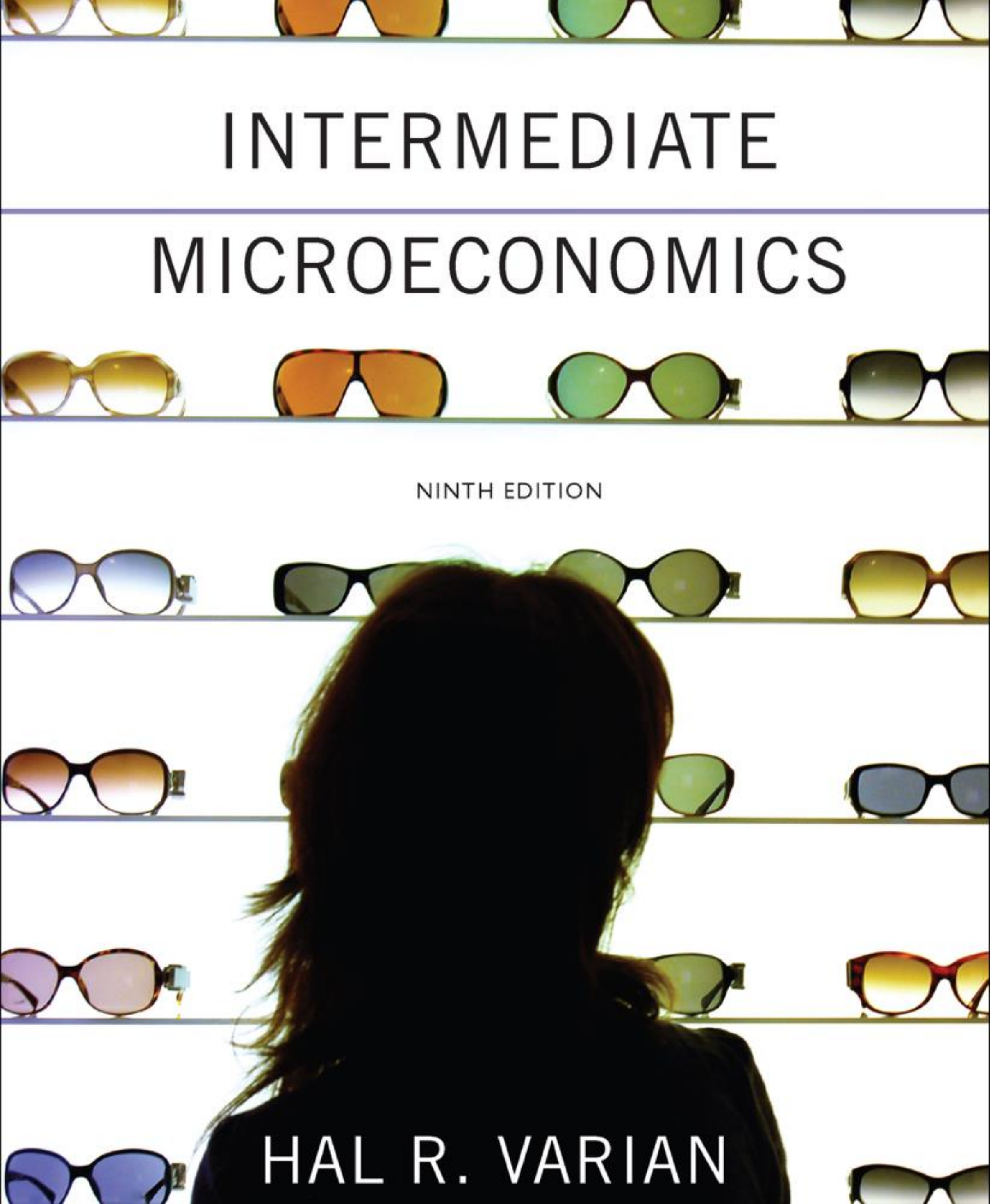


CHAPTER 12

Uncertainty

Uncertainty is Pervasive

What is uncertain in economic systems?

- future health
- future employment
- future wealth
- economic growth or recession

In general, there are different **states of nature** that are (partly) influenced by random events. For instance, for those that have a car, the two states of nature are: car accident versus no car accident.

Uncertainty is Pervasive

What are rational responses to uncertainty?

- buying insurance (e.g., a car insurance)

You can think of an insurance as a **contingent consumption plan**: a specification of what will be consumed in each different state of nature. Contingent means depending on something not yet certain.

In case of a car insurance; how much can be consumed if you get an accident versus if you do not.

Contingent Consumption Plan

Making a **contingent consumption plan** is like choosing an ordinary consumption bundle:

- Instead of deciding between two ordinary goods, you decide on the amount of consumption in the two states of nature.
- Insurance allows you to transfer consumption from one state to the other.

Contingent Consumption Plan

How to find the “optimal” contingent consumption plan?

We can use the framework analyzed in Microeconomics I. That is, subject to a budget constraint, find the consumption plan that maximizes utility:

- Budget constraint: The possible amounts of consumption in each state of nature, which is determined by the amount of insurance.
- Consumption plan: Amount of consumption in each state of nature.
- Utility: Consumers have different preferences over states of nature (risk averse or loving) as they have different preferences over ordinary goods.

Finding the optimal plan implies finding the optimal amount of insurance.

States of Nature: An Example

Two possible states of nature:

- car accident (a)
- no car accident (na)

An accident occurs with probability π_a and does not occur with probability $\pi_{na} = (1 - \pi_a)$.

An accident causes a loss of $L.

Insurance: An Example

Each $1 of accident insurance costs γ

- Hence, γ is the **insurance premium**.
- Total insurance paid to fully cover the loss is γL.

Consumer has $\$m$ of initial wealth.

Consumption in the no-accident state is denoted by C_{na}.

Consumption in the accident state is denoted by C_a.

Budget Constraint: An Example

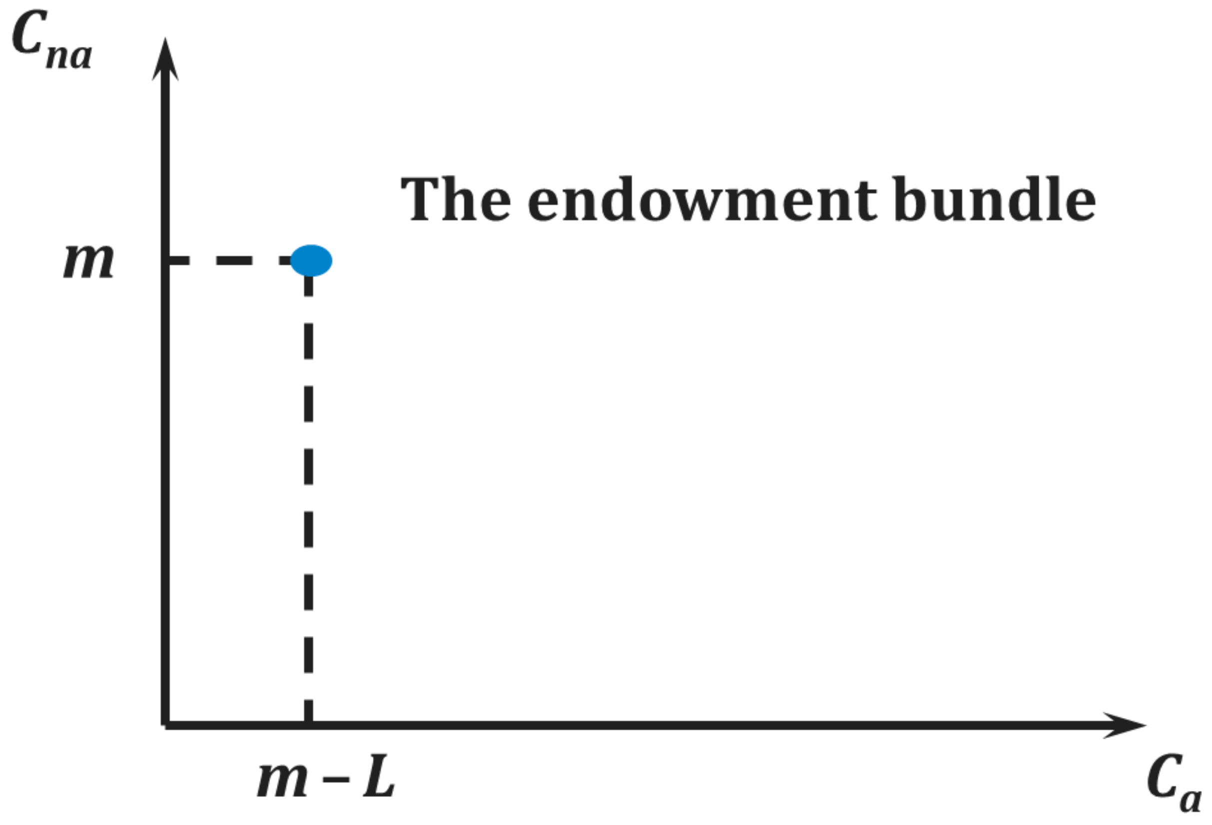
Without insurance, we know that:

$$C_{na} = m$$

$$C_a = m - L$$

This is referred to as the **endowment bundle.**

Budget Constraint: An Example



Budget Constraint: An Example

Buy $\$K$ of accident insurance, then we know that:

$$C_{na} = m - \gamma K$$

$$C_a = m - L - \gamma K + K = m - L + (1 - \gamma)K$$

With full insurance $\$K = L$, we know that:

$$c_{na} = m - \gamma L$$
$$c_a = m - \gamma L$$

Hence, with full insurance we consume the same amount $c_{na} = c_a$ in each state of nature. That is, we consume $c_{na} = c_a$ for sure.

Budget Constraint: An Example

Buy $\$K$ of accident insurance, then we know that:

$$C_{na} = m - \gamma K$$

$$C_a = m - L - \gamma K + K = m - L + (1 - \gamma)K$$

So, $K = (C_a - m + L)/(1 - \gamma)$.

We can now express C_{na} as function of C_a by substituting for K and slightly rewrite:

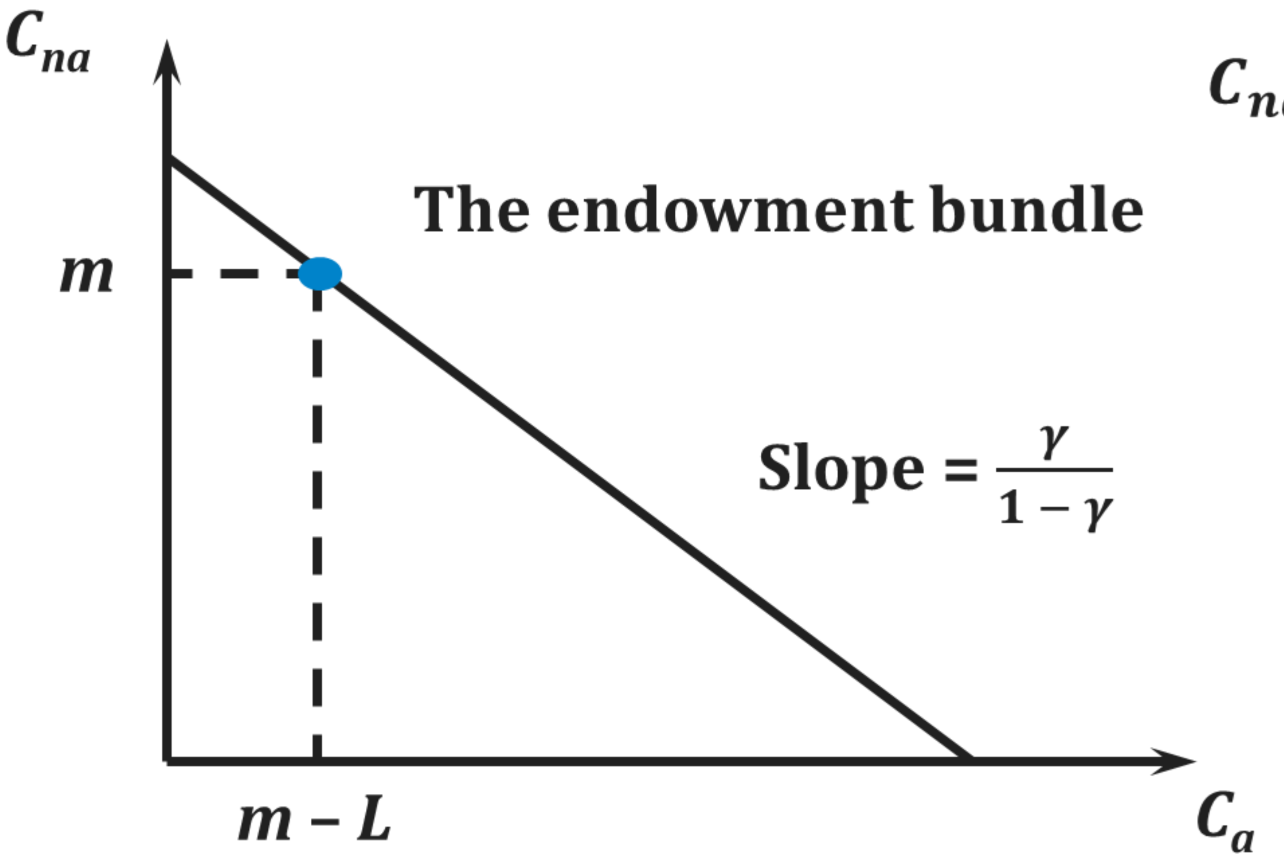
$$C_{na} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_a.$$

Budget Constraint: An Example

Now we have the **budget constraint**: for each consumption level in the accident state of nature c_a we know how much one can consume in the no-accident state of nature c_{na}:

$$c_{na} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} c_a$$

Budget Constraint: An Example



$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$

Preferences under Uncertainty

Now that we understand the budget constraint, we are one step closer to finding the optimal consumption plan (c_{na}^*, c_a^*), and hence the optimal amount of insurance.

To find this plan, we also need to understand preferences under uncertainty.

In general, the utility function will depend upon the levels of consumption c_{na} and c_a and the probabilities of each state of nature π_{na} and π_a:

$$u = u(c_{na}, c_a, \pi_{na}, \pi_a)$$

Expected Utility

A convenient form of the utility function under uncertainty is **expected utility**:

$$u(c_{na}, c_a, \pi_{na}, \pi_a) = \pi_{na} u(c_{na}) + \pi_a u(c_a)$$

Expected utility in the sense that it weighs the utility under each state of nature $u(.)$ by its corresponding probability π.

Is expected utility a reasonable description of consumer preferences?

Preferences under Uncertainty: An Example

No accident: $\pi_{na} = \frac{1}{2}$ and $c_{na} = \$90$.

Accident: $\pi_{a} = \frac{1}{2}$ and $c_{a} = \$0$.

Consider that $u(\$90) = 12$, $u(\$0) = 2$.

Expected utility EU is (without insurance):

$$EU = \frac{1}{2}u(\$90) + \frac{1}{2}u(\$0) = \frac{1}{2}(12) + \frac{1}{2}(2) = 7.$$

Preferences under Uncertainty: An Example

No accident: $\pi_{na} = \frac{1}{2}$ and c_{na}= \$90.

Accident: $\pi_a = \frac{1}{2}$ and c_a= \$0.

Consider that $u(\$90) = 12$, $u(\$0) = 2$.

Expected money EM value is:

$$EM = \frac{1}{2}(\$90) + \frac{1}{2}(\$0) = 45.$$

Preferences under Uncertainty

$EU=7$ and $EM=45$. But what about $u(EM)$?

$u(\mathrm{EM}) > \mathrm{EU}$

$\Rightarrow$ Utility of expected wealth is higher than expected utility of wealth.

$\Rightarrow$ **risk aversion**: prefer the certain $45 above the “lottery”.

$u(\mathrm{EM}) < \mathrm{EU}$

$\Rightarrow$ Utility of expected wealth is lower than expected utility of wealth.

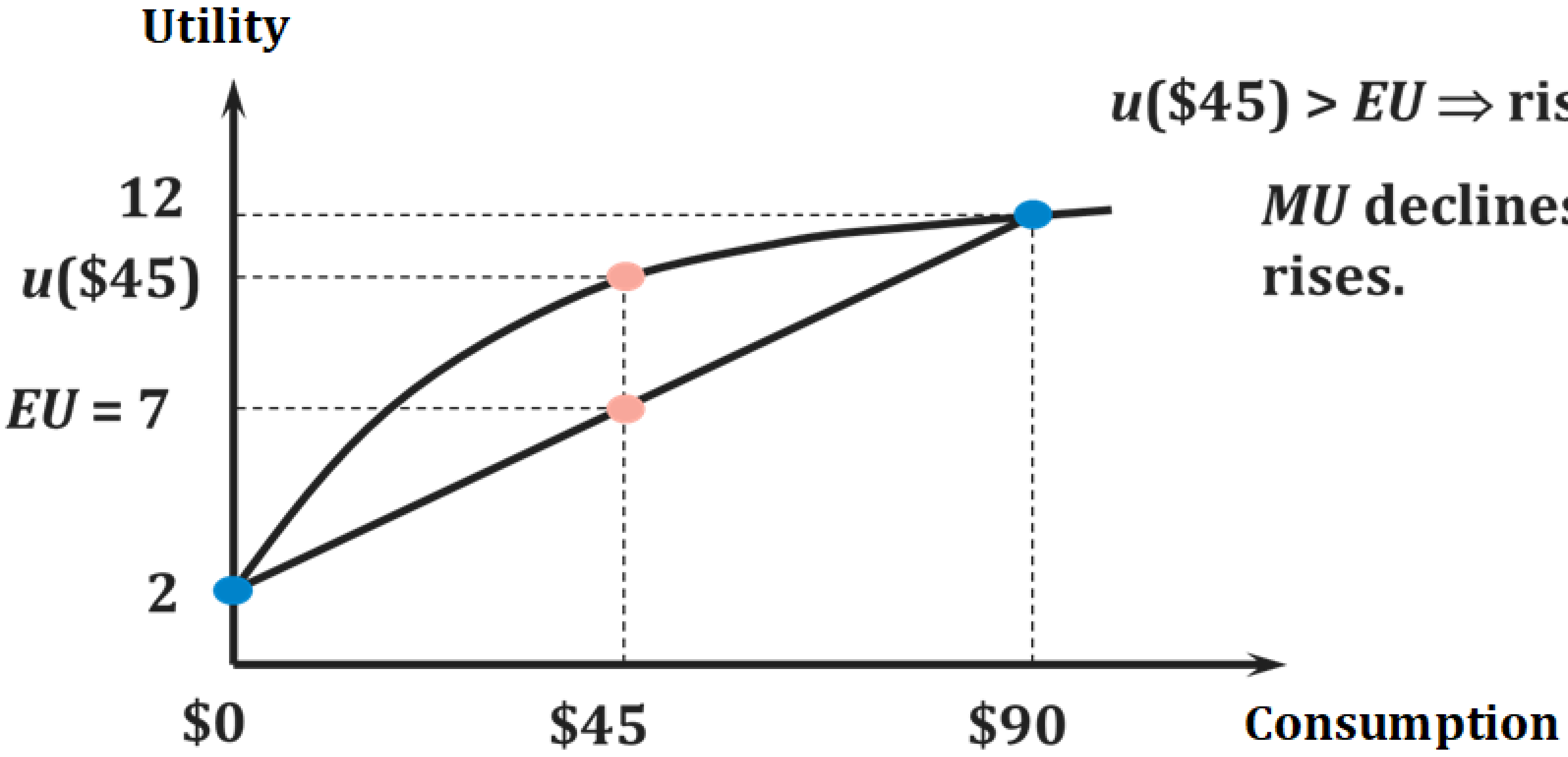
$\Rightarrow$ **risk loving**: prefer the “lottery” above the certain $45.

$u(\mathrm{EM}) = \mathrm{EU}$

$\Rightarrow$ Utility of expected wealth is equal to expected utility of wealth.

$\Rightarrow$ **risk neutrality**: indifferent.

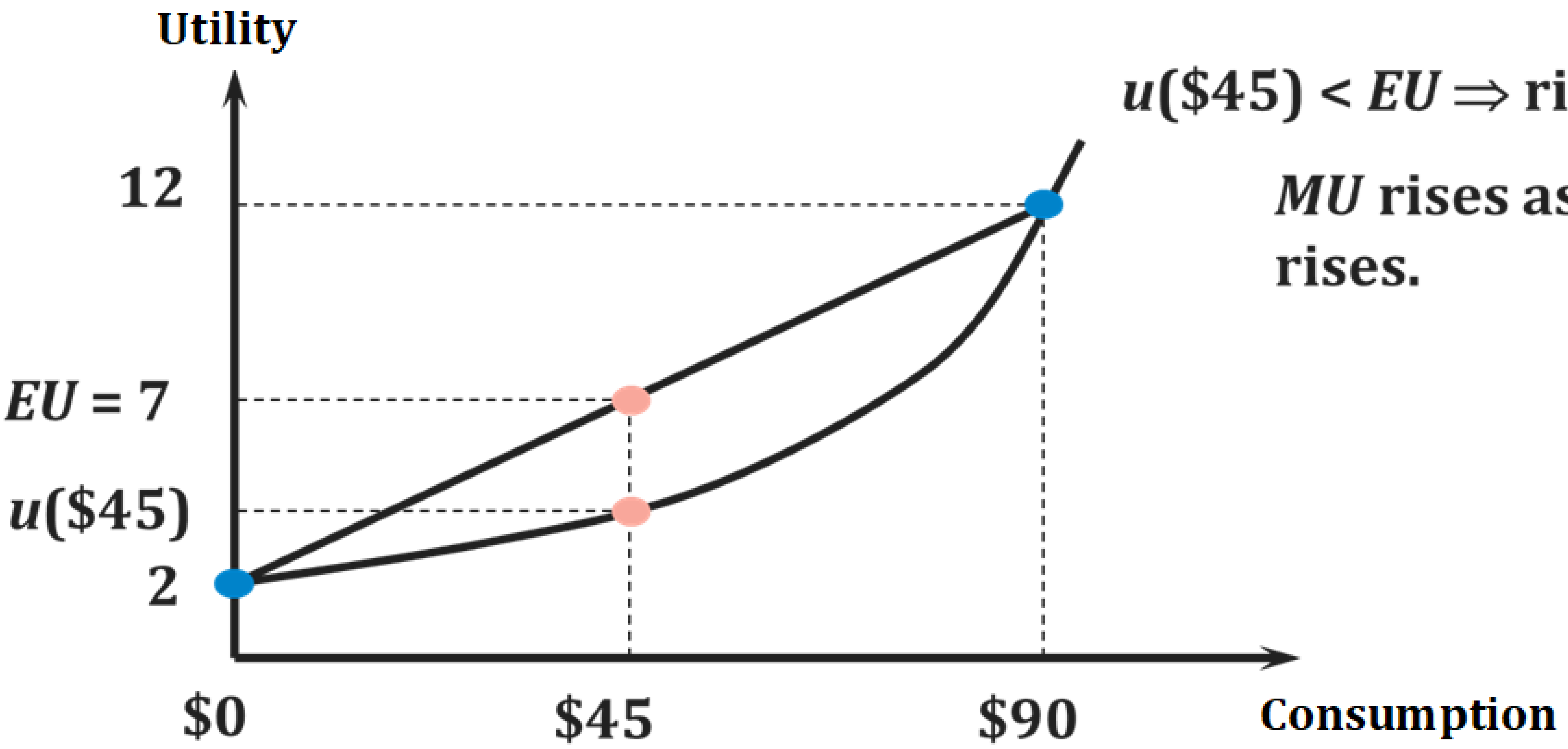
Risk Aversion: Utility Function



$u(\$45) > EU \Rightarrow$ **risk aversion.**

***MU* declines as wealth rises.**

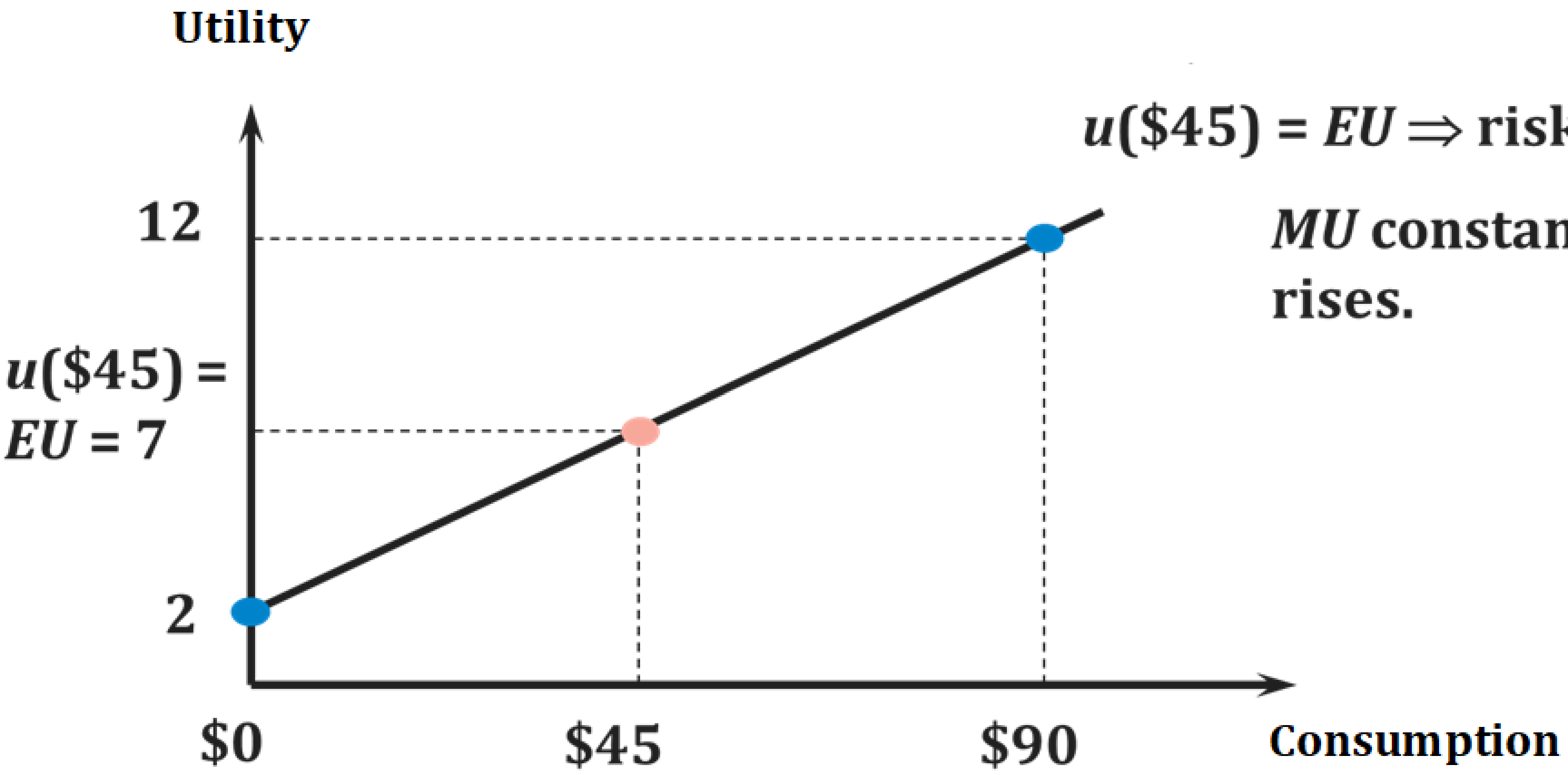
Risk Loving: Utility Function



$u(\$45) < EU \Rightarrow$ risk loving.

MU rises as wealth rises.

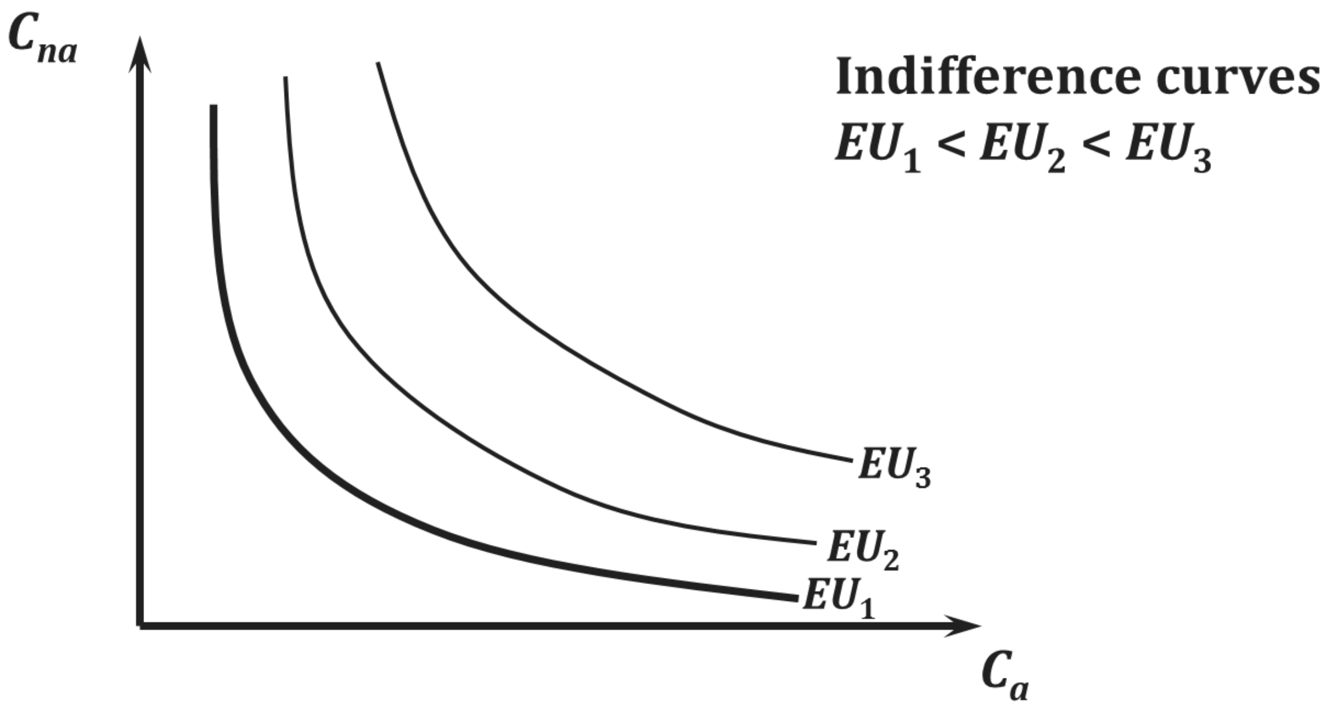
Risk Neutral: Utility Function



$u(\$45) = EU \Rightarrow$ risk neutrality.

MU constant as wealth rises.

Preferences under Uncertainty: Indifference Curve



An indifference curve $c_{na}(c_a)$ is a function that represents all consumption plans c_{na} and c_a that give the same level of expected utility.

Indifference Curve and MRS

Indifference curve: The function $c_{na}(c_a)$ that traces all consumption plans that give the consumer the same expected utility (i.e., that makes the consumer indifferent).

Marginal Rate of Substitution (MRS): If c_a increases, how much should c_{na} decrease as to keep expected utility constant. In mathematics, the MRS is the slope of the indifference curve:

$$MRS = \frac{\partial c_{na}(c_a)}{\partial c_a}$$

Find a useful Expression for MRS

Using the indifference curve for EU_1, we can plug $c_{na} = c_{na}(c_a)$ into the expected utility function and write:

$$EU_1 = \pi_{na} u\big(c_{na}(c_a)\big) + \pi_a u(c_a)$$

And since EU_1 is a fixed number, we know that:

$$\frac{\partial EU_1}{\partial c_a} = 0$$

Let's find that derivative via the chainrule:

$$\frac{\partial EU_1}{\partial c_a} = \pi_{na} \frac{\partial u(c_{na})}{\partial c_{na}} \frac{\partial c_{na}(c_a)}{\partial c_a} + \pi_a \frac{\partial u(c_a)}{\partial c_a} = 0$$

Find a useful Expression for MRS

Note that $MU_x = \frac{\partial u(c_x)}{\partial c_x}$, so that we can write:

$$\frac{\partial EU_1}{\partial c_a} = \pi_{na} MU_{na} \frac{\partial c_{na}(c_a)}{\partial c_a} + \pi_a MU_a = 0$$

Then we can write the MRS as follows:

$$MRS = \frac{\partial c_{na}(c_a)}{\partial c_a} = -\frac{\pi_a MU_a}{\pi_{na} MU_{na}}$$

Interpreting the MRS

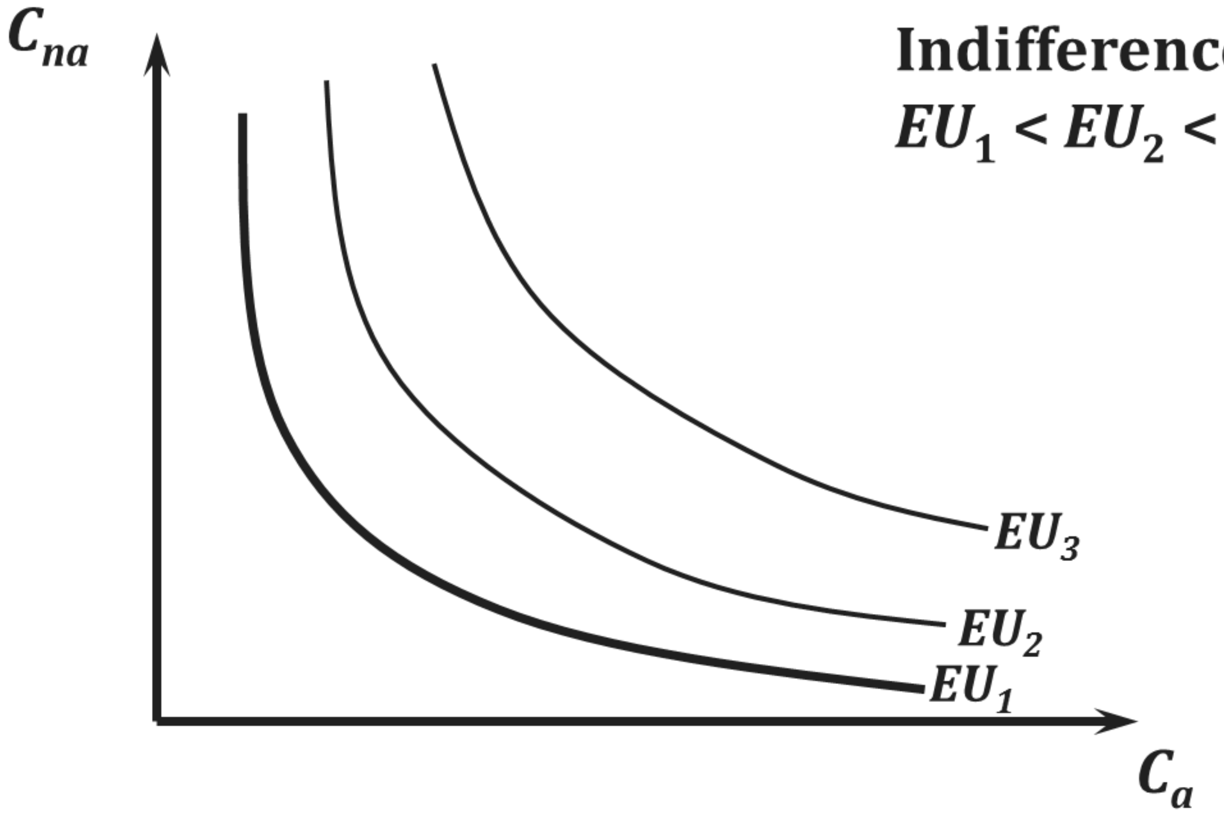
We derived that:

$$MRS = \frac{\partial c_{na}(c_a)}{\partial c_a} = -\frac{\pi_a MU_a}{\pi_{na} MU_{na}}$$

Hence, if $\pi_a MU_a$ is relatively big, then the MRS is strongly negative.

This makes sense: if $\pi_a MU_a$ is relatively big, then if c_a increases, expected utility increases by a lot, and so c_{na} should drop by a lot as to keep expected utility constant.

The Marginal Rate of Substitution (MRS)



Indifference curves

$EU_1 < EU_2 < EU_3$

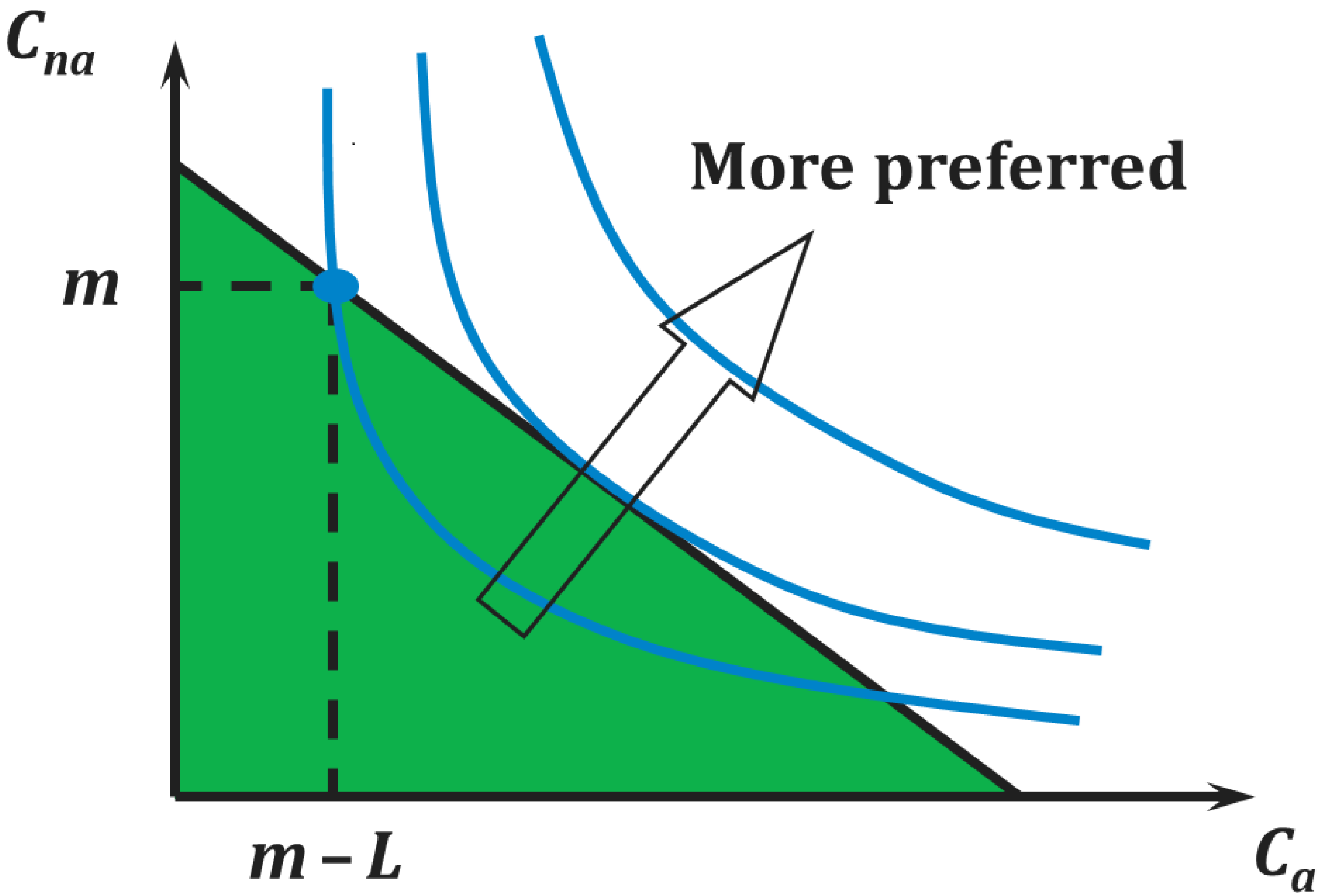
$$\frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

Optimal Consumption Plan

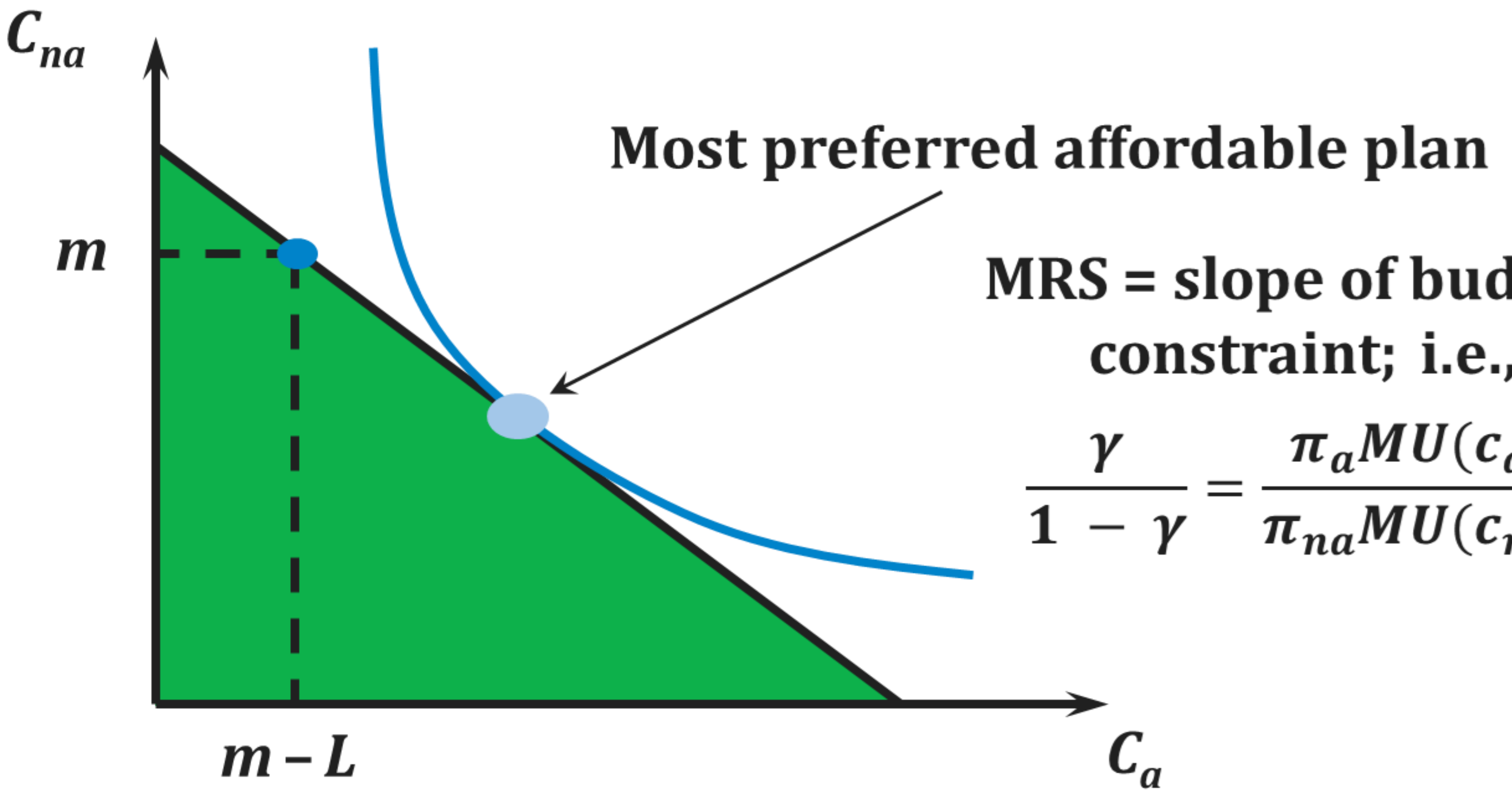
Now that we understand the budget constraint and utility function, we can find the optimal contingent consumption plan (c_{na}^*, c_a^*):

- Subject to the budget constraint, find the consumption plan that maximizes utility.
- This implies finding the optimal amount of insurance.

Optimal Consumption Plan



Optimal Consumption Plan



MRS = slope of budget constraint; i.e.,

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

Fair Insurance Premium

Suppose that entry to the insurance industry is free.

Then profits of the insurer should be approximately zero.

In other words, from a \$$K$ insurance, the profit of the insurer should be

$\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K = 0$. Hence, free entry ensures that $\gamma = \pi_a$.

If $\gamma = \pi_a$ (i.e., price of \$1 insurance = accident probability), the insurance premium is referred to as **actuarially fair**.

Fair Insurance Premium and Optimal Consumption Plan

When insurance is actuarially fair ($\gamma = \pi_a$), the optimal consumption plan and hence optimal amount of insurance satisfies:

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a}{\pi_{na}} = \frac{\pi_a MU_a}{\pi_{na} MU_{na}}$$

Hence, marginal utility of consumption must be the same in both states:

$$MU_a = MU_{na}$$

Fair Insurance Premium and Risk Aversion

With a fair premium, optimal contingent consumption plan implies:

$$MU_a = MU_{na}$$

If consumer is risk averse, then the above condition holds if consumption is the same in both states of nature:

$MU_a = MU_{na}$ if $c_a^* = c_{na}^*$.

Therefore, a risk-averse consumer optimally buys full insurance $\$K = L$, since only then we have that $c_a^* = c_{na}^* = m - \gamma L$.

Fair Insurance Premium and Risk Aversion

Risk-averse consumer buys full insurance: $c_a^* = c_{na}^*$.

With full insurance against a fair premium, the wealth is for sure:
$c_a^* = c_{na}^* = \ m - \gamma L = m - \pi_a L$

And expected wealth equal is to:
$EM = (1 - \pi_a)m + \pi_a(m - L) = m - \pi_a L$

Indeed, with a fair insurance premium one can obtain expected wealth for sure by fully insuring, and a risk-averse consumer prefers utility of expected wealth over expected utility of wealth. Hence, she fully insures. In other words, a risk-averse consumer will never take a fair bet.

Fair Insurance Premium and Risk Neutrality

With a fair premium, optimal contingent consumption plan implies:

$$MU_a = MU_{na}$$

If consumer is risk neutral, then the above condition holds for any levels of consumption:

$MU_a = MU_{na}$ for any level of c_a and c_{na} , since $MU_a = MU_{na}$ always holds with *1 = 1.*

Therefore, a risk-neutral consumer is indifferent towards her level of insurance.

Fair Insurance Premium and Risk Neutrality

Risk-neutral consumer is indifferent between any level of insurance.

Indeed, with a fair insurance premium one can obtain expected wealth for sure by fully insuring. However, one can also not insure and let the “lottery” determine outcomes. A risk-neutral consumer is indifferent between the utility of expected wealth and expected utility of wealth. In other words, a risk-neutral consumer is indifferent between taking a fair bet or not.

Unfair Insurance Premium

Now suppose that the insurer can make a positive profit.

In other words,

$$\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0.$$

Then:

$$\Rightarrow \gamma > \pi_a$$

$$\Rightarrow \frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$$

Unfair Insurance Premium and Risk Aversion

Recall that the optimal consumption plan and hence optimal amount of insurance satisfies:

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}.$$

Since $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $MU(c_a) > MU(c_{na})$.

Hence, with unfair insurance, $c_a^* < c_{na}^*$ for a risk averse consumer. This implies that a risk averse consumer buys less than full unfair insurance.

Diversification

Two firms, A and B.

Shares of both firms cost \$10.

With $\pi_A = 0.5$, A's return (per share) is \$100 and B's return is \$20.

With $1 - \pi_A = 0.5$, A's return (per share) is \$20 and B's return is \$100.

You have \$100 to invest. How?

Diversification

Buy only firm A's stock?

$100/10 = 10 shares.

You earn $1,000 with probability 0.5 and $200 with probability 0.5.

Expected earnings = 0.5 * $1000 + 0.5 * $200 = $600.

Diversification

Buy only firm B's stock?

$100/10 = 10 shares.

You earn $200 with probability 0.5 and $1,000 with probability 0.5.

Expected earnings = 0.5 * $200 + 0.5 * $1000 = $600.

Diversification

Buy 5 shares in each firm?

You earn \$600 with probability 0.5 and \$600 with probability 0.5.

Expected earnings = \$600.

But you earn this \$600 for sure.

Diversification has maintained expected earnings and lowered risk. If risk-averse, you would prefer to diversify:

$$u(\$600) > 0.5 * u(\$1{,}000) + 0.5 * u(\$200)$$

However, typically diversification lowers expected earnings in exchange for lowered risk.

Risk Spreading via Insurance

1,000 persons with initial wealth \$40,000.

Each independently risk a \$10,000 loss.

Loss probability = 0.01.

With no insurance, expected individual wealth is:

$$(0.99)(\$40,000) + (0.01)(\$30,000) = \$39,900.$$

Expected total loss is:

$$(0.01)(\$10,000)(1,000) = \$100,000$$

Risk Spreading via Insurance

Insurance: all 1,000 persons pay a fair premium of $100 into a mutual insurance fund.

The fund has $100,000, which exactly covers the total expected loss.

Individual wealth is $40,000 – $100 = $39,900 for sure, which is equal to expected individual wealth. Each person **spreads** her **risk** over the others.

Insurance with a fair premium is beneficial if risk averse: individual wealth is $39,900 for sure, which is equal to expected wealth. And since risk averse, we know that:

$$u(\$39{,}900) > 0.99 * u(\$40{,}000) + 0.01 * u(\$30{,}000)$$