Advanced Microeconomics – PhD in Economics

Exercise 1 (Supermodular Games)

Consider the following class of symmetric games defined over the positive parameter θ.

		B	
		Invest	Not Invest
A	Invest	$4+\theta, 4+\theta$	$\theta, 2\theta$
	Not Invest	$2\theta, \theta$	$2, 2$

a) Define "supermodularity" of a 2×2 non-cooperative game.

b) Define the range of values of θ such that the associated games are "supermodular".

c) Define the range of values of θ such that the associated games can be solved by dominance.

Resolution

a) A 2×2"supermodular" game features a situation where the pure strategies are complementary, i.e., where the difference in payoffs for each player associated with a switch from a low investment strategy to a strategy of high investment increases with the level of investment made by the opponent.

b) Since the class of games is symmetric, we only need to consider the payoff matrix of a single player, A without loss of generality.

		B Invest	B Not Invest
A	Invest	$4+\theta$	θ
	Not Invest	2θ	2

If we diagonalize this matrix, while keeping the set of Nash equilibria unchanged, we obtain,

		B Invest	B Not Invest
A	Invest	$a_1 = 4-\theta$	0
	Not Invest	0	$a_2 = 2-\theta$

For player A, the differences in payoff related with a switch from "Not Invest" to "Invest" are a_1 and $-a_2$ if B plays "Invest" and "Not Invest", respectively. Hence, the game is "supermodular" if,

$$a_1 > -a_2 \leftrightarrow 4-\theta > -(2-\theta) \leftrightarrow \theta < 3$$

c) Define the range of values of θ such that the associated games may be solved by dominance.

Since the class of games is symmetric, we can answer this question by considering the diagonal payoff matrix of player A. We further notice that we have always $a_1 > a_2$. Consequently, games where "Invest" is a strictly dominating strategy for each player only exist for values θ such that,

$$
\begin{aligned}
&a_1 > 0 > a_2 \text{ or} \\
&4-\theta > 0 > 2-\theta \text{ or} \\
&2 < \theta < 4
\end{aligned}
$$