Homework 3
April 2024
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Exercise 1

Assume an economy where households have an utility function over two consumptions, current consumption, c, and future consumption, c': $u(c, c')$. Assume that each household has current income y, future income y', pays current taxes t and future taxes t'. To simplify the notation assume $we \equiv y - t + \frac{y-t}{1+r}$.

1. Write down the problem of the consumer.

2. Graph the optimal choice of the consumer.

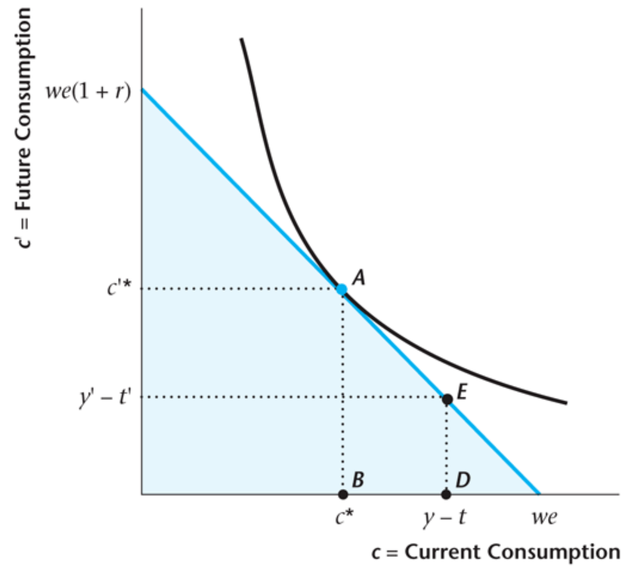
3. Show graphically that a lender is better off when the real interest rate increases. What happens to the lender's savings?

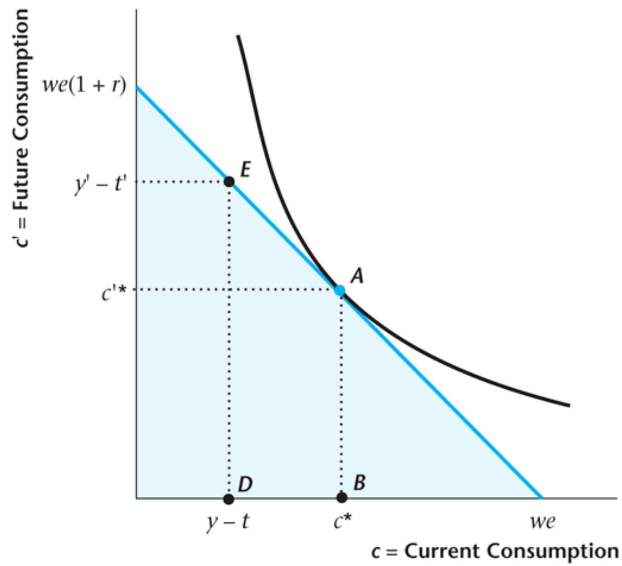
4. Show graphically that a borrower is worse off when the real interest rate increases. What happens to the borrower's savings? Can a borrower become a lender after an interest rate increase?

Solution:

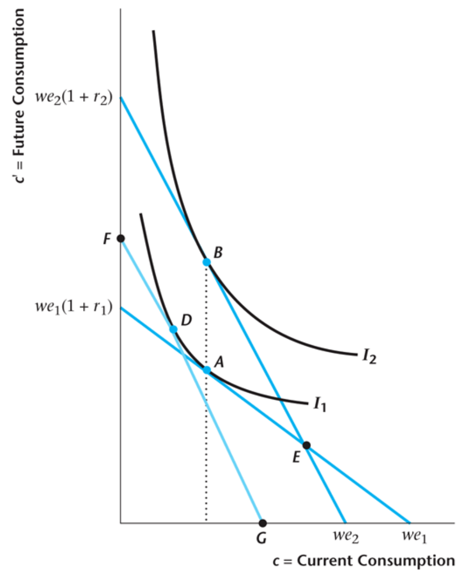
1. $\max_{\{c,c'\}} u(c, c')$ s.t. $c + \frac{c'}{1+r} = we$

2.

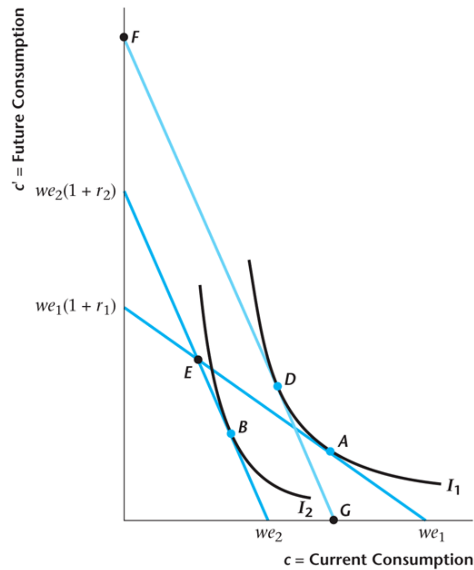




3. An Increase in the Real Interest Rate for a Lender



4. An Increase in the Real Interest Rate for a Borrower



Exercise 2

Consider an economy which lasts for 2 periods. Suppose the representative agent obtains utility from consumption C and suffers disutility from supplying labor N. The representative agent chooses $C_1; C_2; N_1; N_2$ to maximize lifetime utility

$$U(C_1, C_2, N_1, N_2) = \log(C_1) - \frac{(N_1)^a}{a} + \beta \left(\log(C_2) - \frac{(N_2)^a}{a} \right), \; a > 0$$

subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r} = \frac{W_1}{P_1} N_1 + \frac{W_2}{P_2} \frac{N_2}{1+r},$$

where C_1 and C_2 are consumption in period 1 and 2 in real terms, W_1/P_1 and W_2/P_2 are real wages in period 1 and 2, N_1 and N_2 are labor supply in period 1 and 2, r is the real interest rate and β is the discount factor.

1. Set up the Lagrangian for the problem of the representative household and derive the intertemporal Euler equation for consumption, as well as the two optimal conditions for the choice of labor in period 1 and 2. Provide the economic intuition for these conditions in terms of equality of marginal costs and benefits.

2. Suppose that agents have a constant consumption path, but the monetary authorities aim at generating a growing consumption path (that is at decreasing current consumption relative to future consumption). What actions can the monetary authorities take? Briefly explain.

3. Does monetary policy affects the optimal supply of labor at given wages? Briefly explain.

Exercise 3

Consider the following economy. There is a large number of identical firms, except for the fact that a fraction s of them has sticky prices, while the remaining fraction $1-s$ sets prices flexibly. Firm i produces output $Y_t(i)$ using labor $N_t(i)$ according to the following technology

$$Y_t(i) = A_t N_t(i),$$

where A_t denotes labor productivity. Firms are monopolistically competitive and face the following demand for their own product

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \ \epsilon > 1$$

where p_t is the firm's individual price level, P_t is the aggregate/average price level and Y_t is aggregate demand. Let W_t denote the aggregate/average nominal wage in the economy. Firms set the price for their product $P_t(i)$ to maximize profits subject to technology and demand for their product.

1. State the firm's maximization problem and solve for the optimal pricing decision of the individual firms. Give the economic intuition for the optimal price explaining that it gives prices as a markup over nominal marginal cost.

2. Using the optimal condition for labor supply in the previous Exercise 2, show that the real wage in the economy can be written as

$$\frac{W_t}{P_t} = \frac{(Y_t)^a}{(A_t)^{a-1}}$$

3. Use the expression for the real wage to rewrite the firm's optimal pricing decision as a function of the markup, aggregate output, labor productivity and the aggregate price level. Then, derive the expression for the natural level of output. Finally, use the expression for natural output to obtain the firm's optimal pricing decision as a function of aggregate output, natural output and the aggregate price level.

4. Write the loglinear version of the final expression for the desired price found in the previous point and check that it corresponds to the expression in Mankiw's chapter. Then, characterize the (loglinear) price set by flexible price firms and by sticky price firms. Finally, aggregate those prices to derive the (loglinear) expression for the aggregate price level. When doing so, for simplicity set $Y_t^e = Y_t^{n,e}$.

5. Derive the Phillips curve and explain the intuition behind the relation of deviation of output from the natural level and deviation of actual prices from expected prices.