# Advanced Microeconomics – PhD in Economics

**Exercise 2 (Coordination game)**

Consider the following class of symmetric games defined over a positive parameter $\lambda$.

| | | B | |
|---|---|---|---|
| | | Invest | Not Invest |
| A | Invest | $\lambda,\lambda$ | $0,8$ |
| | Not Invest | $8,0$ | $8,8$ |

a) Find the set of Nash equilibrium points in pure strategies for every possible value of $\lambda$.

b) Assess the values of $\lambda$ for which the outcome (Invest, Invest) is payoff dominant and the conditions for which it is "risk dominant".

c) If the players may not communicate before the game is played, what is the most likely outcome if both (Invest, Invest) and (Not Invest, Not Invest) are Nash equilibria?

d) Propose a value for $\lambda$ such that the game might likely end in an out-of-equilibrium outcome. Justify your answer.

Resolution

a) Since the class of games is symmetric, we need only to consider the payoff matrix of a single player, A, w.l.g.

| | | B | |
|---|---|---|---|
| | | Invest | Not Invest |
| A | Invest | $\lambda$ | 0 |
| | Not Invest | 8 | 8 |

If we apply a local displacement of the payoff function to diagonalize the matrix, we obtain,

| | | B | |
|---|---|---|---|
| | | Invest | Not Invest |
| A | Invest | $\lambda - 8$ | 0 |
| | Not Invest | 0 | 8 |

(Not Invest, Not Invest) is *always* a Nash equilibrium. (Invest, Invest) is a Nash equilibrium if $\lambda - 8 > 0 \leftrightarrow \lambda > 8$.

b) This situation is a coordination game, i.e., it has two strict symmetric Nash equilibria if $\lambda > 8$. In this case, (Invest, Invest) payoff dominates (Not Invest, Not Invest). In addition, (Invest, Invest) also *risk* dominates (Not Invest, Not Invest) if the loss from unilateral deviation is higher for the former Nash equilibrium than for the latter one, i.e., if $\lambda - 8 > 8 \leftrightarrow \lambda > 16$.

c) Now we consider the case of a coordination game with two strict symmetric Nash equilibria, i.e., the case with $\lambda > 8$. There are two possibilities,

1. $8 < \lambda < 16$ and (Invest, Invest) payoff dominates (Not Invest, Not Invest), whereas the latter equilibrium risk dominates the former one. In this case, the result is indeterminate.
2. $\lambda > 16$ and (Invest, Invest) is *both* payoff dominant and risk dominant. Hence, it is the solution of the game.

d) If $\lambda > 8$ but close to 8, i.e., if for instance $\lambda = 9$, then (Invest, Invest) is a payoff dominant equilibrium while (Not Invest, Not Invest) is a risk dominant one. Furthermore, the level of risk dominance by the latter equilibrium is given by $\ln\left(\frac{a_{22}-a_{12}}{a_{11}-a_{21}}\right) = \ln\left(\frac{8}{9-8}\right) = \ln 8$, which is strongly positive. Hence, one player might aim (Invest, Invest), while his opponent aims (Not Invest, Not Invest). “Invest” is a much riskier strategy than “Not Invest”.