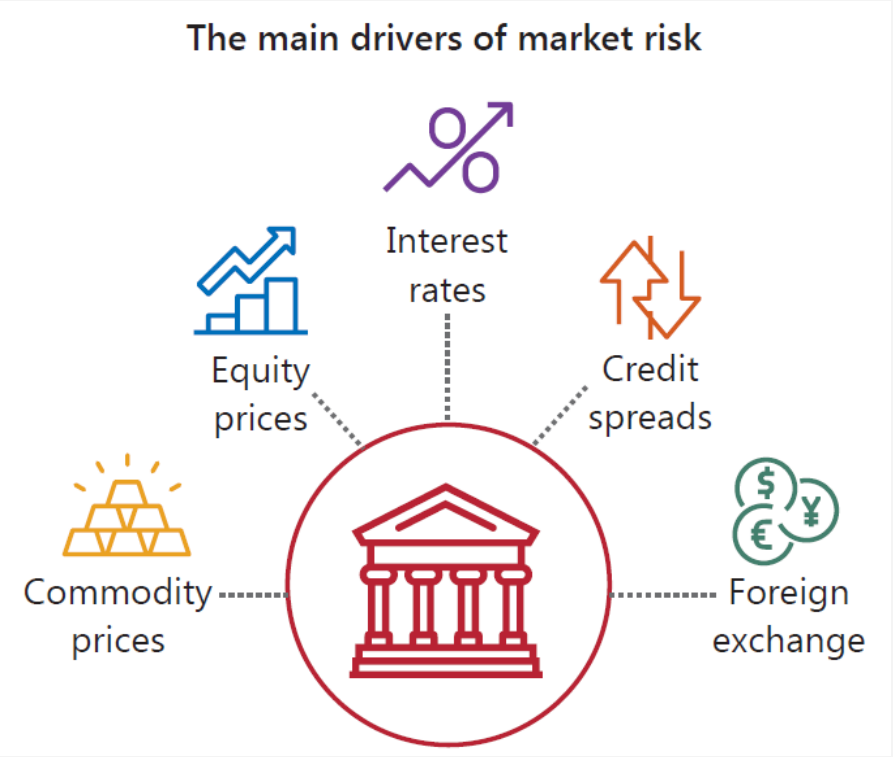
4. Market Risk

Market Risk

- Definition: risk of losses due to the impact of interest rate, exchange rates, stock and bond prices or other financial asset price moves on the value of actively traded portfolios.



Source: BIS (2019), "The Market Risk Framework – in brief".

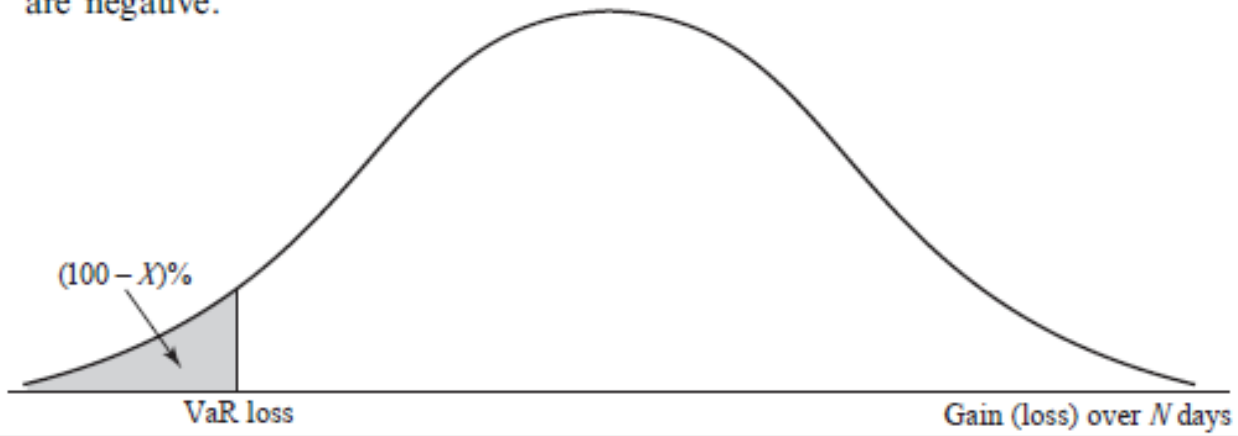
4.1. Value-at-Risk (VaR)

VaR

Please see Jorion (2007) – Chapters 1 and 2

- Usual measure of the market risk of a portfolio: Value-at-Risk (VaR).
- VaR: maximum loss that can occur with X% confidence over a holding period of t days, being X the confidence level (usually high) and t a short number of days, providing a conservative loss measure, corresponding to a highly unlikely but severe scenario.

Figure 22.1 Calculation of VaR from the probability distribution of the change in the portfolio value; confidence level is X%. Gains in portfolio value are positive; losses are negative.



Source: Hull, John (2018)

VaR

- VaR is a measure of unexpected loss that answers to the following question:

how bad things can get for a financial portfolio comprising different types of financial assets under a set of assumptions?

VAR summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence.

Source: Jorion (2007)

- While valuation models focus on the mean of the distribution, VaR gives us the potential variation in prices or returns under very unlikely scenarios, being a summary statistic of the probability density function.

VaR

- For a 5% significance level and a daily horizon, 95 out of 100 days the portfolio won't loose more than the VaR.

- With a confidence level of 1 – *X* (i.e. 1-0,05=95%), the loss in a given time horizon is not expected to exceed the VaR.

- To measure VaR we need to define:
  - Confidence level
  - Time horizon
  - Distribution function

VaR

- **Confidence level**
  - Subjective decision
  - Basel Committee uses a 99% level to compute capital requirements for market risk of trading (marked-to-market) portfolios.
- **Time Horizon**
  - Depends on the portfolio strategy and liquidity
  - Usually daily, weekly or monthly
  - Basel Committee uses a 10-day horizon
  - Rule of thumb: portfolios with higher turnover must use shorter time horizons.
- **Distribution**
  - Can be based on an empirical or a parametric approach.

VaR methods

Please see Jorion (2007), Chapters 4 and 5

(i) Empirical or historical approach

(ii) Parametric approach

- The empirical approach does not assume a theoretical distribution a priori for returns, contrary to the parametric approach, as it uses the past data in a very straightforward way, assuming that past variations replicate in the future.
- It corresponds to generate a number of scenarios for all asset prices included in the portfolio, assuming that each scenario is characterized by the variations in each sample day (being v_i the financial asset value in day i, used to estimate the value in day $n+1$ under the ith scenario):

$$\text{Value under } i\text{th scenario} = v_n \frac{v_i}{v_{i-1}}$$

VaR

- Even though VaR provides a conservative loss measure, it doesn't give us the worst potential loss =>

(i) VaR must be complemented other risk measurement tools, e.g. stress tests or Expected Shortfall (ES);

(ii) Exposure or risk limits based on VaR are not enough and must be added by quantitative limits, according to the risk appetite of the portfolio owner, related to the capital he is willing or able to lose.

- Nonetheless, Basel Committee set capital requirements for market risk based on VaR since 1996, as a conservative multiple (k) of the 10-day 99% VaR, being k chosen on a bank-by-bank basis by regulators, with a minimum of 3.

VaR

- **VaR Advantages**
  - It’s a single number, easily accessible and understood
  - Allows for comparison between different products and strategies
  - Enhances performance evaluation and the settlement of risk limits
- **VaR Disadvantages**
  - NOT a loss forecast
  - NOT the worst case scenario
  - NOT fully objective (depends on time horizon and *a*)
  - NOT the ultimate truth (one may be using the wrong distribution or the wrong period to estimate the parameters)
  - Only works for liquid securities and continuous payoffs
  - Ignores Black Swans

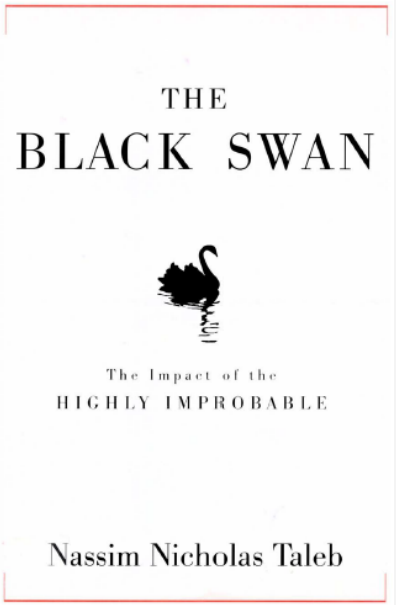
Black Swans

- **Definition**:

(i) An outlier, something completely unexpected according to the past;

(ii) Has an extreme impact (the "Extremistan");

(iii) Even though it is an outlier, economic agents try to find rational explanations for it afterwards, in order to make it predictable in the future.



- **Consequences**:

(i) Being unpredictable, we need to adjust to their existence, instead of trying to predict them;

(ii) Therefore, VaR is not a crystal ball, but just a quantitative tool.

4.2. Expected Shortfall

ES

- Identical VaRs can mean different levels of risk, namely if we have multimodal distributions.

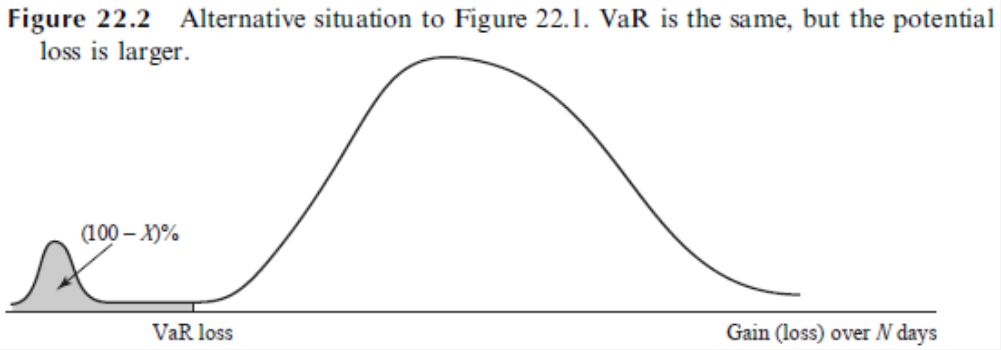


Figure 22.2 Alternative situation to Figure 22.1. VaR is the same, but the potential loss is larger.

Source: Hull, John (2018)

- A measure that deals with this problem is ES.
- While VaR asks the question ‘‘How bad can things get?’’, ES asks:

‘‘If things do get bad, how much can the company expect to lose?’’

- Definition: expected loss during an N-day period conditional on the loss being worse than the VaR.
- Example: with a 99% 10d VaR, ES = average loss over a 10d period when the loss is worse than the 10-day 99% VaR.

4.3. Capital Requirements

Capital Requirements for Market Risk

- **Quantitative Requirements to use VaR:**

(i) Daily calculation

(ii) 99%, 10-day period VaR

(iii) Minimum sample of 1 year, except when higher price volatility justifies a shorter period

(iv) Minimum monthly data update

(v) Minimum weekly frequency for stressed VaR

(vi) VaR is scaled up by a multiplication factor = 3 + additional factor (addend) between 0 and 1, depending on the number of loss excesses observed in the previous 250 business days.

Number of overshootings	addend
Fewer than 5	0,00
5	0,40
6	0,50
7	0,65
8	0,75
9	0,85
10 or more	1,00

Source: European Parliament (2013), CRR.

Capital Requirements for Market Risk

- **Qualitative requirements:**

- Models integrated in bank's daily risk management and internal reports to top management;
- Risk control unit independent from trading and reporting directly to top management, liable for the development, implementation and validation of internal models, producing and analyzing daily reports on model results and presenting proposals on trading limits;
- Board and top management actively involved in risk control processes and daily reports;
- Adequate human resources in trading, risk control, auditing and back-office;
- Internal models with good track record;
- Stress tests - Rigorous and frequent program, with reverse stress tests;
- Internal independent auditing process;
- Minimum yearly internal assessment of the global risk management system.

4.4. Parametric and non-parametric methods of VaR estimation

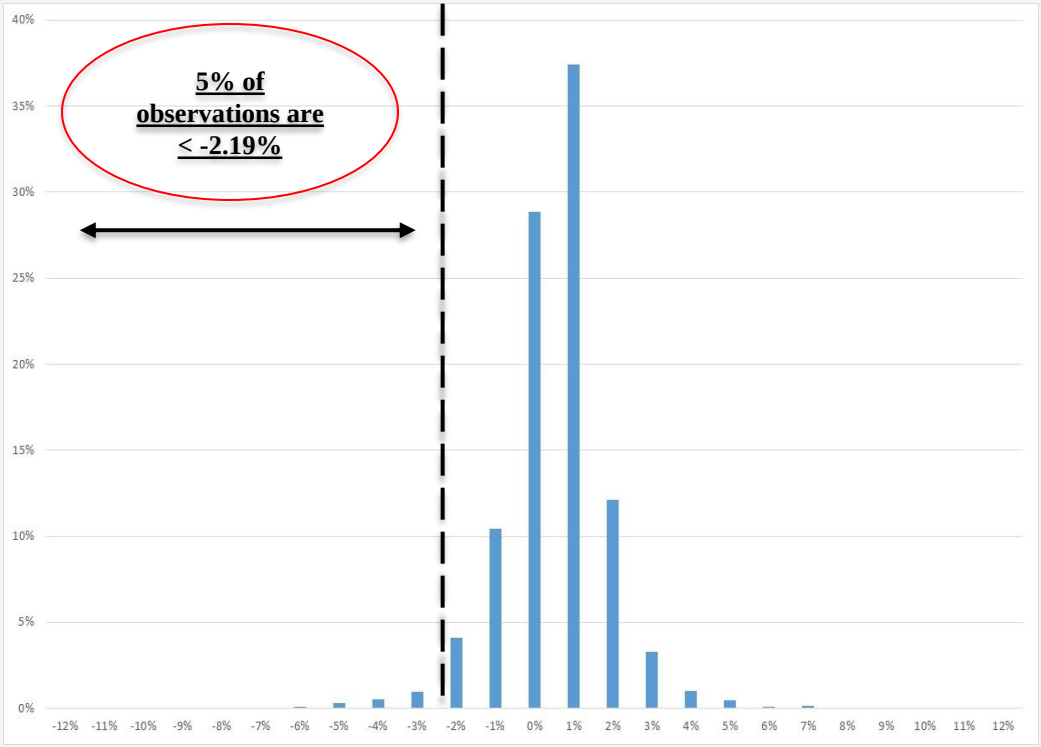
VaR methods

- Generally, the calculation of VaR uses a histogram of the changes in the value of the portfolio (i.e. empirical distribution) for a given pre-defined time horizon and a given α%.
- The higher the volatility, degree of confidence and maturity, the higher will be the VaR.
- The usual assumption is: $N\text{-day VaR} = 1\text{-day VaR} \times \sqrt{N}$
- This assumption is based on the returns being normally distributed and independent => variances are additive over time => volatility grows with the square root of time.
- As the volatility fluctuates along time, the VaR will also change, even when calculated under the same assumptions.
- Volatility also assumes different magnitudes for different classes of financial assets.
- In a portfolio, negative correlations may contribute to mitigate the aggregate volatility.
- VaR can be computed by non-parametric (empirical/historical) or parametric approaches.

VaR methods

(i) Non-parametric or Empirical/Historical approach:

- Empirical distribution of the daily variations of the Nasdaq Index (2705 observations):



- 5% critical level = 135th smallest observation

VaR methods

- 5% critical level = 135th (5% x 2705) smallest observation

- $R^*_{95\%}$ = -2,19% is the daily return for this observation
- μ (average return or average daily growth rate of the index in the full sample) = 0,038%.
- V = 1M€ => daily VaR @ 95% confidence level is:

$$VaR_{95\%}(mean) = -V[\exp(R^*_{95\%}) - \exp(\mu)] = 22.050{,}45€$$

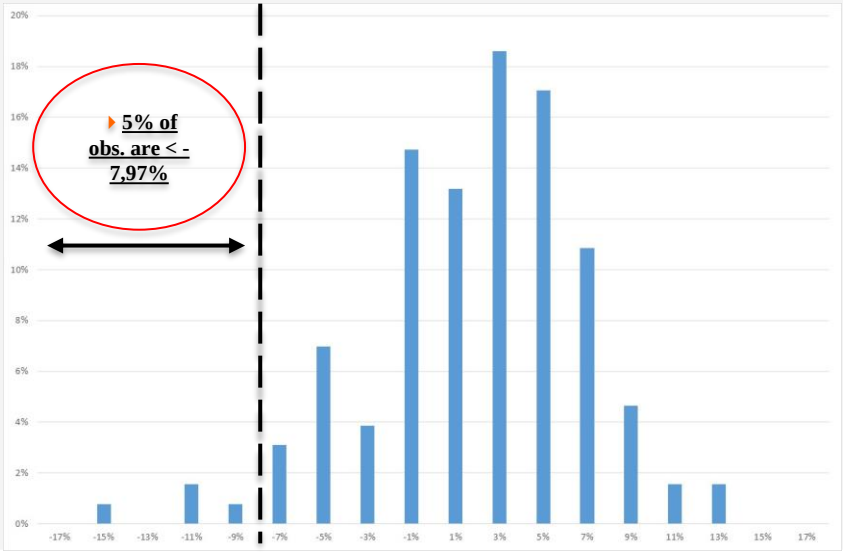
$$VaR\ (current)@\ 95\% = -V[\exp(R^*_{95})] = 21.675{,}25€$$

- **Conclusions:**
  - (i) 95 in 100 days the portfolio won't lose more than $22.050,45 during a 1-day period comparing to the expected return, according to the empirical distribution;
  - (ii) Comparing to the current value of the portfolio, the estimated loss is $21.675,25 (very similar given that the estimated loss is small, as the period considered is also very short).

VaR methods

- Instead of assuming the usual assumption for the calculation of the VaR for a larger period (by multiplying the shorter-period VaR by the square root of time), one may use the same data to get the empirical distribution for larger horizons.

- For monthly variations of the Nasdaq Index, the empirical distribution (129 observations, assuming non-overlapping periods with 21 days per month) is as follows:



VaR methods

- 5% critical level = 6th smallest observation (5% x 129)

R* = -7,97%

- Being μ = 0,79% and V = 1M€ => monthly VaR @ 95% confidence level:

$$VaR\ (mean)\ @\ 95\% = -V[\exp(R^*) - \exp(\mu)] = 84.546{,}87€$$

- In 95 out of 100 days, the portfolio won't lose more than $84.546,87, according to our empirical distribution.
- Following the usual assumption $N\text{-day VaR} = 1\text{-day VaR} \times \sqrt{N}$, by assuming the normality of returns, using the daily VaR (mean) previously computed (22.050,45€), one would have obtained 22.050,45 x sqrt (21) = $101.047,874.
- This difference to the monthly VaR suggests that returns are not normally distributed.

VaR methods

(ii) Parametric approach

- The main alternative to historical simulation is the model building or parametric approach.
- Most common implementation of VaR assumes that returns follow a normal distribution and are i.i.d.

- Returns are normally distributed ⇔ prices follow a log-normal distribution.

- This method also assumes that:

(i) the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio asset is constant;

(ii) the volatility of each risk factor is extracted from the historical observation period.

VaR methods

- The simplest parametric method to calculate VaR is based on the assumption of normally distributed daily returns - delta-normal or variance-covariance method:

$$VaR = \omega' \Sigma \omega \times N^{-1}(X) \times \sqrt{T}$$

where Σ is the variance-covariance matrix of the portfolio's assets and ω corresponds to the weights of each asset in the portfolio.

- If the portfolio has only 1 asset, VaR results only from that asset volatility:

$$VaR = \sigma \times N^{-1}(X) \times \sqrt{T}$$

NASDAQ

	Daily	Monthly
σ	1,37%	5,19%
z_α	-1,645	-1,645
VaR (current) @ 95%	-2,26%	-8,53%

VaR (current) @ 95%	$22 578	$85 299

Volatilities and Correlations

- **Different methods to calculate relevant risk factor volatilities and correlations:**

(i) Simple historic volatility and correlation - the most straightforward method but the effects of a large single market move can significantly change volatilities and correlations over the required forecasting period, as all observations are equally weighted.

(ii) Weighted historical volatility or correlation - this is done to give more weight to recent observations so that large jumps in volatility are not caused by events that occurred some time ago, using 2 main methods.

(1) Exponentially weighted moving averages - the weights are attached according to an exponential function.

(2) Generalized Autoregressive Conditional Heteroscedasticity (*GARCH*) models.