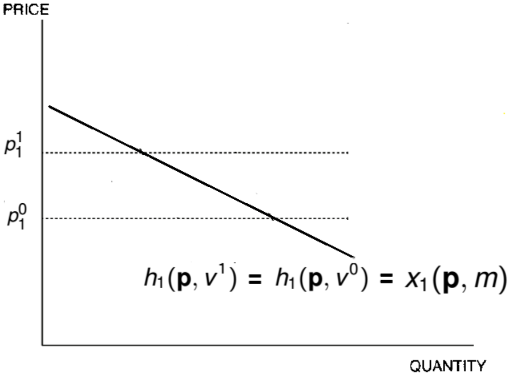
In this case, it is immediate that:

$$CV = \Delta CS = EV.$$

With an income effect of zero we have that ΔCS is an exact measure of welfare change.

Recall that a **quasilinear utility** function has an income effect of zero. Hence, with quasilinear utility ΔCS gives an exact measure of welfare.

Sometimes ΔCS is equal to CV and EV



If we assume that the **income effect is zero**, then $x_1(\mathbf{p}, m) = h_1(\mathbf{p}, v^0) = h_1(\mathbf{p}, v^1)$, and all the measures of welfare are equal.

Exercise

Consider a consumer with utility function $u = 2\sqrt{x_1} + x_2$ and budget constraint $10 = x_1 + 2x_2$.

1. Calculate the CV for an increase in p_1 from 1 to 2.
2. Calculate the EV for an increase in p_1 from 1 to 2.
3. Why is it that $CV = EV$?
4. Calculate ΔCS for an increase in p_1 from 1 to 2.

Homework exercises

Exercises: exercises on the slides