IPM
Course revision

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Course Revision

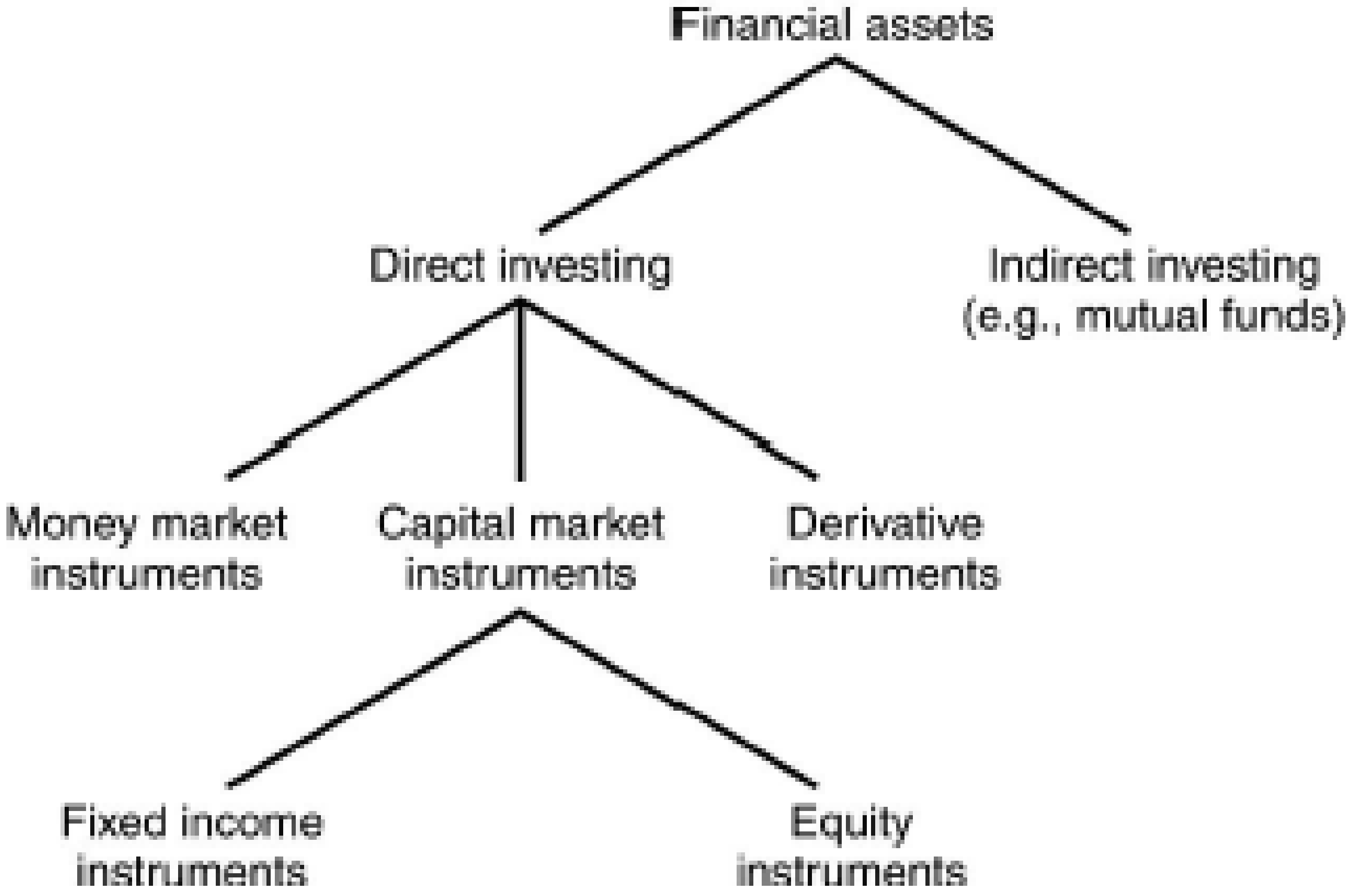
- Financial Market Structures and Instruments
- Theory of Portfolio Management
  - Mean variance
  - Selection Optimal portfolios
- Models of equilibrium
- Market Efficiency & Behavioral Finance
- Portfolio Performance

Financial Market Structures and Instruments

Why financial assets?

- What does a society need to produce wealth?
  - Answer:
- Why do we need financial asset?
  - Answer:

Financial markets & Instruments



- Newly issued securities → Primary Market
- Trading of existing securities → Secondary Market

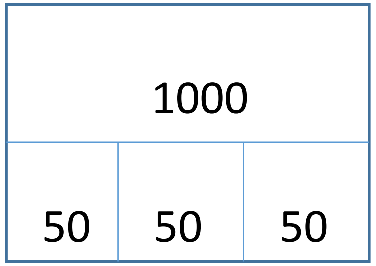
Financial Instruments

Equity

- Common stock
  - limited liability
  - residual claim
- Preferred stock (limited voting power)
- Return:
  - Price change
  - Dividend
  - Foreign exchange

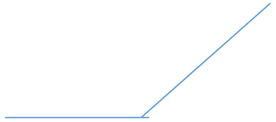
Fixed Income

- Bills (short term, typically zero coupon)
- Bonds (longer term, coupons)
- Issuer:
  - Treasury
  - Municipal (credit risky)
  - Company (credit risky)
- Pricing by NPV

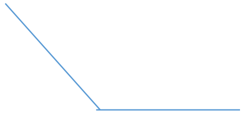


Derivatives

- Futures & forwards
  - Long contract - obligation to buy
  - Short contract – obligation to sell
- Call option
  - Long call – right to…
  - Short call



- Put option
  - Long put -right to…
  - Short put



Money market

- Short term
- “Cash-like”

Funds

- Pool resources
- Price: NAV

Indices

- Quick measure to read off market
- Benchmark for many funds
- Underlying for many derivatives
- Fictive portfolio of assets
  - Example S&P500, PSI, FTSE, DAX, OMXS
- Index weights
  - Price, equal, market cap, fundamental

Type of orders

- Market orders — *Immediately at best price*
- Price-contingent orders
  - Limit orders — *Later at fixed or better*
  - Stop orders — *Later, If threshold hit, then market order*

Bid-ask spread

Bid Price	Ask Price
Exist an offer to buy at this price	Exist an offer to sell at this price
100.00	100.50

Question: If you want to sell this asset using a market order, what is the price you receive?

Answer:

No arbitrage assumption

How can you benefit if an asset has different prices in different markets?

What will happen to prices in efficient markets?

Theory of Portfolio Management

Return and risk

For a discrete random variable that take outcome k with probability p_k

Expected return $$\mathbb{E}(R) = \bar{R} = \sum_{\mathrm{k}} p_{\mathrm{k}} R_{\mathrm{k}}.$$

Variance $$\sigma^2 = \sum_{k=1}^{m} p_k \left(R_k - \bar{R}\right)^2$$

Return and risk for portfolios

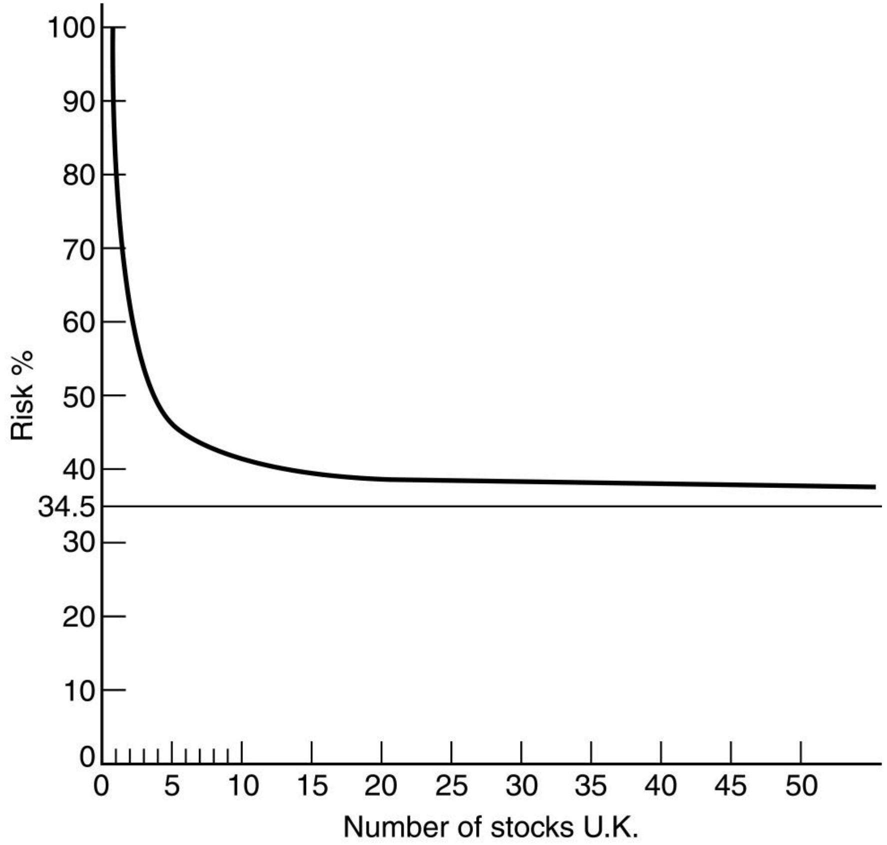
Vector notation

Expected return $E[R_p] = \sum_{i=1}^{n} x_i \bar{R}_i$ $\qquad \bar{R}_p = X' \bar{R}$

Variance $\sigma_P^2 = \mathrm{Var}(R_P) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j$ $\qquad \sigma_P^2 = X'VX.$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad V = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix}$$

Variances of large homogeneous portfolios



$$\text{Var}\left(\frac{1}{n}\sum R_i\right) = \quad = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{\sigma_{ij}}$$

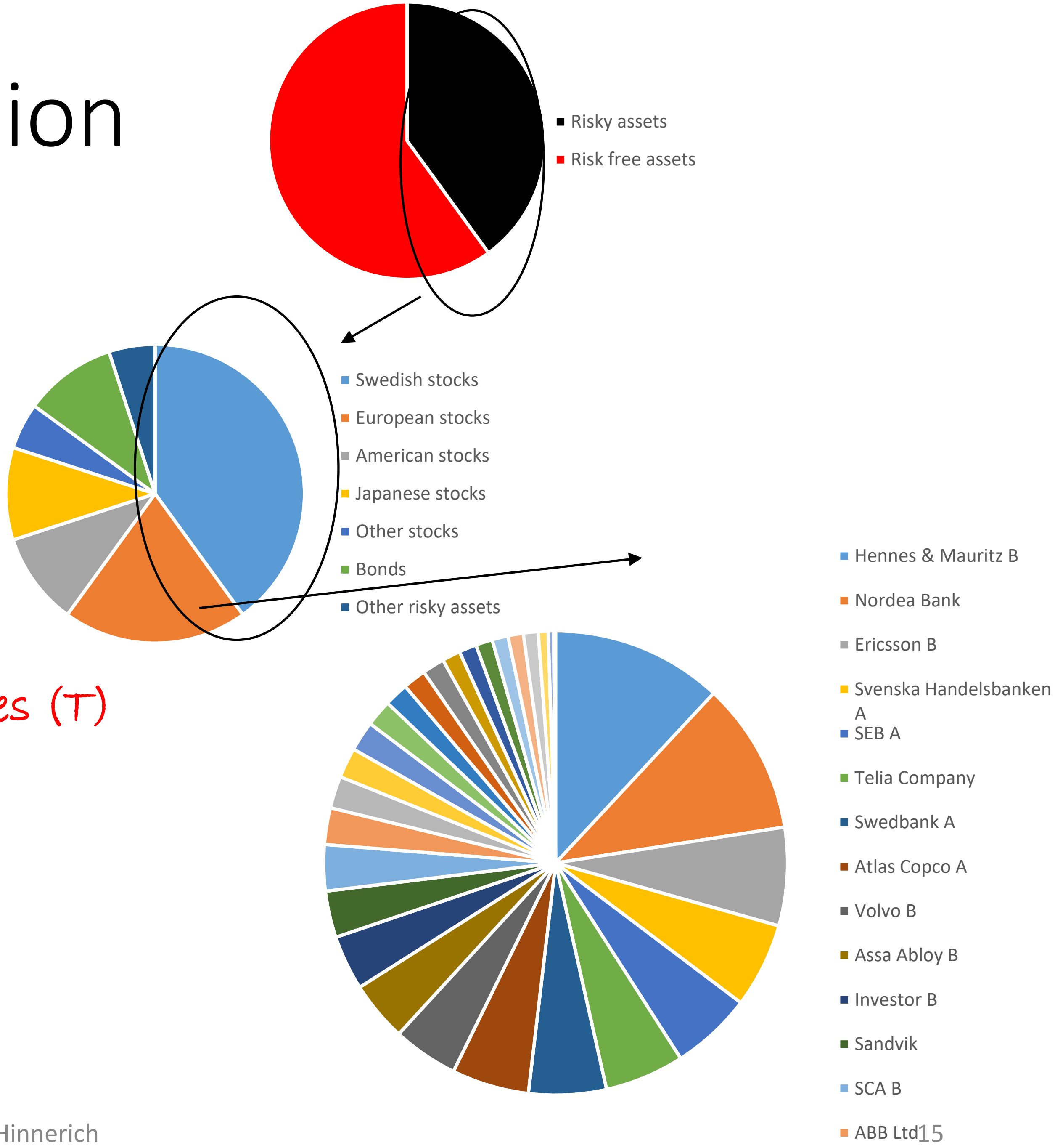
Thus by taking equal proportions of a large number of assets, we obtain a portfolio whose variance is the average covariance of the assets in the pool.

Investor portfolio decision

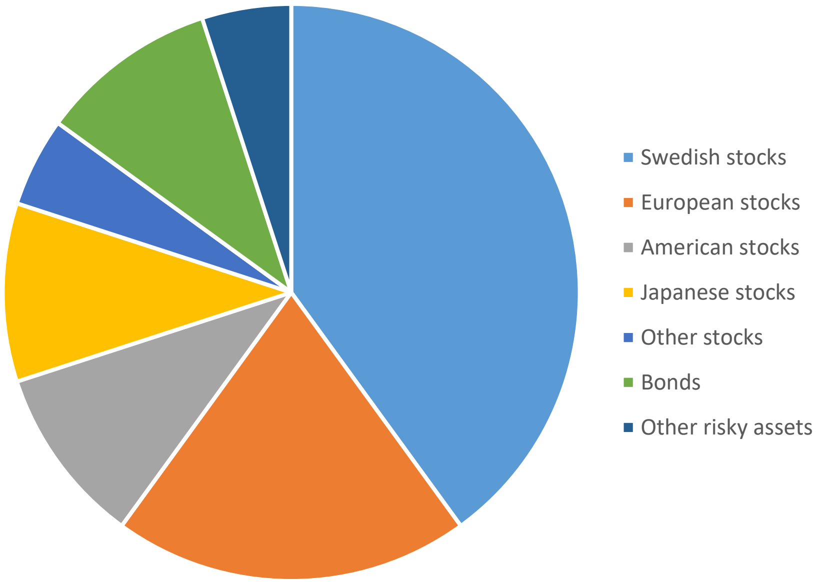
Investor needs to make desicions about:

Top-down

- **Capital allocation** to Risky Assets
  - Decision between risky and non-risky (C)
- **Asset allocation** within Risky Assets
  - Decision between various risky asset classes (T)
- **Security Selection**
  - Decision on instrument level



Asset allocation within Risky Assets
- how to find portfolio P on CAL



Mean-variance efficiency

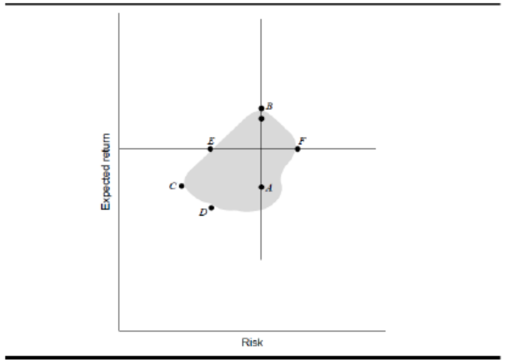


Figure 5.8 Risk and return possibilities for various assets and portfolios.

Definition (Efficiency)

A portfolio (A) is *efficient* provided either

No other portfolio B has at least as much expected return and lower standard deviation, and

$$E(r_A) \geq E(r_B) \quad \underline{and} \quad \sigma_A < \sigma_B$$

No other portfolio B has higher expected return and standard deviation which is smaller or equal.

$$E(r_A) > E(r_B) \quad \underline{and} \quad \sigma_A \leq \sigma_B$$

Investment opportunity set (2 asset case)

- 2 assets both risky C and S

We need to solve

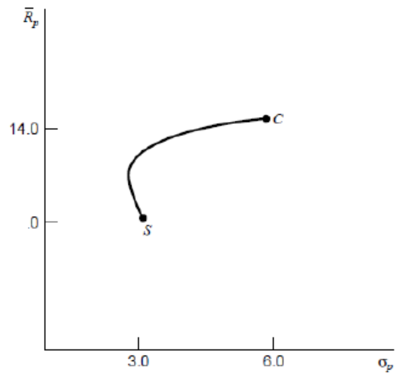
$$\begin{cases} \bar{R}_P = x_C(\bar{R}_C - \bar{R}_S) + \bar{R}_S \\ \sigma_P^2 = x_C^2\sigma_C^2 + (1 - x_C)^2\sigma_S^2 + 2x_C(1 - x_C)\sigma_{CS} \end{cases}$$

- Solution

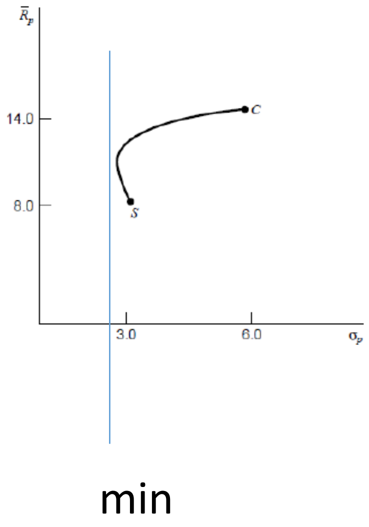
$$x_C = \frac{\bar{R}_P - \bar{R}_S}{\bar{R}_C - \bar{R}_S}.$$

- If one substitutes this back into the expression for variance, one gets the **investment opportunity curve**

$$\sigma_P^2 = \alpha\bar{R}_P^2 + \beta\bar{R}_P + \gamma$$



Minimum variance portfolio



- The variance of all combinations of C and S is given by

$$\sigma_P^2 = x_C^2\sigma_C^2 + (1 - x_C)^2\sigma_S^2 + 2x_C(1 - x_C)\sigma_{CS}$$

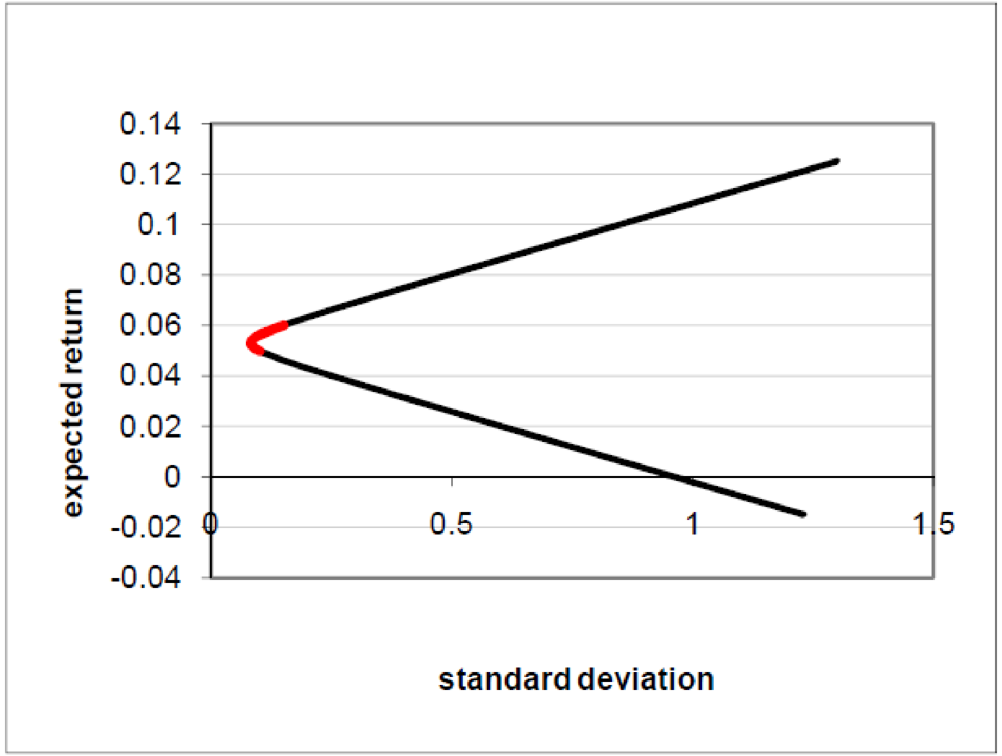
- To find the minimum variance portfolio (MV) we can solve

$$\frac{\partial \sigma_P^2}{\partial x_c} = 0$$

to find the value of x_C^* that gives least variance,

$$x_C^{MV} = \frac{\sigma_S^2 - \sigma_{CS}}{\sigma_C^2 + \sigma_S^2 - 2\sigma_{CS}} = \frac{\sigma_S^2 - \sigma_C\sigma_S\rho_{CS}}{\sigma_C^2 + \sigma_S^2 - 2\sigma_C\sigma_S\rho_{CS}}.$$

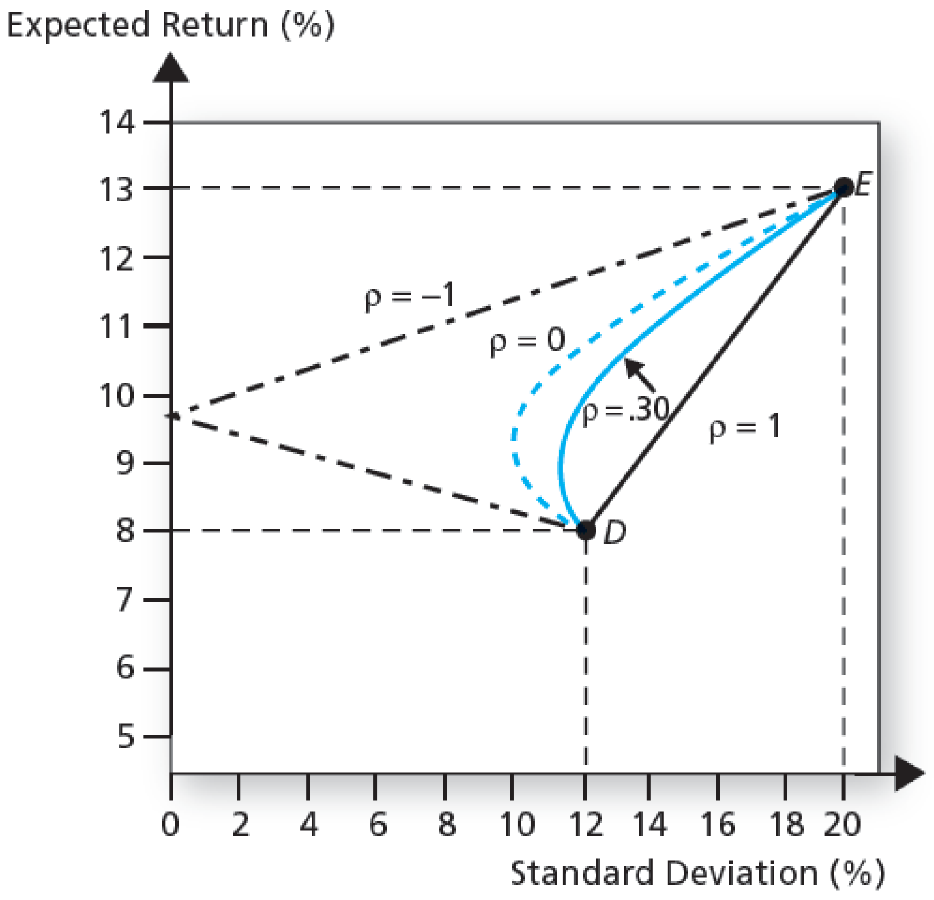
Allowing for shortselling



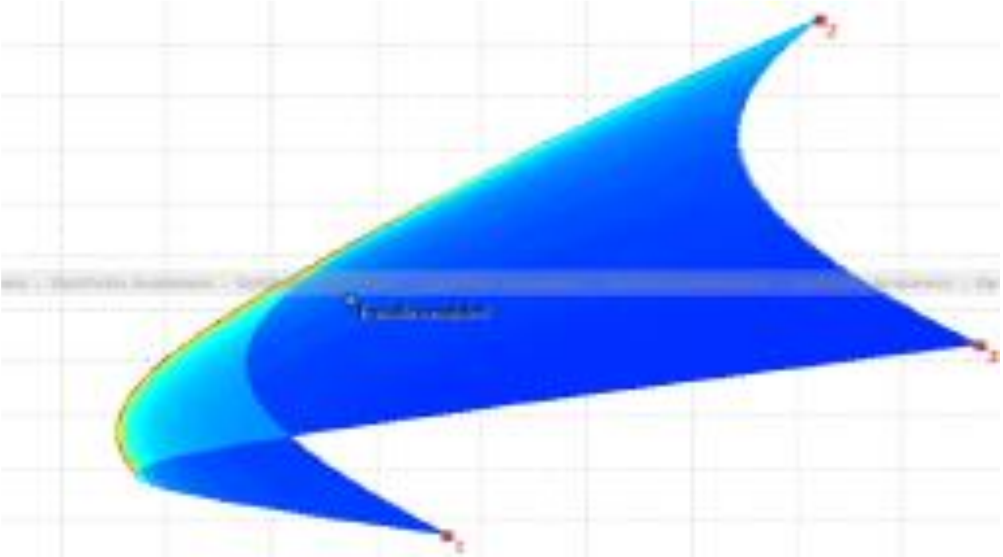
- The red portion is the opportunity curve without shortselling.

2 risky assets –different correlation

Smaller correlation gives larger diversification effect

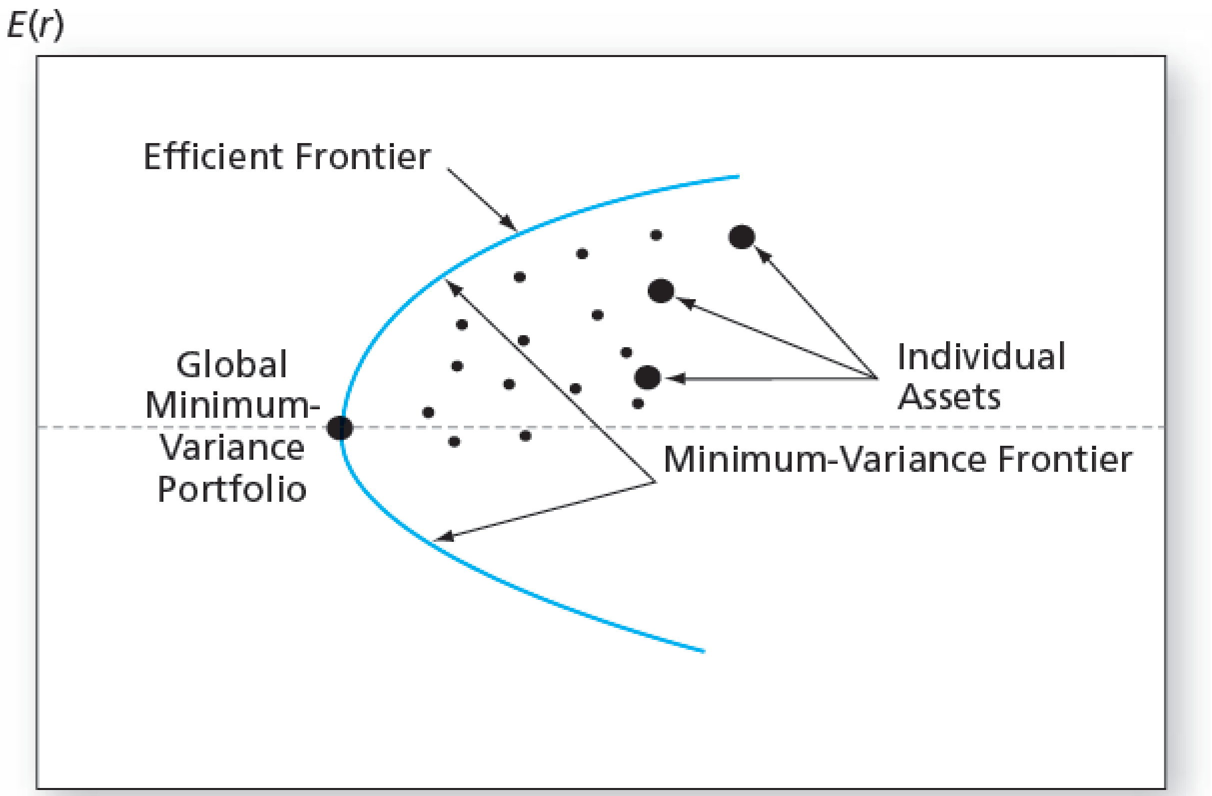


3 risky assets



Several risky assets

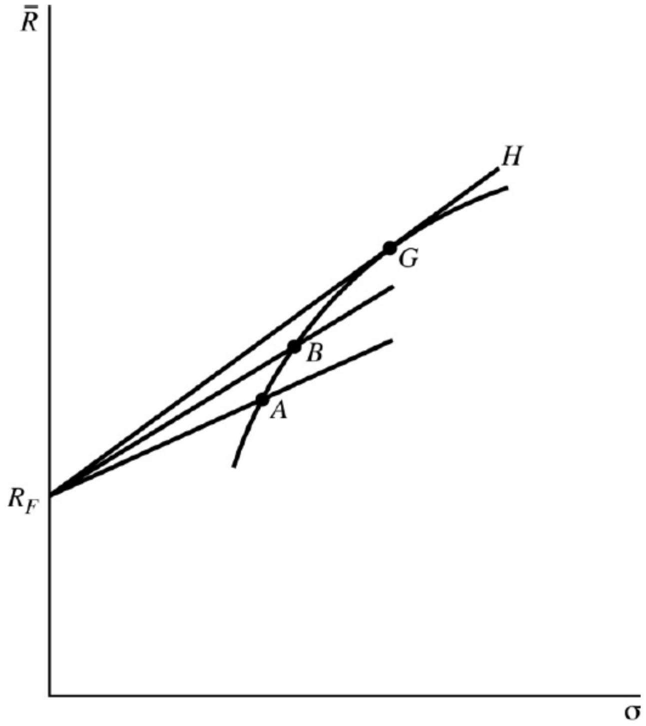
Question: Suppose you are the boss of the stock department. Which risky portfolio should you invest into?



$$\sigma_P^2 = \frac{A\bar{R}_P^2 - 2B\bar{R}_P + C}{AC - B^2}$$ (Unlimited shortselling)

$$A = \mathbb{1}'V^{-1}\mathbb{1} \qquad B = \mathbb{1}'V^{-1}\bar{R} \qquad C = \bar{R}'V^{-1}\bar{R}\ .$$

Which is the optimal risky portfolio?



Question: Which risky portfolio do the board-members prefer A, B or G and why?

Hint: What is the slope of the straight lines

Choose the risky portfolio T

(Scenario1)

Maximize the slope

$$\max_{x_A, x_B} \quad \theta = \frac{\bar{R}_p - R_f}{\sigma_p}$$

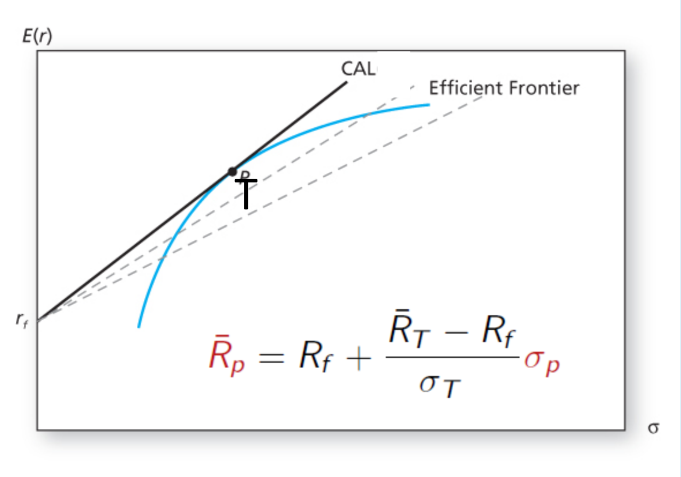


Figure 7.11 The efficient frontier of risky assets with the optimal CAL

$$\bar{R}_p = x_A \bar{R}_A + x_B \bar{R}_B$$

$$\sigma_p = \left(x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{AB}\right)^{\frac{1}{2}}$$

$$x_A + x_B = 1$$

Insert expressions into objective function apply FOC and

define lambda $z_i = \lambda x_i$, for $i = A, B$, to get:

$$\begin{cases} \bar{R}_A - R_f = z_A \sigma_A^2 + z_B \sigma_{AB} & (1) \\ \bar{R}_B - R_f = z_A \sigma_{AB} + z_B \sigma_B^2 & (2) \end{cases}$$

In vector notation

$$\max \quad \theta(X) = \frac{X'\bar{R} - R_f}{(X'VX)^{\frac{1}{2}}} \quad \text{subject to } X'\mathbb{1} = 1 \ .$$

with solution

$$\begin{pmatrix} \bar{R}_A - R_f \\ \bar{R}_B - R_f \end{pmatrix} = \begin{pmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{pmatrix} \begin{pmatrix} z_A \\ z_B \end{pmatrix}$$

$$(\bar{R} - R_f \mathbb{1}) = VZ \quad \Leftrightarrow \quad Z = V^{-1} (\bar{R} - R_f \mathbb{1})$$

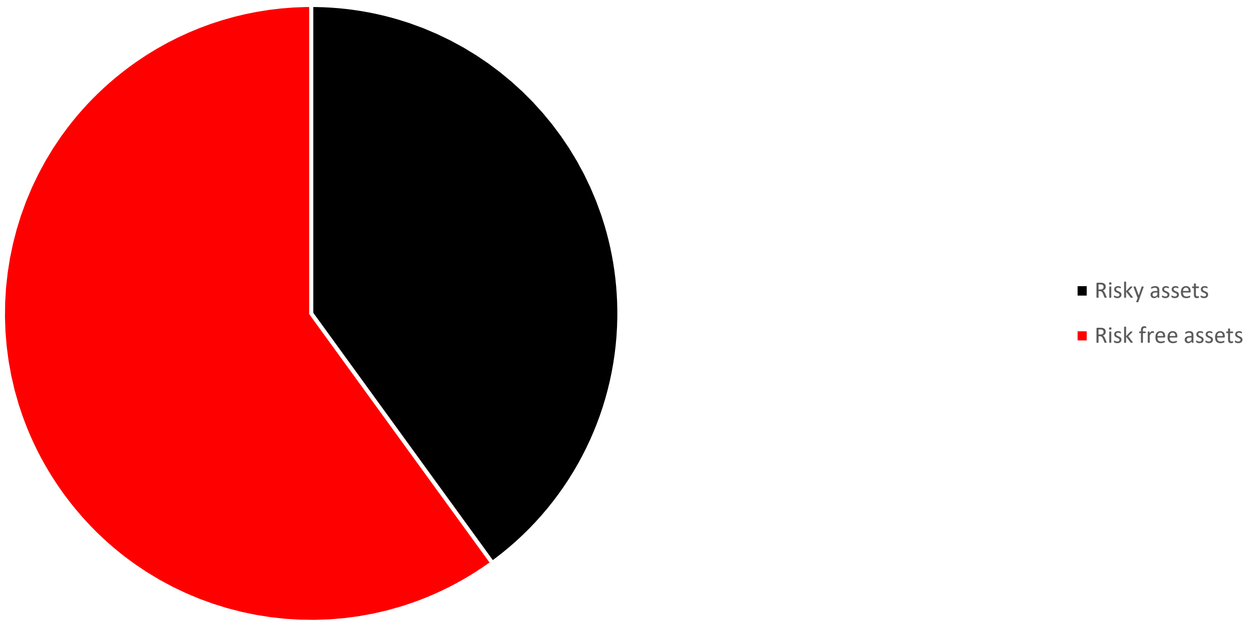
$$x_i = \frac{z_i}{\sum_{j=1}^{n} z_j}$$

Alternatives to find the optimal risky portfolio T

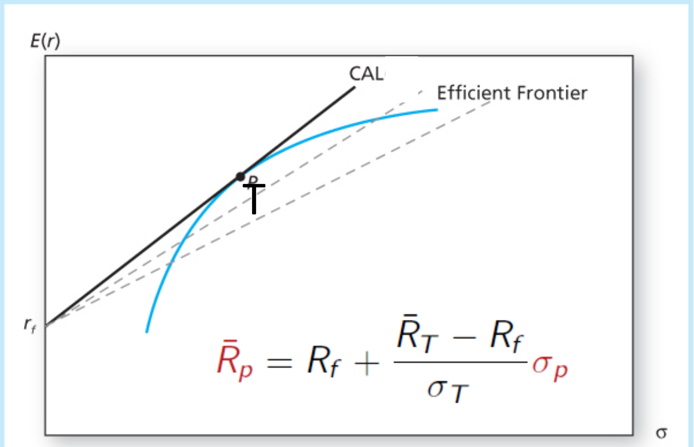
- Ranking method (sharpe) in the constant correlation model
  - Works for both shortselling allowed and not allowed

- Ranking method (Treynor) in the single factor model
  - Works for both shortselling allowed and not allowed

Capital allocation to Risky Assets



The allocation to risky assets



- The board members will get the risky portfolio T
- from the stock-department.

- The board members will now decide how large proportion to put at risk (to put in portfolio T) and how much to put in the risk-free asset F. (i.e. should decide at which point on CAL to invest into)

- This must depend on the risk aversion – more risk avert -> less into the risky asset… need to take investor preferences into account – use utility function!

Utility

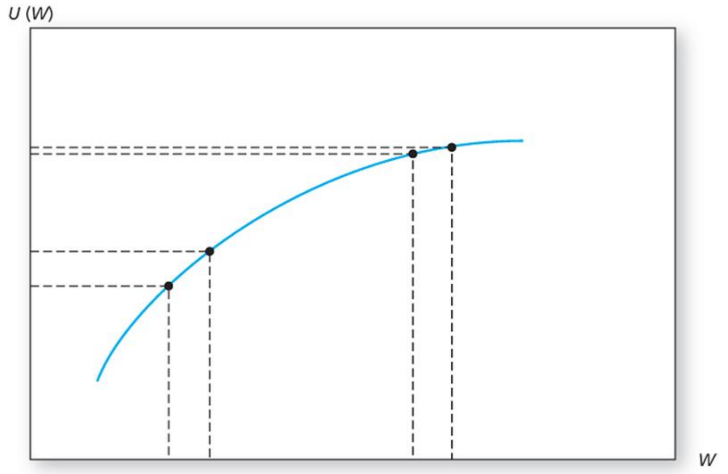
- Utility is the **satisfaction** from wealth (the goods and services that can be bought for the money)

- Risk-averse investors **prefer more** compared to less

- Risk-averse investors prefer more at a **decreasing speed** (decreasing marginal utility) i.e. concave utility

Suppose we have $W_A < W_Y < W_B$, and p is such that

$$(1-p)U(W_A) + pU(W_B) < U(W_Y) \ .$$



Other aspects of utilities

- Preferences
- Rational investors and the 4 axioms
- Equivalent utility functions
- Certainty equivalent
- Indifference pricing
- Utility risk premia
- Absolute risk aversion
- Relative risk aversion
- …

Stating the Asset Allocation problem (general)

Some common utility functions for risk averse investors

- $U(W) = \log(W)$, log utility
- $U(W) = 1 - e^{-W}$, exponential utility
- $U(W) = aW - bW^2$, with $b > 0, W \leq \frac{a}{2b}$, quadratic utility

Can be picture this in the expected-return standard deviation space?

Investors would like to maximize the expected future utility

$$\max_{p} \quad \mathbb{E}\left[U(W)\right]$$
$$\text{s.t.} \quad p \in EF$$

EF=Efficient frontier

Risk Tolerans functions

Risk Tolerance function (RTF)

The RTF $f : (\sigma, \bar{R}) \to \mathbb{R}$ is defined as

$$f(\sigma, \bar{R}) = E(U(W)).$$

RTF indifference curves are the level curves for which

$$f(\sigma, \bar{R}) = K$$

for some fixed expected utility level K.

Theorem

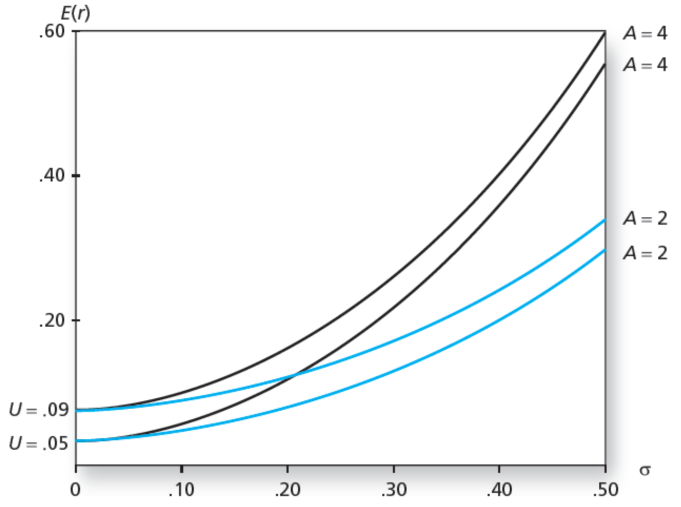
The RTF resulting from a second-order Taylor approximation of a generic utility function U is equivalent to

$$f(\bar{R}, \sigma) = \bar{R} - \frac{1}{2} r_0 \left[\bar{R}^2 + \sigma^2\right] \ ,$$

where r_0 is the coefficient of relative risk aversion evaluated at W_0.

One famous variation is the investors maximizing Markowits mean-variance function:

$$E[R] - \frac{A}{2}\ \sigma^2$$



Stating the Asset Allocation problem (In Markowitz)

Choose the portfolio on the capital allocation line (CAL) that maximize expected utility.

Markowitz example:

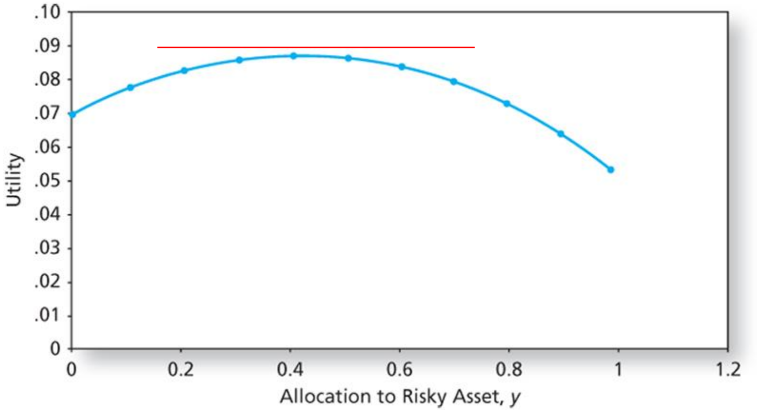
maximize $U=E[r_p] - \frac{1}{2} A \sigma_p^2$

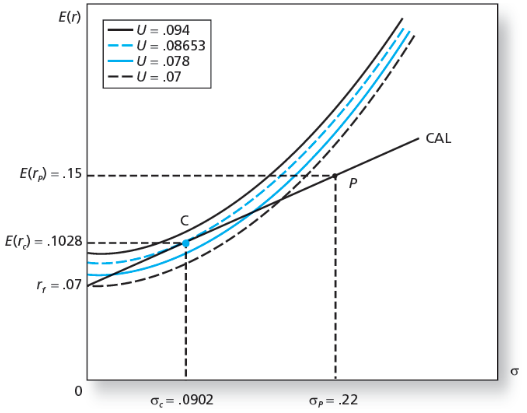
Such that (CAL): $E[r_p]=r_f+x(E[r_T]-r_f)$ and $\sigma_p = x\sigma_T$

Insert restriction into objective fcn, apply foc:

Optimal amount in risky portfolio T is: $x^* = \frac{E[r_p]-rf}{A\sigma_p^2}$

and the rest in the risk free asset.



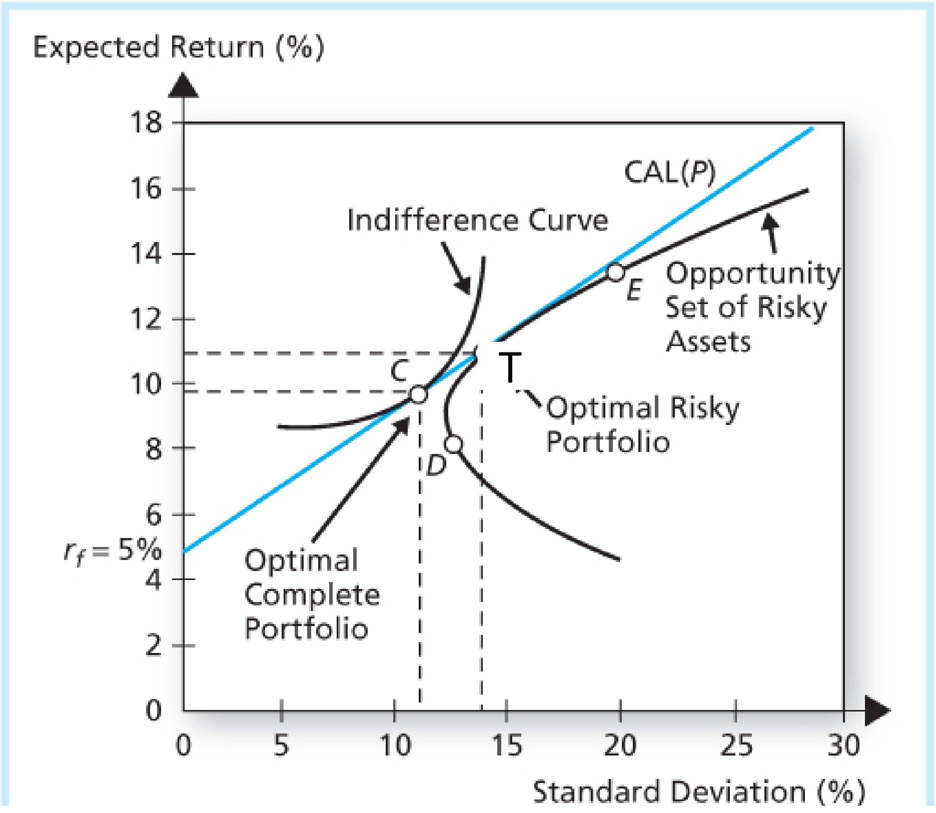


In conclusion

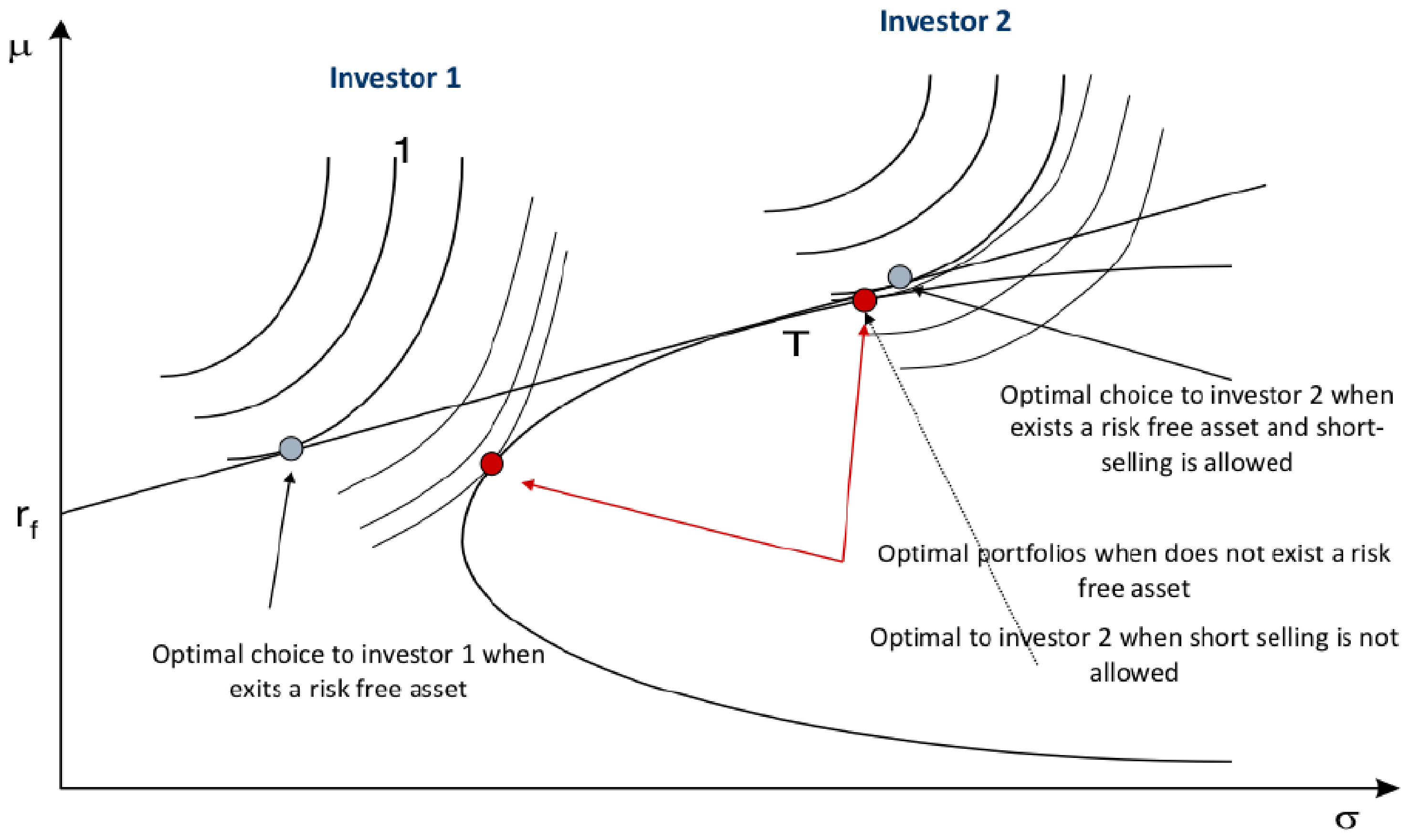
1. Calculate risk-return combinations of several risky assets (efficient frontier)

2. Identify optimal portfolio T of risky assets by picking steepest CAL same for all investors

3. Determine the allocation btw risky and riskfree assets (C)
   depends on investor preferences



Optimal portfolio with and without riskfree



Q1: Which investor is more riskaverse 1 or 2 ?

Q2: If investor one can access risk free asset what is the overall optimal portfolio ?

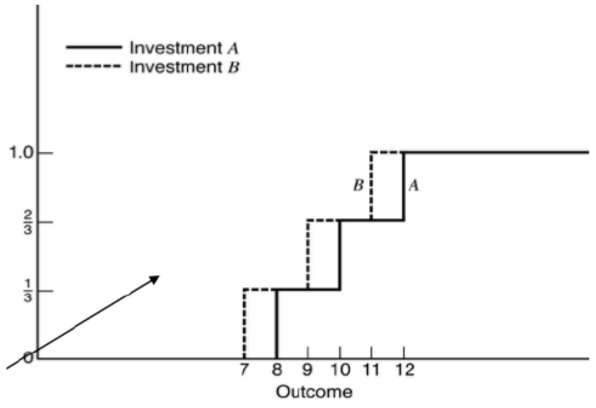
Q3 If investor one cannot access risk free asset what is the overall optimal portfolio ?

Alternatives to utility

- Maximizing long term growth
  - Kelly: geometric mean (log utility) $\mathbb{E}(\log(1+r))$.
  - Samuelsson: arithmetic mean $\mathbb{E}(1+r)$.
- Stochastic dominance
  - First order
  - Second order
  - There are higher orders too

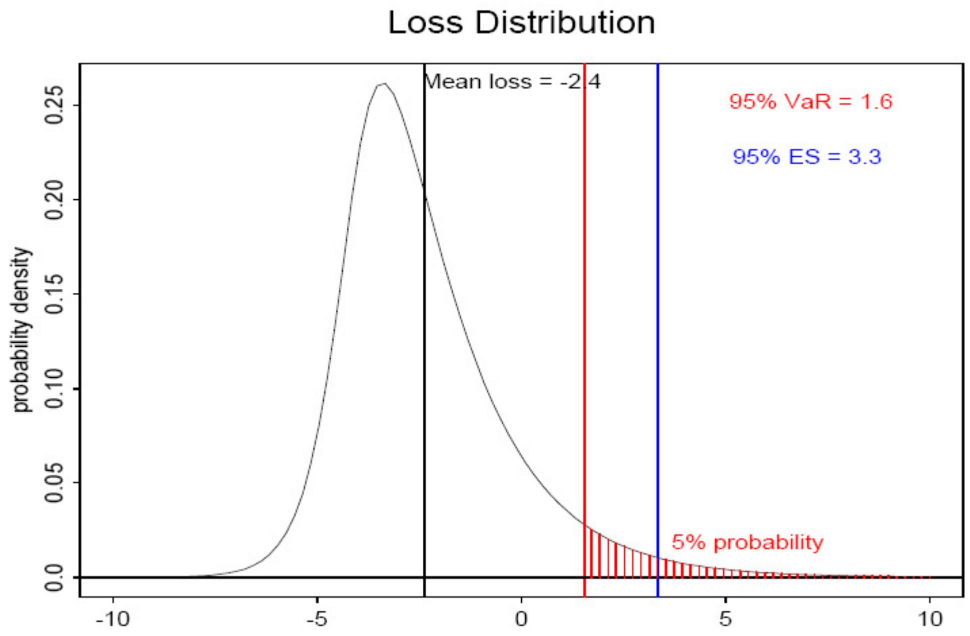
Z has first-order stochastic dominance (FOSD) over X if

$$\Pr(R_X \leq a) \geq \Pr(R_Z \leq a), \quad \text{for all } a,$$
$$\Pr(R_X \leq b) > \Pr(R_Z \leq b), \quad \text{for some } b.$$



Alternative risk measures

- Value at Risk
- Conditional expected shortfall



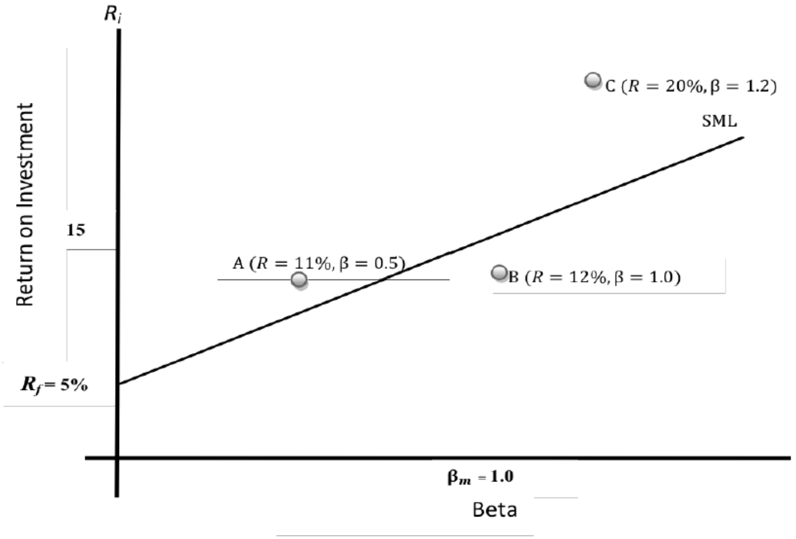
Models of Equilibrium

CAPM

$$\bar{R}_P^e = R_f + \frac{R_M - R_f}{\mathrm{Var}(R_M)} \mathrm{Cov}(R_M, R_P).$$

$$\beta_P = \frac{\mathrm{Cov}(R_P, R_M)}{\mathrm{Var}(R_M)} .$$

Security market line



Alpha = expected return minus CAPM. Asset A and C have positive alpha (good buy)

Assumptions:

1) no transaction costs
2) Assets are divisible
3) No tax effects
4) Not moving the market
5) Mean variance investors
6) Short sales allowed
7) Risk free asset
8) Same time horizon
9) Same input estimates
10) All assets are marketable

CAPM extensions:
no riskfree asset (2 factor)
No borrowing
Different lending/ borrowing rates

Arbitrage Pricing Model (APT)

1 Consider a multi-factor model and take a number of uncorrelated indices I_k which are random, to set

$$R_i = a_i + \sum_{k=1}^{K} b_{ik} I_k + c_i .$$

2 Suppose that, *in equilibrium*, there are no idiosyncratic terms, i.e. $c_i = 0$ for all i.

3 *We can apply the principle of no arbitrage to the returns.*

4 then

$$\bar{R}_i = \lambda_0 + \sum_{k=1}^{K} b_{ik} \lambda_k .$$

where $\lambda_0 = R_f$, and $\lambda_k = \mathbb{E}(I_k) - R_f$ for $k > 0$

Whilst individual securities have residual risk, well-diversified large portfolios will not $\Longrightarrow$ So we can apply the results to large portfolios but not to individual stocks.

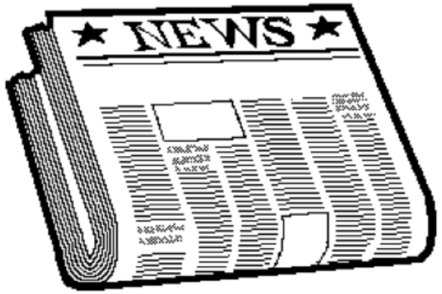
Market Efficiency and Behavioral Finance

Market efficiency

Hypothesis:

Prices of securities fully reflect available information

- Strong – all information, public or private, is already reflected in prices
- Semi-strong – all public information is already reflected in prices.



- Weak – all information in historical prices (trading data) is already reflected in prices.

Anomalies:

- January effect
- Twin shares
- Size effect
- Rebound effect
- Crashes
- Bubbles

Behavioural Finance

Individual biases

- Overconfidence
- Loss Aversion
- Inertia
- (Mis)using info
    - Anchoring
    - Availability bias
    - Representative bias
    - Conservatism bias

Portfolio issues

- Framing
- Mental Accounting
- Naïve diversification
- Home bias

Portfolio Performance Analysis & International investments

Optional

Performance evaluation

Performance measures:

- Sharpe's measure $$\frac{(\overline{r_P} - \overline{r_f})}{\sigma_P}$$
- Treynor's measure $$\frac{(\overline{r_P} - \overline{r_f})}{\beta_P}$$
- Jensen's measure $$\alpha_P = \overline{r_P} - \left[\overline{r_f} + \beta_P(\overline{r_M} - \overline{r_f})\right]$$

Disclaimer

- This revision brings up many but not all of the important concepts and theories we worked with in the course, but of course it is impossible to fit an entire course into 1-2 hours so the course has been on more material than what we covered today.
- This course cover various theoretical and practical aspects of investing, but should not be used as private investment advice. The lecturers are not in any way responsible for any students investments decisions.

At last…

- Bruno and I hope that our lectures and exercises have contributed to your understanding and interest in this topic and hopefully facilitated your own learning from the book.

- Good luck on the exam!

- Reminder: all material is copy right and for your own use- you are not allowed to distribute it!

- Thanks for listening.