1.1. (3 points)

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve the FOCs for x1 and x2 to reach

$$x_1 = \frac{m}{p_1}\frac{\alpha}{\alpha+\beta}$$
$$x_2 = \frac{m}{p_2}\frac{\beta}{\alpha+\beta}$$

1.2. (3 points)

$$\frac{dx_1}{d\alpha} = \frac{m}{p_1}\frac{\beta}{(\alpha+\beta)^2} > 0 \text{ and } \frac{dx_1}{d\beta} = -\frac{m}{p_1}\frac{\alpha}{(\alpha+\beta)^2} < 0$$
$$\frac{dx_2}{d\alpha} = -\frac{m}{p_2}\frac{\beta}{(\alpha+\beta)^2} < 0 \text{ and } \frac{dx_2}{d\beta} = \frac{m}{p_2}\frac{\alpha}{(\alpha+\beta)^2} > 0$$

Hence, if α goes up then x_1 goes up and x_2 goes down.

In turn, if β goes up then x_1 goes down and x_2 goes up.

2.1. (3 points)

Here we use the Slutsky equation. In class we used a graph with indifference curves and budget lines to answer this question.

The Slutsky equation is:

$$\underbrace{\frac{\partial x}{\partial p}}_{TE} = \underbrace{\frac{\partial h}{\partial p}}_{SE} - \underbrace{\frac{\partial h}{\partial m}x}_{IE}$$

If the Marshallian demand curve is less steep than the Hicksian demand curve, it must be that the Marshallian demand *drops* by more than the Hicksian demand if there is an *increase* in the price (since we draw *x* on the horizontal axis and *p* on the vertical axis).

This implies that:

$$\underbrace{\frac{\partial x}{\partial p}}_{TE} < \underbrace{\frac{\partial h}{\partial p}}_{SE}$$

Hence it must be that:

$$\underbrace{\frac{\partial h}{\partial m}x}_{IE} > 0$$

Hence it must be that the income effect is bigger than zero: if income goes up, consumption of *x* goes up. This implies that *x* is a normal good.

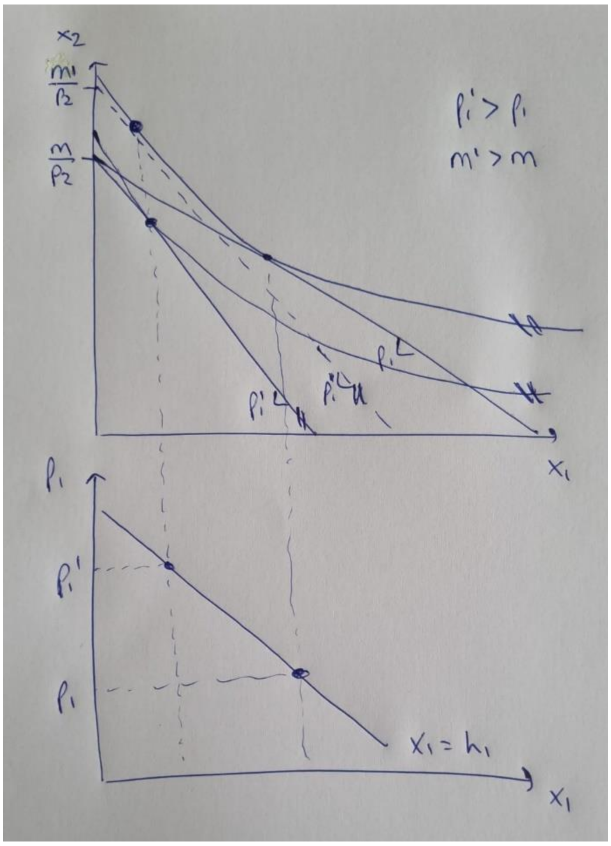
2.2. (2 points)

Incorrect. The Hicksian demand curve reflects how prices change demand while utility is kept fixed. This is the substitution effect, and since the substitution effect is negative because of the convexity assumption on preferences, the Hicksian demand function has to be downwards sloping.

2.3. (2 points)

Incorrect. Duality only ensures that the Marshallian and Hicksian demand function are equal at *one* certain price, not at every price. Only if the income effect is zero we have that the Marshallian and Hicksian demand functions are equal at every price.

3.1. (2 points)



Conclusion: the two demand curves are the same given that with this utility function the income effect is zero. Indeed, you should have drawn two indifference curves that are parallel to each other, and hence have the same slopes for a given level of x1, so that the income effect is zero.

3.2. (3 points)

To find the compensating variation you can follow three steps:

1. Solve the UMP at original prices and income to find indirect utility

$v(p^0, m) = 14$

2. Solve the EMP at new prices and v^0 to find minimum expenditure

$e(p^1, v^0) = 48$

3. Calculate CV by subtracting income in step 1 from minimum expenditure in step 2.

$CV = e(p^1, v^0) - m = 48 - 40 = 8$

3.3. (2 points)

The compensating variation is the area to the left of the Hicksian demand curve. The change in consumer surplus is the area to the left of the Marshallian demand curve. Since the Hicksian and Marshalian demand coincide as the income effect is zero, the compensating variation and change in consumer surplus are the same.

This is useful since in practice we do not observe utility and therefore cannot estimate the Hicksian demand curve, but we do observe income and may be able to estimate the Marshallian demand curve. If we can do the latter, we can retrieve the area to left of the Marshallian demand curve and hence the change in consumer surplus. If income effects are zero, we can argue that this is an exact measure of welfare as its equal to the compensating variation.