2. Credit Risk

2.1. Main Definitions and Measures

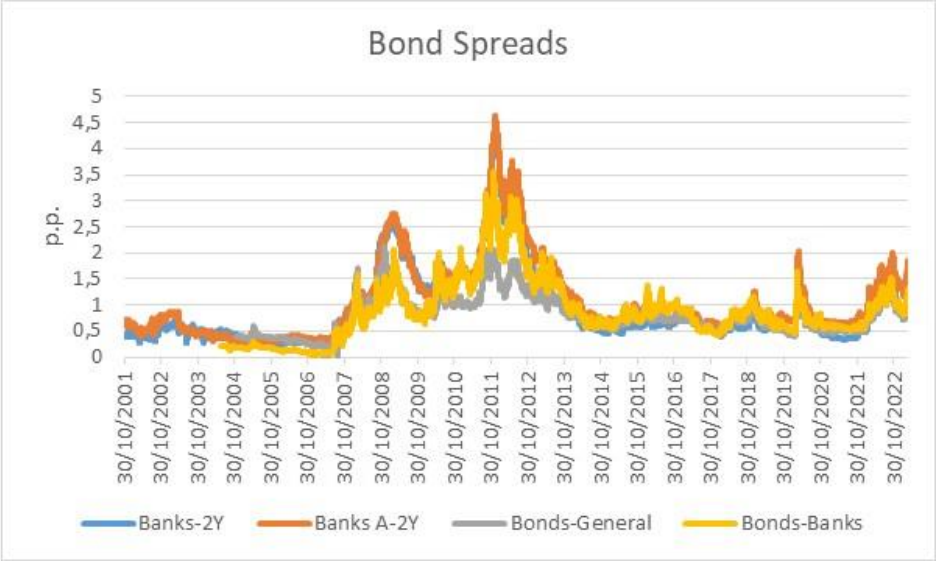
2.1.1. Expected and Unexpected Losses

Introduction

- **Credit Risk -** the risk of a loss due to a credit event, i.e. the risk of promised future cash-flows not being paid in full due to the default of debtors (quantified by the PD).
- **Traditional credit analysis** - evaluates the ability of a borrower to honor its financial obligations (default risk), as well as the loan purpose and the guarantees.
- **Credit Risk types** - the market usually distinguishes sovereign, banking, counterparty, corporate and individual/household credit risk.
- **Sovereign Risk** - credit risk in Government bonds.
- **Counterparty risk** - may also be seen as a type of credit risk, being more likely in the derivatives markets.
- Default events are rare but they may occur unexpectedly and involving significant losses.
- All loans and debt securities have credit risk – the risk-free asset is a theoretical concept.

Introduction

- In bonds, credit risk may provoke price decreases even if a default doesn't happen, namely when rating downgrades occur or are anticipated => **bond spreads may be used to monitor credit risk of a given issuer or sector:**



"Credit risk is the risk of default or of reductions in market value caused by changes in the credit quality of issuers or counterparties".

in Duffie, Darrell and Kenneth J. Singleton (2003), "Credit Risk", Princeton University Press.

Introduction

- Traditional banking was very much an "**expert system**", where bankers allocated credit in their sectors using subjective judgements.
- A popular framework was the "**five C's**" – character, capital, capacity, collateral, and cycle – weighted as deemed appropriate, lacking robust analytical analysis.
- As regulation has imposed higher standards in credit risk management and measurement, along with the development of credit derivatives, **credit risk entered an era of quantification and modeling**.
- **Sound practices** now involve measuring the credit risk at facility and portfolio levels to estimate the amount of capital that banks need to hold as a buffer for extreme losses.

Introduction

- **Credit risk may be assessed on an individual or a portfolio basis**, using internal models or information from the markets, e.g. stock and bond prices and external ratings.
- **Portfolio credit risk models usually correspond to Credit-VaR models**, that compute credit losses if a very unlikely and severe scenario occurs, through the estimation of a probability density function for credit losses.
- **Credit risk assessment is relevant at the origination but also for monitoring purposes**, focusing on debtors and collaterals, in order to quantify the PD and the LGD.

Introduction

- **Credit Risk components:**

(1) **Arrival risk** – uncertainty about whether a default will occur in a given time horizon.

(2) **Timing risk -** uncertainty about the precise time of default.

(3) **Recovery risk** – describes the uncertainty about the severity of losses if a default happens, usually expressed as a % of the exposure amount.

(4) **Default correlation risk** – the risk of default of several debtors simultaneously.

- **External Ratings** – the most straightforward way to assess credit risk of bonds, but they only order the risk of debt issuers => they don't provide explicitly any PD measure.

- However, major rating agencies (S&P, Moodys and Fitch) disclose historical frequencies of default for each rating classification and different maturities, as well as the historical frequencies of transition between ratings => these frequencies of default may be used as a rough proxy for the PD.

Introduction

- Several decision variables impact on credit risk, e.g. LTV, DSTI and Maturity.
- Quantification of risk is key to decide loans, including pricing decisions, as well as to calculate capital requirements and impairments.
- This quantification is done through credit risk models that place borrowers in different risk classifications, each of them having a term structure of PDs attached.
- Credit risk materializes in impairments, NPLs and write-offs (usually after the loans are fully provisioned), being impairments an estimate for the expected credit loss => credit risk impacts on the NI and also on the net value of assets.
- Besides the impact of credit risk on impairments, it also affects key management indicators, namely when it materializes in NPL, in two different ways:

(I) level of NPLs– asset quality indicators (usually the focus);
(II) level of coverage of NPLs by provisions.

Introduction

- **Stages in credit risk materialization:**

(i) first signs of difficulties => impairments, early recognizing partial losses => lower NI

(ii) NPL => higher impairments and capital requirements

(iii) Unable to return to performing => recovery process

(iv) Non-recovered amount => full impairment and write-off

2.1.1. Expected and Unexpected Losses

Expected and Unexpected Losses

- **Expected Loss: EL = PD x LGD x EAD.**

being EAD the Exposure at Default

- **Unexpected Loss (UL):** losses beyond EL that may happen under a very unlikely and severe scenario.
- **EL -** covered by the spreads charged by banks on loans, as well as by provisions for impairments.
- **UL –** covered by equity, through capital requirements

Loan Impairments

- **Impairments** - EL in loans (and other assets not at fair value, i.e. valued according to market prices, e.g. real estate properties, bonds in held-to-maturity portfolios or financial participations), ruled by IFRS 9 (replaced IFRS39 in 1.1.2018; IFRS: International Financial Reporting Standards), introducing the Expected Credit Loss (ECL) framework.
- **Purpose -** Anticipate the recognition of losses as promptly as possible, even before the loans become non-performing, smoothing the impact of potential losses along time, through provisions (a cost) whose flow is the transmission channel of credit risk to the Net Income (the stock reduces the loan values in the balance sheet).
- **Calculation** – depends on the magnitude of the exposures:

(i) **Individual** – exposures above a given threshold set by the bank, with impairments calculated by credit analysts, based on an individual assessment of expected cash-flows.

(ii) **Collective** – exposures below a given threshold set by the bank, with impairments calculated by using internal estimates of PD and LGD.

Loan Impairments

- **Recognition of Loan Impairments – 3 stages:**

(i) **Stage 1 -** when a loan is originated or purchased => 1y Expected Credit Loss (ECL)

(ii) **Stage 2 -** significant increase in credit risk (SICR) => Lifetime PD and ECL

(iii) **Stage 3 -** the loan is considered credit-impaired. e.g a non-performing loan (NPL) for >90 days or forborne loans considered as NPL => PD = 100%.

- **SICR:** banks have to identify relevant factors that suggest a significant increase in credit risk (impairment signs), in order to conclude about whether loans should be classified based on 3 elements:

(i) quantitative;

(ii) qualitative;

(iii) 30 days past due.

- This assessment has to be forward looking, considering different scenarios and quantitative and qualitative factors, set by the bank.

Loan Impairments

- **Quantitative factors:**

  - PD increase (i.e. internal rating downgrade) above a given internal threshold.

- **Qualitative factors (according to IFRS9):**

(i) Macroeconomic deterioration:

General economic and/or market conditions

National or local economic conditions relevant to the asset class;

Unemployment rate;

Property prices for mortgages;

Industry (or sector);

Loan Impairments

(ii) Companies:

Borrower requests for waivers/forbearance;
Breach of contract or covenants;
Credit rating;
Significant increase in credit spread
Debt service capacity;
Financial performance;
Cash flow or liquidity issues, e.g. delay in servicing of trade creditors
Net worth;
Operating performance of the borrower, namely decrease in turnover;
Loss of customers or market share;
Diversion of cash flows from earning assets to support non-earning assets;
Prospects of the guarantors;

- Significant change in collateral value (which is expected to increase PD);

Country risks.

Loan Impairments

(iii) Mortgage loans:

Likely or actual borrower bankruptcy

Payment delays and past due information

Previous arrears in the last 12 months

Decrease in rents received;

Absence of refinancing options.

(iv) Consumer and revolving loans (e.g. overdrafts and credit cards):

Early delinquency (for example, one payment in arrears);

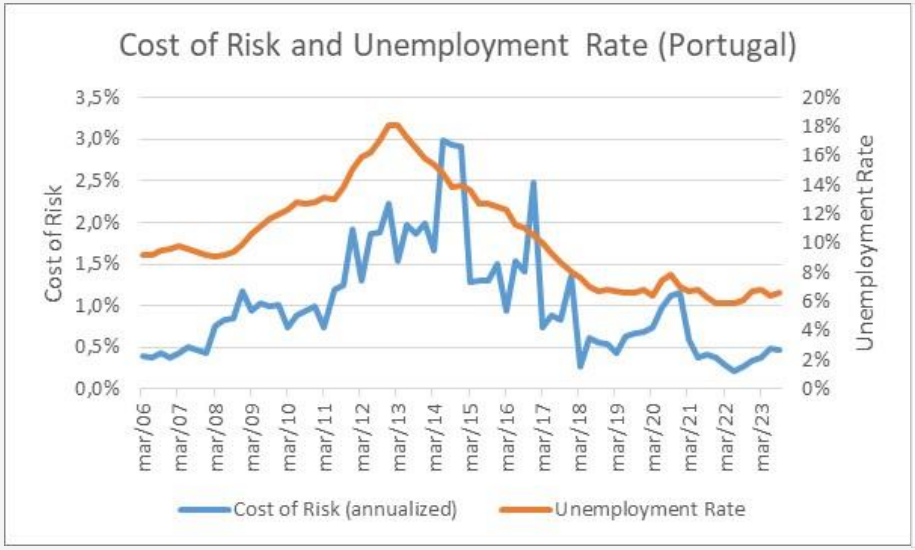
Continual high utilization of facilities;

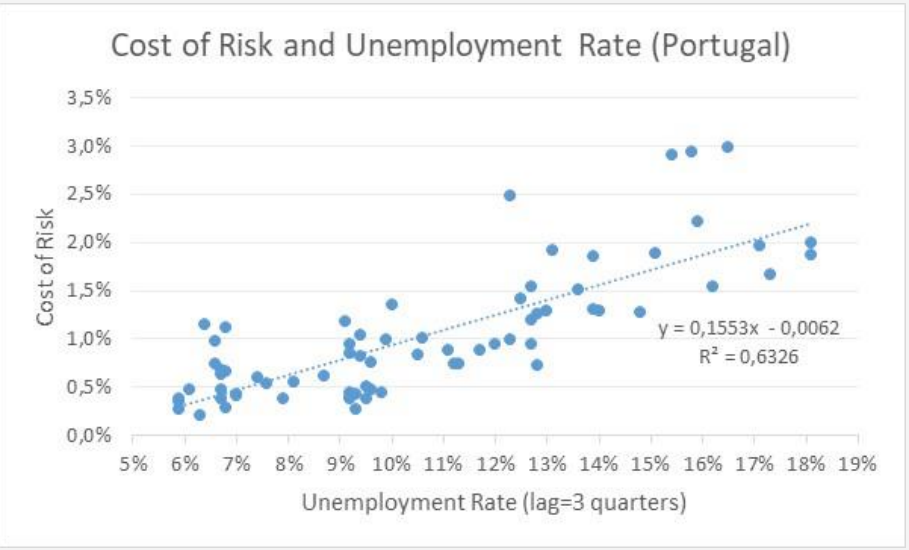
Steady increase in total debt for the client;

Income less than total debt repayments.

Loan Impairments

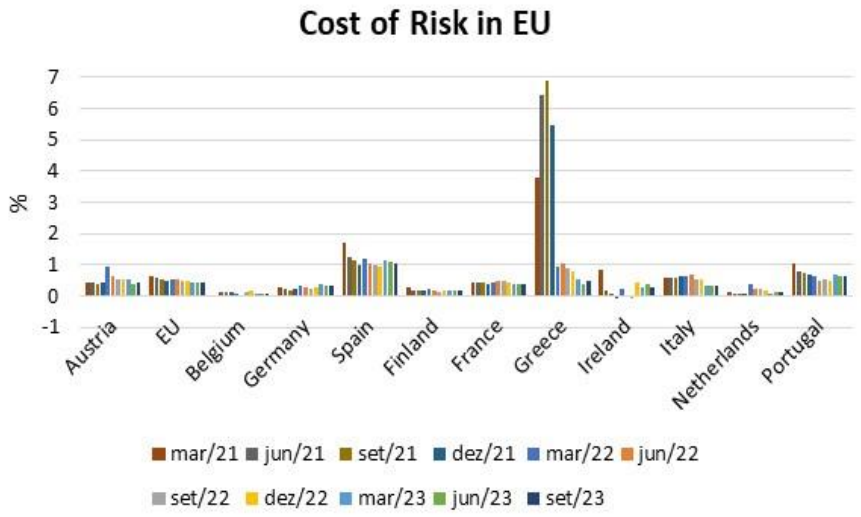
- **The impact of credit risk on loan impairments is usually illustrated by the cost of risk:** Cost of Risk = Impairments Flow/Total Loans.
- The cost of risk is usually highly correlated with unemployment (in Portugal with a lag of 3 quarters).

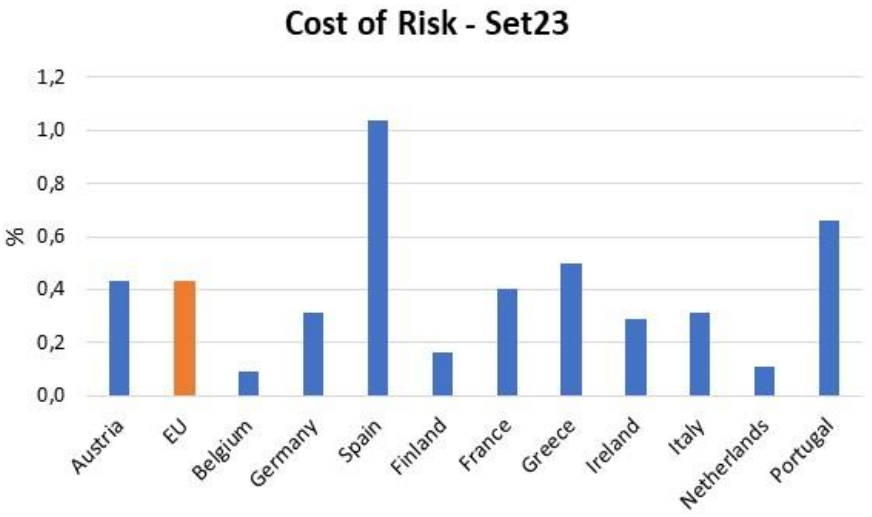




Loan Impairments

- **Most countries tend to exhibit maximum levels for the cost of risk of 2%, even though during and after recessions this level is often overcome, as in Greece.**
- **In years of economic growth, the cost of risk tends to be not above 0,5%** and in some cases may even be negative (if impairment reversions exceed new impairments).





Source: EBA and own calculations.

Loan Impairments

- Restructuring of debt is very close to default if it results from the inability of the borrower to face payment obligations unless its debt structure changes.
- **Forbearance** – a loan is considered as forborne or in forbearance when the debtor is in financial difficulties and:

(a) a modified contract is classified as NPE or would be classified as non-performing in the absence of modification/restructuring;
(b) the modification made to a contract involves a total/partial cancellation by write-offs of the debt.

- Forborne exposures can be either in performing or non-performing portfolios.
- **Cure period -** The forbearance classification shall be discontinued when all the following conditions are met (to be assessed at least on a quarterly basis):

(a) the contract is considered as performing;
(b) a minimum 2y probation period has passed since the forborne loan was considered performing;
(c) regular payments of more than an insignificant aggregate amount of principal or interest have been made during at least half of the probation period;
(d) none of the exposures to the debtor is more than 30 days past-due at the end of the probation period.

Capital Requirements

- **2 approaches in the calculation of Pillar 1 capital requirements for credit risk:**

(i) **Standardized** – corresponds roughly to Basel I, added by the differentiation of capital requirements as a function of the external ratings of counterparties:

- non-rated companies kept a risk weight of 100%;
- preferential treatment of mortgage loans was also kept (now with a risk weight of 35%, vis-à-vis 50% before);
- the differentiation between OECD member countries and others was eliminated.

(ii) **IRB** – involves the integration of internal credit risk models in the loan granting decisions processes for the different portfolios, with these models supplying estimates for PD and LGD (for the corporate segment, there are two IRB sub-approaches – basic and advanced, with the former requiring only the PD estimation). Banks have to subject their models and the corresponding integration in the decision processes (**use test**) to approval by the supervisor.

Standardized approach

- **Risk weights for exposures to sovereigns, banks and non-financial companies:**

Basel II

	Sovereigns	Banks	Non Financial Companies
AAA to AA	0%	20%	20%
A+ to A-	20%	50%	50%
BBB+ to BBB-	50%	100%/50%	100%
BB+ to B-	100%	100%	100%
<B-	150%	150%	150%
Non-rated	100%	100%/50%	100%

CEBS

Credit Quality Step	Fitch's assessments	Moody's assessments	S&Passessments	Corporate	Institution (includes banks)			Sovereign
					Sovereign method	Credit Assessment method		
						Maturity > 3 months	Maturity 3 months or less	
1	AAA to AA-	Aaa to Aa3	AAA to AA-	20%	20%	20%	20%	0%
2	A+ to A-	A1 to A3	A+ to A-	50%	50%	50%	20%	20%
3	BBB+ to BBB-	Baa1 to Baa3	BBB+ to BBB-	100%	100%	50%	20%	50%
4	BB+ to BB-	Ba1 to Ba3	BB+ to BB-	100%	100%	100%	50%	100%
5	B+ to B-	B1 to B3	B+ to B-	150%	100%	100%	50%	100%
6	CCC+ and below	Caa1 and below	CCC+ and below	150%	150%	150%	150%	150%

Notes:

- Exposures to the ECB shall be assigned a 0 % risk weight.
- Since Jan19, at national discretion, a lower risk weight may be applied to banks' exposures to their sovereign (or central bank) denominated in domestic currency and funded in that currency. Where this discretion is exercised, other national supervisory authorities may also permit their banks to apply the same risk weight to domestic currency exposures to this sovereign (or central bank) funded in that currency => Currently, Exposures to Member States' central governments and central banks denominated and funded in the domestic currency of that central government and central bank are assigned a risk weight of 0 %.

- Risk weights to regional and local governments and banks may be calculated according two alternative methodologies:
  - Risk weights immediately above the one applicable to the respective government (100% if non-rated or central banks from countries rated between BB+ and B-);

Standardized approach

- Exposures to the following multilateral development banks have a 0 % risk weight:

(a) International Bank for Reconstruction and Development (IBRD);
(b) International Finance Corporation (IFC);
(c) Inter-American Development Bank (IADB);
(d) Asian Development Bank;
(e) African Development Bank;
(f) Council of Europe Development Bank;
(g) Nordic Investment Bank;
(h) Caribbean Development Bank;
(i) European Bank for Reconstruction and Development (EBRD);
(j) European Investment Bank (EIB);
(k) European Investment Fund (EIF);
(l) Multilateral Investment Guarantee Agency;
(m) International Finance Facility for Immunization;
(n) Islamic Development Bank.

Standardized approach

- **Exposures to the following international organizations also get a 0 % risk weight:**

(a) EU
(b) International Monetary Fund (IMF);
(c) Bank for International Settlements (BIS);
(d) European Financial Stability Facility;
(e) European Stability Mechanism;

Standardized approach

- Some changes in the coefficients of the standardized approach were decided afterwards, to be implemented in 1.1.2022 (later postponed in 1 year, with a phasing-in period of 5 years), trying to decrease excessive variability of RWAs (e.g. BCBS (2017), "Basel III: Finalising post-crisis reforms", Dec.).

Corporate exposures:

100% previously

External rating of counterparty	AAA to AA–	A+ to A–	BBB+ to BBB–	BB+ to BB–	Below BB–	Unrated
"Base" risk weight	20%	50%	75%	100%	150%	100%

- **Corporate SME** (annual consolidated sales <= 50M€ for the most recent year) – risk weight = 85% (between investment and speculative grade corporates, due to higher risk, but also higher granularity.

Interbank exposures:

50% previously

External rating of counterparty	AAA to AA–	A+ to A–	BBB+ to BBB–	BB+ to B–	Below B–
"Base" risk weight	20%	30%	50%	100%	150%
Risk weight for short-term exposures	20%	20%	20%	50%	150%

Standardized approach

- **Subcategories of Specialised lending:**

(i) **Project finance** - the lender looks primarily to the revenues generated by a single project, both as the source of repayment and as security for the loan, being usually for large, complex and expensive installations such as power plants, chemical processing plants, mines, transportation infrastructure, environment, media, and telecoms.

(ii) **Object finance** - funding of the acquisition of equipment (e.g. ships, aircraft, satellites, railcars, and fleets) where the repayment of the loan is dependent on the cash flows generated by the assets that have been financed and pledged or assigned to the lender;

(iii) **Commodities finance** - short-term lending to finance reserves, inventories, or receivables of exchange-traded commodities (e.g. crude oil, metals, or crops), where the loan will be repaid from the proceeds of the sale of the commodity and the borrower has no independent capacity to repay the loan.

Standardized approach

- **Risk Weights**

(i) Specialised lending

- Object and commodities finance - 100%;
- Project finance - 130% during the pre-operational phase, 100% during the operational phase, and 80% during the operational phase of projects deemed to be high quality, i.e. able to meet their financial commitments in a timely manner and its ability to do so is assessed to be robust against adverse changes in the economic cycle and business conditions.

(ii) **commercial real estate:**

If repayment doesn't depend on cash-flows generated by the property (depends on LTV and the borrower's risk):

	LTV ≤ 60%	LTV > 60%
Risk weight	Min (60%, RW of counterparty)	RW of counterparty

If repayment depends on cash-flows generated by the property (depends only on the LTV, with higher weights, due to higher risk):

	LTV ≤ 60%	60% < LTV ≤ 80%	LTV > 80%
Risk weight	70%	90%	110%

Source: BCBS (2017), "Basel III: Finalising post-crisis reforms", Dec.

Standardized approach

(iii) Risk weights for residential mortgage loans (after Basel III, which brought a set of several risk weights, instead of the former differentiation between LTV > or < 75%, with risk weights of 75% and 35%, respectively):

- if repayment doesn't depend on cash-flows generated by the property:

	LTV ≤ 50%	50% < LTV ≤ 60%	60% < LTV ≤ 80%	80% < LTV ≤ 90%	90% < LTV ≤ 100%	LTV > 100%
Risk weight	20%	25%	30%	40%	50%	70%

- if repayment depends on cash-flows generated by the property:

	LTV ≤ 50%	50% < LTV ≤ 60%	60% < LTV ≤ 80%	80% < LTV ≤ 90%	90% < LTV ≤ 100%	LTV > 100%
Risk weight	30%	35%	45%	60%	75%	105%

Source: BCBS (2017), "Basel III: Finalising post-crisis reforms", Dec.

Standardized approach

(iv) Other Retail exposures - 75%

- **Definition:**

(i) revolving credits and lines of credit (including credit cards, charge cards and overdrafts), personal term loans and leases (eg instalment loans, auto loans and leases, student and educational loans, personal finance) and small business facilities and commitments.

(ii) exposure <= 1M€

(iii) granularity requirement - aggregated exposure to a counterparty cannot exceed 0.2% of the overall regulatory retail portfolio (unless national supervisors have determined another method to ensure satisfactory diversification of the regulatory retail portfolio).

IRB Approach

- **For Credit Risk, pre-established formulas were set** in Basel II, based on the parameters for the EL – PD and LGD.
- **For corporate exposures, there are 2 IRB approaches:**
  - **Foundation** – only the PD (and the EAD, for revolving exposures) has to be estimated
  - **Advanced** – PD, LGD and EAD have to be estimated
- **In IRB Foundation, the LGD is pre-defined according to the type of exposure:**
  - Loans with receivables as collaterals - 20%
  - Loans with real estate collaterals - 20%
  - Loans with other eligible physical collaterals – 25%
  - Non-collateralized loans to non-financial companies – 40%
  - Subordinated assets – 75%
  - Other assets - 45%

IRB Approach

- The LGD applicable to a collateralized transaction (LGD*) must be calculated as the exposure weighted average of the LGD applicable to the unsecured part of an exposure (LGD_U) and the LGD applicable to the collateralized part of an exposure (LGD_S):

$$LGD^{*} = LGD_{U} \cdot \frac{E_{U}}{E \cdot (1 + H_{E})} + LGD_{S} \cdot \frac{E_{S}}{E \cdot (1 + H_{E})}$$

- For different types of collaterals:

$$LGD^{*} = LGD_{U} \cdot \frac{E_{U}}{E \cdot (1 + H_{E})} + \sum_{i} LGD_{Si} \cdot \frac{E_{Si}}{E \cdot (1 + H_{E})}$$

E - current value of the exposure (in the case of securities lent, the exposure value has to be increased by applying the appropriate haircuts - H_E).

E_S – collateralized part of the exposure, i.e. the current value of the collateral received after the application of the haircut applicable for the type of collateral (H_c) and for any currency mismatches between the exposure and the collateral. E_S is capped at the value of . $E^{*}(1+E_H)$.

E_U – uncollateralized part of the exposure.

E_{Si} –the current value of the collateral *i* received after the application of the haircut applicable for that type of collateral (H_c)

LGD_{Si} – LGD applicable to that type of collateral (H_c).

IRB Approach

- Haircuts (H_C):

Type of collateral	LGD_S	Haircut
Eligible financial collateral	0%	As determined by the haircuts that apply in the comprehensive formula of the standardised approach for credit risk (paragraph 163 for jurisdictions that allow the use of ratings for regulatory purposes and paragraph 164 for jurisdictions that do not). The haircuts have to be adjusted for different holding periods and non-daily remargining or revaluation according to paragraphs 169 to 172 of the standardised approach.
Eligible receivables	20%	40%
Eligible residential real estate / commercial real estate	20%	40%
Other eligible physical collateral	25%	40%
Ineligible collateral	N/A	100%

IRB Approach

- **Floors for LGD in IRB Advanced:**

LGD parameter floors		
	LGD	
	Unsecured	Secured
Corporate	25%	Varying by collateral type: • 0% financial • 10% receivables • 10% commercial or residential real estate • 15% other physical

- The LGD floor for a partially secured exposure is calculated as a weighted average of the unsecured LGD floor for the unsecured portion and the secured LGD floor for the secured portion:

$$Floor = LGD_{U\ floor} \cdot \frac{E_U}{E \cdot (1 + H_E)} + LGD_{S\ floor} \cdot \frac{E_S}{E \cdot (1 + H_E)}$$

PDs

- **PDs are usually estimated by econometric models, providing internal rating classifications characterized by different term structures of PDs.**
- Ratings are the basis for credit approval, pricing, monitoring and loan loss provisioning, including external ratings.
- Internal ratings are based on models for **3 different types of non-financial companies:**
  - ***middle market*** **(non-listed medium to large size companies)** – models relate financial ratios to past loan behavior based on the FI's credit experience or representative external databases, with no structural background, just resulting from the statistical results.
  - **listed companies** – besides the same type of non-structural models based on stock prices, also using data from financial statements (for a shadow PD or to get data on the liabilities).
  - **small business** – similar to middle market, but including variables related to the entrepreneur.

PDs

- **Statistical methods to estimate PDs include:**
  - Linear regression;
  - Discriminant analysis;
  - Logit and Probit models;
  - Panel models;
  - Cox proportional hazards model;
  - Neural Networks.
- **Credit analysts can override internal ratings** in the corporate segment, following the qualitative assessment of management, business perspectives, quantitative information still not reflected on financial statements or from the customer relationship with the bank (e.g. sudden increase in the utilization of credit lines), or from external sources (e.g. commercial information, central credit risk database).
- PDs in individual loans tend to differ for different types of loans (e.g. PDs are usually lower for residential mortgage loans), while for corporate PDs tend to be the same.

LGD

- **2 main LGD estimation methods:**

(i) **Market LGD** - observed from market prices of defaulted bonds and marketable loans soon after default events - 1-Price (as a % of EAD, often after 1 month). This is the methodology used most by the rating agencies.

(ii) **Workout LGD** – 1-NPV of estimated cash flows from the workout process.

- Different LGDs are usually associated with different collateral types or debt seniority, being also correlated with PDs, as higher ratings exhibit lower LGDs.
- LGD also fluctuates with the business cycle.
- LGD for loans corresponds to the workout LGD = 1 - NPV of recoveries

EAD

- **Fixed-credit facilities** (e.g. term loans): amount outstanding (although EADs slightly above 100% are not uncommon given interest accrual).

- **Revolving facilities** (e.g. credit lines, liquidity facilities and overdrafts):

  = drawn amount + estimate of the % of the amount available in the credit line to be used at the time of default (often referred as the Credit Conversion Factor).

Exposure classes for IRB

- **Corporate** – includes specialized credit:

(i) project finance - cash-flows generated by a single project;

(ii) object finance - cash-flows generated by a single asset;

(iii) commodity finance - cash-flows generated by the sale of goods whose acquisition is financed;

(iv) income-producing real estate

(v) high-volatility commercial real estate

- **Sovereign**
- **Banks**

Exposure classes for IRB

- **Retail**

(i) Residential Mortgage Loans

(ii) Revolving Loans:

- credit lines
- credit cards
- Overdrafts

(iii) Other:

- Small business with exposure <= 1M€
- Consumer loans.

Exposure classes for IRB

- All IRB formulas are based on the Gaussian copula model of time to default (where the joint distribution of 2 loans is a bivariate normal and the defaults are triggered by a common factor (single-factor Vasicek formula), providing the Worst Case Default Rate (WCDR):

$$\mathrm{WCDR} = \mathrm{N}\left[\left(\mathrm{N}^{-1}(\mathrm{PD}) + \sqrt{\mathrm{p}}\,\mathrm{N}^{-1}(0.999)\right) / \sqrt{1-\mathrm{p}}\right]$$

- Particular cases:
  - $\rho = 0 \Rightarrow V(X,T) = N(N^{-1}[Q(T)]) = Q(T) = PD \Rightarrow$ Credit-VaR = EL
  - $\rho \to 1 \Rightarrow V(X,T) \to N(\infty) \to 1 \Rightarrow$ Credit-VaR -> LGD x EAD (>EL, as PD<1) => a portfolio more correlated or less diversified has a higher capital requirement.
- As capital requirements are a proxy for the UL, i.e. the difference between the loss under a very unlikely scenario and the EL, **IRB formulas are WCDR x LGD – EL.**
- Additionally, maturity and debtor size adjustments are made for corporate exposures, while the correlation coefficient either depends on the PD and LGD, or is fixed, according to the level of the portfolio granularity (e.g. higher correlation for residential mortgage loans than for revolving exposures).

Corporate

Risk-weights:

$$\text{Correlation (R)} = 0.12 \cdot \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)} + 0.24 \cdot \left[1 - \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)}\right]$$

$$\text{Maturity adjustment (b)} = \left[0.11852 - 0.05478 \cdot \ln(PD)\right]^2$$

$$\text{WCDR} = \text{N}\left[\left(\text{N}^{-1}(\text{PD}) + \sqrt{\text{p}}\,\text{N}^{-1}(0.999)\right) / \sqrt{1-\text{p}}\right]$$

$$\text{Capital requirement}^{13,14}\text{(K)} = \left[LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot LGD\right] \cdot \frac{\left(1+(M-2.5) \cdot b\right)}{(1-1.5 \cdot b)}$$

$$\text{Risk-weighted assets (RWA)} = K \cdot 12.5 \cdot EAD$$

being N[x] the standardized normal distribution value in x, $G(z)$ the inverse of N[x], R *the* correlation coefficient between exposures and M the maturity (in years).

Size adjustment for corporates:

$$\text{Correlation (R)} = 0.12 \cdot \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)} + 0.24 \cdot \left[1 - \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)}\right] - 0.04 \cdot \left[1 - \frac{(S-5)}{45}\right]$$

Banks

Risk weights:

- Similar to Corporate, but with a multiplier of 1.25 applied to the correlation parameter:

$$\text{Correlation (R\_FI)} = 1.25 \cdot \left[0.12 \cdot \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)} + 0.24 \cdot \left(1 - \frac{\left(1-e^{-50 \cdot PD}\right)}{\left(1-e^{-50}\right)} \right) \right]$$

- For these exposures, as well as for corporates, PD is subject to a floor of 0.03%, LGD is set at 45% for senior claims and 75% for subordinated claims.

Residential Mortgages

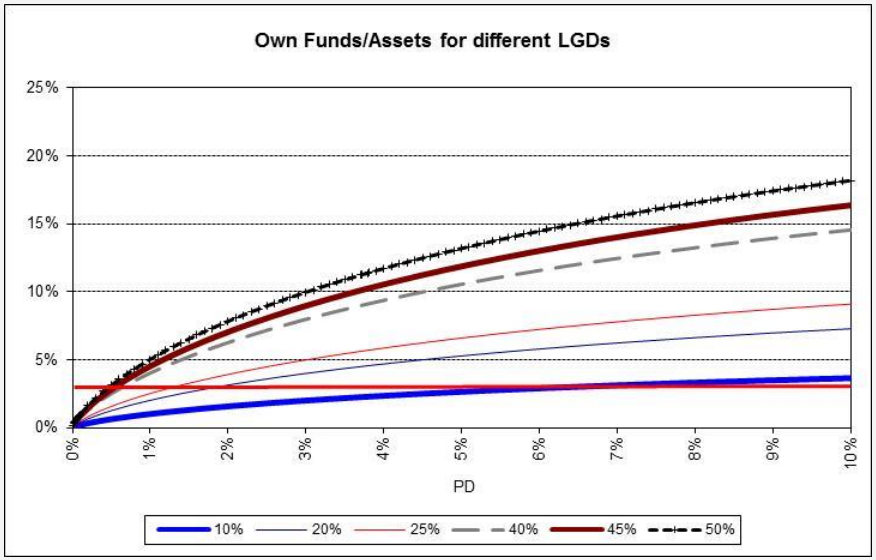
Risk-weights:

Correlation (R) = 0.15

Capital requirement (K) = $\left[LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999) \right] - PD \cdot LGD \right]$

Risk-weighted assets = $K \cdot 12.5 \cdot EAD$

$$\mathrm{WCDR} = \mathrm{N}\left[\left(\mathrm{N}^{-1}(\mathrm{PD}) + \sqrt{\mathrm{p}}\mathrm{N}^{-1}(0.999)\right) / \sqrt{1-\mathrm{p}}\right]$$



Retail Revolving

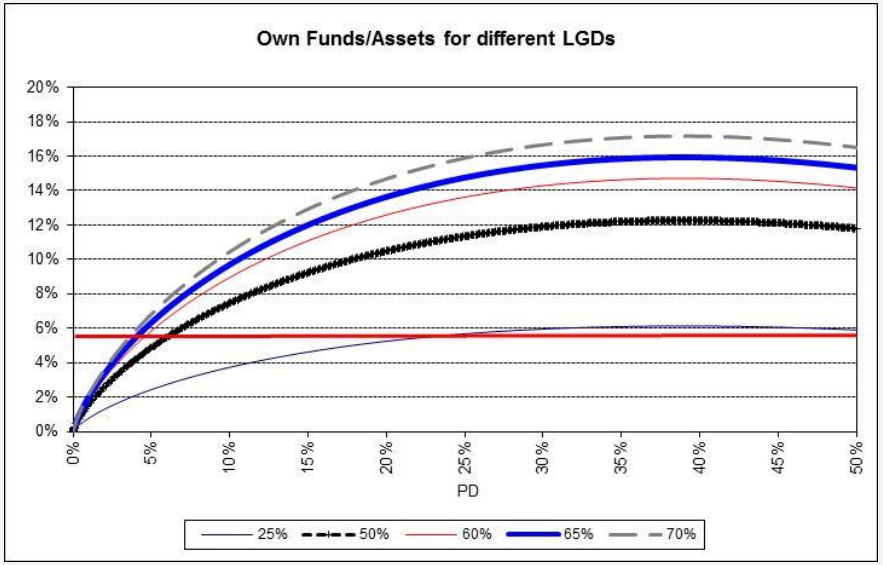
Risk-weights:

$$WCDR = N\left[\left(N^{-1}(PD) + \sqrt{p}N^{-1}(0.999)\right) / \sqrt{1-p}\right]$$

Correlation (R) = 0.04

Capital requirement (K) = $\left[LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot LGD\right]$

Risk-weighted assets = $K \cdot 12.5 \cdot EAD$



Other Retail

Risk-weights:

$$\text{Correlation } (R) = 0.03 \cdot \frac{\left(1 - e^{-35 \cdot PD}\right)}{\left(1 - e^{-35}\right)} + 0.16 \cdot \left(1 - \frac{\left(1 - e^{-35 \cdot PD}\right)}{\left(1 - e^{-35}\right)}\right)$$

$$\text{Capital requirement } (K) = \left[LGD \cdot N\left[\frac{G(PD)}{\sqrt{(1-R)}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot LGD\right]$$

$$\text{Risk-weighted assets} = K \cdot 12.5 \cdot EAD$$

Example

- Gaussian Copula vs Basel requirements for a “Other Retail” exposure:

- Value = 100 M€
- 1y PD = 2%
- $\rho = 0.1$
- RR = 60%
- WCDR (99,9%): $V(X, T) = N\left(\frac{N^{-1}[Q(T)] + \sqrt{\rho}\, N^{-1}(X)}{\sqrt{1-\rho}}\right)$ → $V(0.999, 1) = N\left(\frac{N^{-1}(0.02) + \sqrt{0.1}\, N^{-1}(0.999)}{\sqrt{1-0.1}}\right) = 0.128$

- 1y Credit-VaR = *WCDR* x *LGD* x *EAD* = 0,128 x (1- 0.6) x 100 M€ = 5,13 M€
- 1y EL = PD x LGD x EAD = 002 x (1- 0,6) x 100 M€ = 0.8 M€
- 1y Economic Capital = 1y Credit-VaR – 1y – 1y EL = 5,13 – 0,8 = 4,33
- K = 4,12 (see spreadsheet).

Risk Weight Floors

- **Basel III imposed floors to credit risk weights using IRB**, to avoid capital requirements to fall below 72,5% of the capital requirements derived under the standardised approach.
- There is a phasing-in period (2022-2027), starting with a floor of 50%.

	Pre-floor RWAs	Standardised RWAs	72.5% of standardised RWAs
Credit risk	62	124	89.9
- of which Asset Class A	45	80	58
- of which Asset Class B	5	32	23.2
- of which Asset Class C (not modelled)	12	12	8.7
Market risk	2	4	2.9
Operational risk (not modelled)	12	12	8.7
Total RWA	76	140	101.5

Date	Output floor calibration
1 Jan 2022	50%
1 Jan 2023	55%
1 Jan 2024	60%
1 Jan 2025	65%
1 Jan 2026	70%
1 Jan 2027	72.5%

Source: BCBS (2017), “Basel III: Finalising post-crisis reforms”, Dec.

Credit Risk Mitigation

- Exposures may be collateralised by cash or securities, a guarantee provided by a third party or a credit derivative:

(1) Collateralized transactions:

- **simple approach** - replaces the risk weight of the counterparty by the risk weight of the collateral for the collateralised portion of the exposure (with a 20% floor), for a set of eligible collaterals (that have to be revalued at least every 6 months and be pledged for at least the life of the exposure);
- (i) **comprehensive approach** - more precise reduction of exposures by the collateral, considering a volatility-adjusted value of the collateral.

(2) On-balance sheet netting - capital requirements based on credit exposures, net of the collateral value.

(3) Guarantees and credit derivatives - replaces the risk weight of the debtor by the risk weight of the guarantor or the credit derivative counterparty.

Credit Risk Mitigation

- **Simple approach - Eligible collaterals:**

(i) Cash
(ii) Gold
(iii) Debt securities

- Government Debt: rating >= BB-
- Other entities: rating >= BBB-
- Short-term debt: rating A-3/P-3
- Non-rated debt: senior debt issued by a bank, listed on a recognized exchanged, with similar debt issued with an investment grade rating and liquidity considered by the supervisor as adequate.

- **Comprehensive approach – exposure amount after risk mitigation,** taking into account currency risk and the market risk of the collateral**:**

$$E^* = max\left\{0, E \cdot \left(1 + H_e\right) - C \cdot \left(1 - H_c - H_{fx}\right)\right\}$$

E^*	=	the exposure value after risk mitigation
E	=	current value of the exposure
H_e	=	haircut appropriate to the exposure
C	=	the current value of the collateral received
H_c	=	haircut appropriate to the collateral
H_{fx}	=	haircut appropriate for currency mismatch between the collateral and exposure

Credit Risk Mitigation

- **Supervisory haircuts (Hc and He):**

Supervisory haircuts for comprehensive approach

Jurisdictions that allow the use of external ratings for regulatory purposes — Table 14

Issue rating for debt securities	Residual maturity	Sovereigns[73]	Other issuers[74]	Securitisation exposures[75]
AAA to AA–/A-1	≤ 1 year	0.5	1	2
	>1 year, ≤ 3 years	2	3	8
	>3 years, ≤ 5 years		4	
	>5 years, ≤ 10 years	4	6	16
	> 10 years		12	
A+ to BBB–/ A-2/A-3/P-3 and unrated bank securities per para. 148(c)(ii)	≤ 1 year	1	2	4
	>1 year, ≤ 3 years	3	4	12
	>3 years, ≤ 5 years		6	
	>5 years, ≤ 10 years	6	12	24
	> 10 years		20	
BB+ to BB–	All	15	Not eligible	Not eligible
Main index equities (including convertible bonds) and gold		20		
Other equities and convertible bonds listed on a recognised exchange		30		
UCITS/mutual funds		Highest haircut applicable to any security in which the fund can invest, unless the bank can apply the look-through approach (LTA) for equity investments in funds, in which case the bank may use a weighted average of haircuts applicable to instruments held by the fund.		
Cash in the same currency[76]		0		

Supervisory haircuts for comprehensive approach

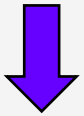
Jurisdictions that do not allow the use of external ratings for regulatory purposes — Table 15

	Residual maturity	Issuer's risk weight (only for securities issued by sovereigns[77]			Other investment-grade securities, consistent with paragraphs 148(d)(iii)[78]	
		0%	20% or 50%	100%	Non-securitisation exposures	Senior securitisation exposures with risk weight < 100%
Debt securities	≤ 1 year	0.5	1	15	2	4
	>1 year, ≤ 3 years	2	3	15	4	12
	>3 years, ≤ 5 years				6	
	>5 years, ≤ 10 years	4	6	15	12	24
	> 10 years				20	
Main index equities (including convertible bonds) and gold		20				
Other equities and convertible bonds listed on a recognised exchange		30				
UCITS/mutual funds		Highest haircut applicable to any security in which the fund can invest, unless the bank can apply the look-through approach (LTA) for equity investments in funds, in which case the bank may use a weighted average of haircuts applicable to instruments held by the fund.				
Cash in the same currency[79]		0				
Other exposure types		30				

- **Hfx = 8%**

Credit Risk Mitigation

- **Example:**
  - Exposure = 800M€
  - Debtor rating: B- => risk-weight =150%
  - Collateral: 700 M€ of BBB bonds with a maturity = 12 years => risk-weight =100%
  - **RWA – simple approach:**
  - the risk-weight corresponding to the debtor will be replaced by the risk-weight of the collateral, in the collateralized amount
  - in the remaining amount, the risk-weight keeps unchanged (150%)



  - **RWA = 100% x 700M + 150% x 100M = 850M€**

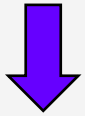
Credit Risk Mitigation

- **RWA – comprehensive approach:**

$$E^* = max\left\{0, E \cdot \left(1 + H_e\right) - C \cdot \left(1 - H_c - H_{fx}\right)\right\}$$

RWA = 150% x [800M - 700M x (1-0,2)] = 360M€

- Non- collateralized: RWA = 150% x 800 M€ = 1200 M€
- Collateralized with 700 M€ of BBB bonds =850 M€



- Collateralization with 700 M€ of cash: RWA = 0% x 700M + 150% x 100 = 150M€

- Collateralization allowed a reduction of the RWA between 350 M€ (with BBB bonds) and 1050M€ (with cash).

2.1.2. Credit Risk in Banking Decision Procedures

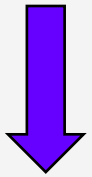
Origination

- Credit Risk assessment in origination involves several steps:

1. Customer’s financial capacity
2. Concentration limits
3. Credit Rules
4. Minimum spread setting and adequacy

Financial Capacity

- The 1st step in credit risk analysis is the assessment of the ability to generate enough cash-flows to face the credit installments.



- The bank must impose credit limits to its larger customers/counterparties (in the corporate and institutional sectors), in line with their ability to absorb debt and the weight in that debt targeted by the bank.
- The limits must depend on the maturities (larger limits for larger maturities).
- For companies, the financial capacity is inferred from the cash-flows exhibited in the financial statements.
- For individual customers, that analysis is based on income and asset information, when the customer applies to a loan, or by bank's estimates (income models) based on relationship data.

Concentration limits

- Credit limit systems are based on common principles:

(i) Diversification of exposures across various dimensions such as customers, industries and regions.

(ii) Avoid lending to any borrower an amount that would increase its debt beyond its borrowing capacity (the equity of the borrower sets up some reasonable limit to its debt, given acceptable levels of debt/equity ratios and repayment ability.

(iii) Set up a maximum risk level, for example defined by the credit standing of borrowers, above which lending is prohibited.

Credit Rules

- Banks usually set credit rules, aka decision filters, leading to the automatic loan approval or rejection.
- Regarding the former, the credit rules are motivated by commercial reasons (e.g. automatic offer of credit cards to private banking customers).
- Concerning the latter, banks usually reject loans whenever the applicants display negative credit events (e.g. non-performing loans).

Minimum spread

- **Minimum spread to be charged in a loan:** $s = Ca + ROE \cdot K + Cf + EL$

with

s = minimum spread (e.g. over the Euribor)

Ca = administrative costs (% total credit)

ROE = return on equity (long-term goal)

K = capital requirement for the loan

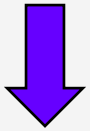
Cf = funding cost (spread over the Euribor)

EL = Expected Loss (PD x LGD x EAD)

EAD - For fixed-credit facilities (term loans), EAD is simply the amount outstanding. For revolving facilities (e.g. lines of credit, liquidity facilities and overdrafts), EAD is the drawn amount plus an estimate of the amount of the remainder limit likely to be drawn at the time of default. These estimates are often referred to as the Credit Conversion Factor or Loan Equivalent.

Minimum spread

- Conceptually, all loans can be accepted, as long as the spread charged to the performing customers is high enough to compensate for the losses with the remaining customers.
- However, if these minimum spreads become too high, given market conditions, they may lead to adverse selection, i.e. to grant loans charging very high spreads to customers who are willing to accept those loans because they know that they will not redeem those loans.



- A maximum EL (risk classification) must be defined as a cut-off level, in order to mitigate adverse selection risk and avoid the adoption of a less conservative credit policy than in the past, leading to a disruption of the PD estimates.
- The PDs are obtained from rating classifications provided by rating agencies or internal credit risk models, which are characterized by a term structure of PDs.

Credit Approval

- These processes vary across banks and types of transaction:

(i) **Retail exposures** - the process relies on procedures that need to accommodate a large volume of transactions, namely credit scorings and delegations (but for many years, retail credit decisions were made by local bank relationship managers based on qualitative factors.

(ii) **Large transactions** - the process involves credit committees, joining the business line, the risk managers and the general top management, with the business line proposing new transactions, together with a risk analysis, and the committee reviewing the deal.

Key Indicators

- Besides the impact of credit risk on impairments, it also affects key management indicators, namely when it materializes in NPL, in two different ways:

(i) level of NPLs– asset quality indicators (usually the focus);
(ii) level of coverage of NPLs by provisions.

(i) level of NPLs

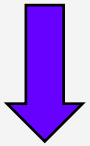
- Definition of credit risk indicators has faced harmonization and transparency problems in EU and worldwide => **3 major issues**:

(a) Installments past-due vs EAD
(b) Default definition – past-due time to be considered? 30 days? 60? 90?
(c) Restructured loans

- For capital requirements, exposures are either performing or non-performing (NPE).
- Restructured loans may demand higher provisions/impairments and impact on capital requirements and NPE or NPL ratios, if they are considered as NPE, following EBA rules.

Key Indicators

- In the past, simpler measures only considered installments past-due, excluding the remaining EAD and the restructured loans:



- 30/60/90 days past due (DPD) capital and interest (as % of total credit) =>
  - ⇒ 30 days past-due was the credit risk ratio commonly used in Portugal.
  - ⇒ Some countries opted for less strict ratios – 60 or 90 DPD.

- Until EBA harmonization efforts took effect in 2014, several definitions were used in Portugal and other EU countries, sometimes simultaneously and even being contradictory.

Key Indicators

- **NPE** (Art. 178 of CRR, Reg. 575/2013 and EBA/GL/2016/07, 28/09/2016):

(i) > 90 DPD (national authorities may replace for 180 days when exposures are guaranteed by residential mortgages and, in loans to SMEs, by commercial mortgages, as well as Public sector entities).

(ii) the debtor is considered as unlikely to pay, with no recourse to additional bank decisions, e.g.:

(a) suspension of interest payments;

(b) loan restructuring, including the forgiveness or the postponement of capital redemption or the payment of interest or fees;

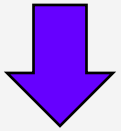
(c) request the debtor insolvency.

Key Indicators

- "Unlikely to pay" (UTP) definition gives some room for interpretation => banks are required to have clearly defined internal criteria to identify UTP indicators/events, implemented homogeneously in all parts of the group.
- **Cross-default**
  - for reporting purposes: NPLs > 20% of the exposure to the customer, except for retail loans.
  - for UTP assessment: banks are expected to set a threshold
- **Economic Groups** - when a debtor belongs to a given group, the need to consider exposures to other group entities also as non-performing must be assessed.
- **Cure Period** – at least 3 months.

Key Indicators

- NPE include the defaulted and impaired exposures, as well forborne exposures where the customer is considered UTP.
- A distressed restructuring is an indication of UTP when it is likely to result in a diminished financial obligation caused by the material forgiveness, or postponement, of principal, interest or, where relevant, fees.
- Diminished financial obligation – assessment based on a comparison between the NPV of expected cash flows before and after the changes in the terms of the contract (both discounted using the original effective interest rate).



- If this difference > a given threshold => exposure must be classified as defaulted.
- The threshold must be set by banks.

Key Indicators

- Banks still have to assess exposures for possible other indications of UTP => if there are reasonable doubts regarding the likeliness of repayment of the obligation according to the new arrangement in full in a timely manner, the obligor should be considered defaulted.

- Indicators to be used:

(i) large balloon payments

(ii) a significant grace period

(iii) the exposure has been restructured multiple times

Key Indicators

(ii) level of coverage of problem loans by provisions

- **Coverage ratio of Past Due Loans** = Provisions (Impairments)/Past Due Loans

- **Texas ratio** = NPLs/(tangible equity + loan loss reserves): compares problem loans with the financial resources a bank has to absorb further losses from its troubled assets.

Lower Texas ratio ⇔ more robust position

External Ratings

- **Banks often employ external ratings issued by international rating agencies** to bonds and their issuers, to assess the risk of their exposures to these securities.
- **Rating agencies have a sound reputation due to their long history**, comprehensiveness and consistency of their assessments in providing credit risk information to the markets, being considered as unbiased analysts of that risk => their ratings are widely accepted by market participants and many investors are restricted to hold only rated instruments.
- **There are 3 main rating agencies – S&P, Moody's and Fitch, with DBRS following.**
- **According to S&P, a rating is an "opinion of the general creditworthiness of an obligor**, or the creditworthiness of an obligor with respect to a particular debt security or other financial obligation, based on relevant risk factors".
- **The rating is always asked by the issuer and updated along time and can be withdrawn following an issuer's request.**

External Ratings

- **The long-term ratings of the main external rating agencies are split by 9 main performing classes**, each of them (excluding AAA) with rating modifiers +/ /- (S&P and Fitch) or 1/2/3 (Moody's).
- In addition, there is a default classification (D).
- Often, for statistical purposes, ratings between CCC and C are merged into a single class.
- Ratings are usually split by market participants as "Investment Grade" (the better ratings) and "Speculative Grade" ("junk") bonds.

	S&P/Fitch	Moody's
Investment Grade	AAA	Aaa
	AA	Aa
	A	A
	BBB	Baa
Speculative Grade	BB	Ba
	B	B
	CCC	Caa
	CC	Ca
	C	C

External Ratings

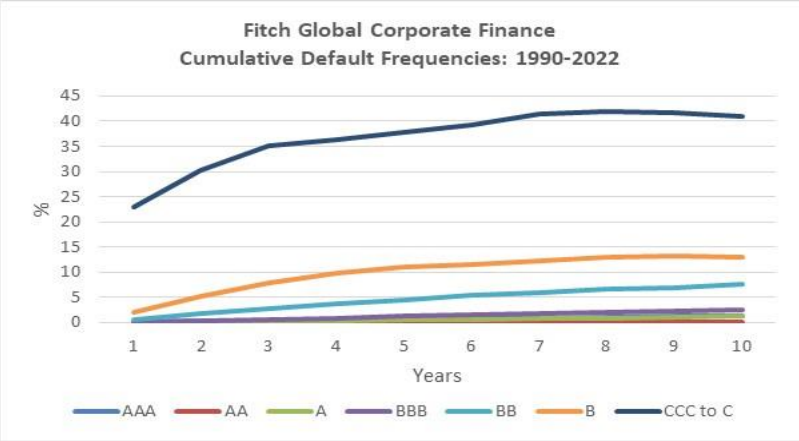
- **In addition to the 9 main performing classes**, rating agencies use rating modifiers in their classification, to have a thinner rating scale, except for the top and the 2 bottom performing rating classes (+/ /- by S&P and Fitch and 1/2/3 by Moody's).

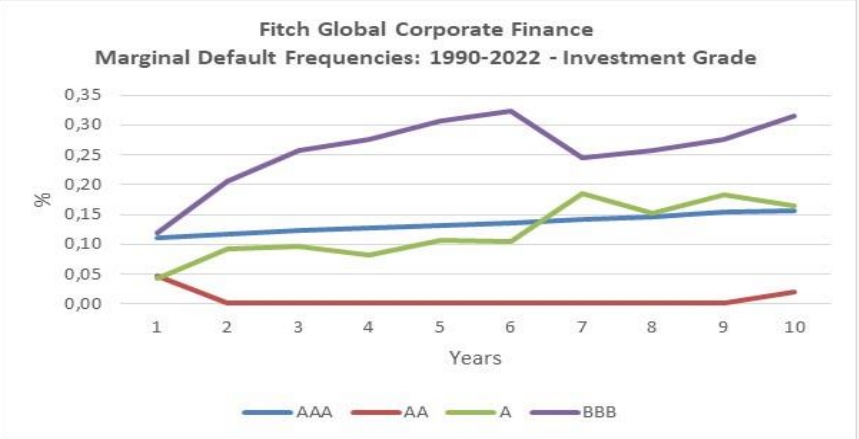
S&P	Moody's	Quality
AAA	Aaa	Prime
AA + AA AA –	Aa1 Aa2 Aa3	High grade
A + A A – BBB + BBB BBB –	A1 A2 A3 Baa1 Baa2 Baa3	Medium grade
BB+ BB BB –	Ba1 Ba2 Ba3	Non-investment grade, 'junk'
B + B B – CCC + CCC	B1 B2 B3 Caa1 Caa2	Highly speculative
CCC – CC C	Caa3 Ca	Near default, limited recovery
D	C	In default

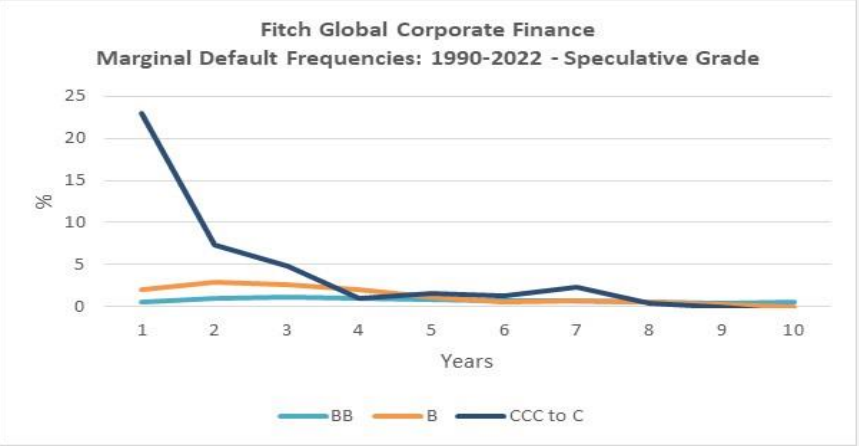
Source: Daníelsson, Jón, "Global financial systems"

External Ratings

- **Default Frequencies may be Cumulative or Marginal:**







Source: Fitch (2023), "2022 Transition and Default Studies", Mar.

External Ratings

- **Stylized Facts:**

(1) Curves often follow the rating order (though sometimes it doesn't happen).

(2) There is a significant difference between investment and speculative grade frequencies, with the former being very close among themselves and close to zero up to 5 years.

(3) Marginal frequencies are not smooth, even if cumulative curves suggest the contrary.

(4) Smooth marginal frequencies of default may be obtained by fitting smooth functions on cumulative curves.

(5) Marginal curves of the riskiest classes have a very pronounced curvature, but they tend to very low figures after 4 years ⇔ if a speculative grade doesn't default in the initial years, it tends to be performing and may even be upgraded.

Transition Matrices

- **Stylized facts:**

(i) Transition matrices illustrate the significant stability of rating classifications, being this stability higher for higher ratings, as the highest frequencies are found in the "main diagonal" and the 2nd highest values are adjacent to this "main diagonal";

(ii) PDs are higher for worse ratings

(iii) Default frequencies are broadly increasing with the ratings' deterioration.

Average one-year letter rating migration rates, 1920-2022*

From\To	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	WR	Def
Aaa	87.1%	7.5%	0.8%	0.2%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%
Aa	1.0%	84.5%	7.6%	0.7%	0.1%	0.0%	0.0%	0.0%	6.0%	0.1%
A	0.1%	2.6%	85.8%	5.2%	0.6%	0.1%	0.0%	0.0%	5.5%	0.1%
Baa	0.0%	0.2%	3.9%	84.1%	4.1%	0.6%	0.1%	0.0%	6.7%	0.2%
Ba	0.0%	0.1%	0.4%	6.0%	74.6%	6.6%	0.7%	0.1%	10.4%	1.1%
B	0.0%	0.0%	0.1%	0.6%	5.5%	72.2%	6.3%	0.4%	11.8%	3.0%
Caa	0.0%	0.0%	0.0%	0.1%	0.4%	6.0%	69.9%	2.8%	14.0%	6.8%
Ca_C	0.0%	0.0%	0.1%	0.1%	0.4%	2.5%	9.6%	45.2%	17.2%	25.0%

Transition Matrices

Average one-year alphanumeric rating migration rates, 1983-2022*

From\To	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	Caa1	Caa2	Caa3	Ca_C	WR	Def
Aaa	87.2%	5.2%	2.2%	0.5%	0.3%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.3%	0.0%
Aa1	1.6%	77.0%	7.9%	5.6%	1.4%	1.0%	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.1%	0.0%
Aa2	1.0%	4.2%	74.8%	9.7%	3.2%	1.5%	0.4%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%
Aa3	0.1%	1.0%	4.0%	76.0%	8.7%	3.3%	0.8%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.3%	0.0%
A1	0.0%	0.1%	1.0%	5.0%	77.2%	7.3%	2.5%	0.6%	0.4%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.2%	0.1%
A2	0.1%	0.0%	0.2%	0.9%	5.7%	77.3%	7.1%	2.4%	0.9%	0.3%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.5%	0.0%
A3	0.0%	0.0%	0.1%	0.3%	1.4%	6.1%	76.7%	6.6%	2.4%	0.8%	0.3%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.9%	0.0%
Baa1	0.0%	0.0%	0.1%	0.1%	0.2%	1.4%	6.2%	77.3%	6.4%	2.0%	0.5%	0.3%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	4.9%	0.1%
Baa2	0.0%	0.0%	0.0%	0.1%	0.2%	0.5%	1.7%	6.5%	76.9%	6.0%	1.2%	0.5%	0.4%	0.3%	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	5.2%	0.2%
Baa3	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.4%	1.6%	8.7%	73.9%	4.5%	1.9%	0.9%	0.7%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	6.1%	0.3%
Ba1	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.6%	2.3%	10.0%	65.6%	5.6%	3.9%	1.5%	0.6%	0.5%	0.1%	0.2%	0.0%	0.1%	8.1%	0.5%
Ba2	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.3%	0.6%	3.6%	8.1%	64.7%	6.2%	3.7%	1.3%	0.8%	0.3%	0.2%	0.1%	0.1%	8.8%	0.7%
Ba3	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.4%	0.8%	2.7%	7.0%	64.8%	6.9%	3.0%	1.8%	0.7%	0.4%	0.1%	0.1%	9.6%	1.3%
B1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.6%	2.7%	6.9%	63.7%	6.3%	4.4%	1.3%	0.7%	0.2%	0.2%	10.3%	1.8%
B2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%	0.6%	2.1%	7.4%	62.4%	8.0%	3.5%	1.8%	0.4%	0.5%	9.7%	2.8%
B3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.8%	2.5%	6.3%	60.7%	7.5%	3.3%	1.0%	0.8%	12.2%	4.3%
Caa1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.6%	1.3%	7.5%	59.2%	9.4%	2.8%	1.2%	13.9%	3.8%
Caa2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.1%	0.3%	0.7%	1.9%	6.5%	59.8%	6.3%	2.7%	14.7%	6.8%
Caa3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.9%	2.9%	9.9%	46.5%	8.8%	14.3%	16.3%
Ca_C	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.4%	1.5%	1.7%	3.2%	5.8%	38.1%	19.6%	29.2%

Source: Moody's (2023), "Default Trends – Global".

Default Frequencies

- Default frequencies also tend to change along time, namely for lower ratings.

Annual issuer-weighted corporate default rates by letter rating, 1920-2022

Year	Aaa	Aa	A	Baa	Ba	B	Caa-C	IG	SG	All
1920	0.0%	0.0%	0.3%	0.9%	2.2%	4.4%	0.0%	0.4%	3.0%	1.2%
1921	0.0%	0.2%	0.4%	0.6%	0.4%	2.7%	13.3%	0.4%	2.2%	1.1%
1922	0.0%	0.2%	0.2%	1.1%	1.1%	1.7%	7.6%	0.5%	1.6%	1.0%
1923	0.0%	0.0%	0.0%	0.6%	0.9%	2.3%	5.9%	0.2%	1.7%	0.8%
1924	0.0%	0.4%	0.0%	0.1%	2.1%	2.7%	12.9%	0.1%	2.9%	1.2%
1925	0.0%	0.0%	0.1%	0.7%	1.7%	2.6%	14.4%	0.3%	2.6%	1.2%
1926	0.0%	0.4%	0.1%	0.1%	1.4%	2.9%	3.7%	0.2%	1.9%	0.8%
1927	0.0%	0.0%	0.2%	0.0%	1.3%	2.0%	12.9%	0.1%	1.8%	0.7%
1928	0.0%	0.0%	0.0%	0.0%	0.2%	1.3%	10.5%	0.0%	0.9%	0.4%
1929	0.0%	0.3%	0.0%	0.4%	0.8%	0.9%	9.7%	0.2%	1.4%	0.7%
1930	0.0%	0.0%	0.0%	0.4%	0.9%	3.2%	7.7%	0.2%	2.2%	1.0%
1931	0.0%	0.0%	0.3%	1.1%	3.0%	9.5%	31.7%	0.5%	7.9%	3.6%
1932	0.0%	0.7%	1.1%	0.9%	6.1%	14.0%	34.1%	0.9%	11.0%	5.0%
1933	0.0%	0.0%	0.3%	1.0%	11.7%	18.1%	23.9%	0.3%	15.6%	6.5%
1934	0.0%	0.0%	0.3%	0.9%	2.5%	4.2%	16.5%	0.0%	5.9%	3.4%
1935	0.0%	0.0%	1.4%	1.9%	5.1%	4.3%	13.0%	1.3%	8.2%	3.9%
1936	0.0%	0.0%	0.5%	0.3%	1.2%	2.4%	7.0%	0.5%	2.7%	1.6%
1937	0.0%	0.0%	0.5%	1.0%	1.0%	2.7%	9.1%	0.8%	2.7%	1.7%
1938	0.0%	0.9%	1.8%	2.0%	1.0%	1.5%	12.9%	1.8%	2.6%	2.1%
1939	0.0%	0.0%	0.0%	1.0%	0.8%	1.7%	8.1%	0.4%	1.8%	1.2%
1940	0.0%	0.0%	0.0%	1.4%	0.4%	3.3%	11.9%	0.8%	3.8%	2.5%
1941	0.0%	0.0%	0.0%	0.0%	1.0%	0.8%	5.1%	0.0%	1.7%	1.1%
1942	0.0%	0.0%	0.0%	0.0%	0.0%	0.8%	2.0%	0.0%	0.7%	0.5%
1943	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	0.0%	0.0%	0.8%	0.4%
1944	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	2.6%	0.0%	0.7%	0.4%
1945	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.6%	0.0%	0.8%	0.3%
1946	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1947	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	2.6%	0.0%	0.8%	0.3%
1948	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1949	0.0%	0.0%	0.0%	0.0%	1.4%	1.3%	8.8%	0.0%	1.9%	0.8%
1950	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1951	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%	0.4%	0.2%
1952	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1953	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1954	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	7.1%	0.0%	0.5%	0.2%
1955	0.0%	0.0%	0.0%	0.0%	0.0%	1.6%	0.0%	0.0%	0.5%	0.2%
1956	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1957	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	0.0%	0.0%	0.4%	0.1%
1958	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1959	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1960	0.0%	0.0%	0.0%	0.0%	1.3%	0.0%	0.0%	0.0%	0.7%	0.2%
1961	0.0%	0.0%	0.0%	0.0%	0.8%	0.0%	8.7%	0.0%	1.1%	0.4%
1962	0.0%	0.0%	0.0%	0.0%	1.7%	1.5%	0.0%	0.0%	1.5%	0.5%
1963	0.0%	0.0%	0.0%	0.0%	1.2%	1.5%	0.0%	0.0%	1.2%	0.4%
1964	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1965	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1966	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.0%	0.0%	0.4%	0.1%
1967	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1968	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.0%	0.0%	0.4%	0.1%
1969	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1970	0.0%	0.0%	0.0%	0.5%	4.2%	19.4%	50.0%	0.3%	8.7%	3.6%
1971	0.0%	0.0%	0.0%	0.0%	0.9%	0.0%	12.5%	0.0%	1.2%	0.3%
1972	0.0%	0.0%	0.0%	0.0%	0.0%	0.9%	37.5%	0.0%	1.9%	0.5%
1973	0.0%	0.0%	0.0%	0.5%	0.0%	3.8%	37.5%	0.2%	1.3%	0.5%

Source: Moody's (2023), "Default Trends – Global".

Year	Aaa	Aa	A	Baa	Ba	B	Caa-C	IG	SG	All
1974	0.0%	0.0%	0.0%	0.0%	0.5%	7.2%	0.0%	0.0%	1.3%	0.3%
1975	0.0%	0.0%	0.0%	0.0%	1.0%	6.2%	0.0%	0.0%	1.7%	0.4%
1976	0.0%	0.0%	0.0%	0.0%	1.0%	0.0%	0.0%	0.0%	0.9%	0.2%
1977	0.0%	0.0%	0.0%	0.3%	0.5%	3.2%	33.3%	0.1%	1.4%	0.4%
1978	0.0%	0.0%	0.0%	0.0%	1.1%	5.4%	0.0%	0.0%	1.8%	0.4%
1979	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	0.0%	0.4%	0.1%
1980	0.0%	0.0%	0.0%	0.0%	0.0%	5.0%	33.3%	0.0%	1.6%	0.3%
1981	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%	0.0%	0.7%	0.2%
1982	0.0%	0.0%	0.3%	0.3%	2.8%	2.2%	21.4%	0.2%	3.5%	1.0%
1983	0.0%	0.0%	0.0%	0.0%	1.2%	2.2%	45.1%	0.0%	4.3%	1.0%
1984	0.0%	0.0%	0.0%	0.6%	0.5%	5.2%	18.2%	0.2%	3.1%	0.9%
1985	0.0%	0.0%	0.0%	0.0%	0.9%	7.1%	6.3%	0.0%	3.7%	0.9%
1986	0.0%	0.0%	0.0%	0.8%	2.4%	10.3%	16.2%	0.2%	6.1%	1.8%
1987	0.0%	0.0%	0.0%	0.0%	3.0%	5.3%	9.8%	0.0%	4.3%	1.4%
1988	0.0%	0.0%	0.0%	0.0%	1.4%	5.6%	12.5%	0.0%	3.7%	1.3%
1989	0.0%	0.5%	0.0%	0.5%	3.0%	7.5%	20.4%	0.3%	5.9%	2.2%
1990	0.0%	0.0%	0.0%	0.3%	3.5%	13.7%	45.1%	0.1%	10.4%	3.5%
1991	0.0%	0.0%	0.0%	0.2%	3.8%	13.1%	16.0%	0.1%	9.1%	2.8%
1992	0.0%	0.0%	0.0%	0.0%	0.3%	7.6%	15.7%	0.0%	4.9%	1.3%
1993	0.0%	0.0%	0.0%	0.0%	0.6%	4.3%	14.0%	0.0%	3.4%	0.9%
1994	0.0%	0.0%	0.0%	0.0%	0.0%	4.2%	5.3%	0.0%	2.3%	0.6%
1995	0.0%	0.0%	0.0%	0.0%	0.3%	3.8%	11.5%	0.0%	3.1%	0.9%
1996	0.0%	0.0%	0.0%	0.0%	0.0%	1.5%	10.0%	0.0%	1.7%	0.5%
1997	0.0%	0.0%	0.0%	0.0%	0.2%	2.0%	9.2%	0.0%	1.9%	0.6%
1998	0.0%	0.0%	0.0%	0.1%	0.8%	3.8%	8.3%	0.0%	3.0%	1.1%
1999	0.0%	0.0%	0.0%	0.1%	1.4%	5.0%	15.2%	0.0%	5.3%	2.1%
2000	0.0%	0.0%	0.0%	0.3%	1.4%	5.6%	18.0%	0.1%	6.2%	2.5%
2001	0.0%	0.0%	0.2%	0.2%	1.2%	8.7%	28.7%	0.1%	9.3%	3.5%
2002	0.0%	0.0%	0.2%	0.8%	1.8%	4.5%	26.7%	0.4%	7.8%	2.9%
2003	0.0%	0.0%	0.0%	0.0%	0.9%	2.7%	20.3%	0.0%	5.3%	1.8%
2004	0.0%	0.0%	0.0%	0.0%	0.4%	0.8%	11.4%	0.0%	2.4%	0.8%
2005	0.0%	0.0%	0.0%	0.2%	0.0%	0.8%	7.2%	0.1%	1.7%	0.6%
2006	0.0%	0.0%	0.0%	0.0%	0.2%	1.1%	5.8%	0.0%	1.7%	0.6%
2007	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	4.9%	0.0%	1.0%	0.4%
2008	0.0%	0.5%	0.4%	1.0%	2.3%	4.0%	10.9%	0.6%	5.5%	2.5%
2009	0.0%	0.0%	0.2%	0.9%	1.8%	6.9%	26.5%	0.4%	12.1%	5.0%
2010	0.0%	0.0%	0.2%	0.1%	0.0%	0.4%	8.7%	0.1%	3.1%	1.2%
2011	0.0%	0.2%	0.0%	0.4%	0.2%	0.3%	6.2%	0.2%	2.1%	0.9%
2012	0.0%	0.0%	0.0%	0.1%	0.1%	0.5%	8.0%	0.0%	2.8%	1.2%
2013	0.0%	0.0%	0.1%	0.1%	0.6%	1.0%	6.3%	0.1%	2.7%	1.2%
2014	0.0%	0.0%	0.1%	0.1%	0.1%	0.5%	4.8%	0.1%	2.0%	1.0%
2015	0.0%	0.0%	0.0%	0.0%	0.3%	2.4%	6.6%	0.0%	3.7%	1.8%
2016	0.0%	0.0%	0.0%	0.0%	0.1%	1.6%	9.1%	0.0%	4.6%	2.2%
2017	0.0%	0.0%	0.0%	0.0%	0.5%	0.4%	7.6%	0.0%	3.6%	1.7%
2018	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	5.2%	0.0%	2.5%	1.2%
2019	0.0%	0.0%	0.0%	0.1%	0.0%	1.2%	6.5%	0.1%	3.2%	1.5%
2020	0.0%	0.0%	0.0%	0.1%	0.1%	4.0%	12.8%	0.1%	6.9%	3.2%
2021	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	3.0%	0.0%	1.8%	0.8%
2022a*	0.0%	0.0%	0.0%	0.0%	0.3%	3.3%	3.8%	0.0%	2.9%	1.3%
2022b*	0.0%	0.0%	0.0%	0.9%	4.2%	5.0%	3.9%	0.5%	4.3%	2.3%
Mean**	0.0%	0.1%	0.1%	0.3%	1.0%	3.1%	10.3%	0.1%	2.9%	1.2%
Median**	0.0%	0.0%	0.0%	0.0%	0.5%	2.0%	7.6%	0.0%	1.9%	0.8%
Std Dev**	0.0%	0.2%	0.3%	0.4%	1.6%	3.7%	11.1%	0.3%	2.9%	1.3%
Min**	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Max**	0.0%	0.9%	1.6%	2.0%	11.7%	19.4%	50.0%	1.6%	15.8%	8.5%

Default Frequencies

- Default levels of sovereign issuers are lower than for corporates.

Sovereign One-Year Transition Matrix — 2022 Cohort

(%)	AAA	AA	A	BBB	BB	B	CCC to C	D	WD
AAA	100.00	–	–	–	–	–	–	–	–
AA	–	100.00	–	–	–	–	–	–	–
A	–	6.67	93.33	–	–	–	–	–	–
BBB	–	–	5.26	89.47	–	–	–	–	5.26
BB	–	–	–	–	95.45	4.55	–	–	–
B	–	–	–	–	3.57	75.00	14.29	7.14	–
CCC to C	–	–	–	–	–	14.29	57.14	14.29	14.29

Sovereign Average Cumulative Default Rates: 1995–2022

(%)	Year One	Year Two	Year Three	Year Four	Year Five	Year Six	Year Seven	Year Eight	Year Nine	Year Ten
AAA	–	–	–	–	–	–	–	–	–	–
AA+	–	–	–	–	–	–	–	–	–	–
AA	–	–	–	–	–	–	–	–	–	–
AA–	–	–	–	–	–	–	–	–	–	–
A+	–	–	–	–	–	–	–	–	1.39	1.54
A	–	–	–	1.08	2.25	3.53	4.94	6.49	6.76	8.45
A–	–	–	–	–	–	–	–	–	–	–
BBB+	–	–	1.01	1.04	1.09	1.14	1.22	1.37	1.54	1.82
BBB	–	–	–	–	–	–	–	–	–	–
BBB–	–	1.06	1.67	2.33	3.07	3.90	4.26	4.72	5.17	5.66
BB+	–	–	–	–	–	–	0.80	1.67	1.72	1.82
BB	0.92	1.92	3.03	4.26	4.55	5.00	5.33	5.63	5.80	5.80
BB–	–	–	–	–	0.65	1.34	2.86	3.82	4.27	5.77
B+	0.56	1.76	3.70	5.96	7.75	9.16	10.08	12.26	14.29	12.09
B	2.63	4.42	8.33	12.18	14.58	16.67	19.01	19.09	20.20	22.22
B–	2.70	10.78	16.67	18.89	17.28	17.81	14.71	16.67	18.75	20.00
CCC to C	25.45	33.33	38.46	35.29	39.39	43.75	44.83	42.31	37.50	36.36
Investment Grade	–	0.14	0.29	0.45	0.63	0.84	0.97	1.12	1.30	1.49
Speculative Grade	2.41	4.26	6.12	7.36	8.24	9.23	9.90	10.63	11.24	11.54
All Sovereigns	0.96	1.76	2.56	3.11	3.53	4.00	4.31	4.65	4.97	5.19

Source: Fitch Ratings (2023), " Sovereign 2023 Transition and Default Study", 19 May.

PDs

- **Cumulative probability of surviving *t* years**: *P(t)*

- **Unconditional marginal probability of survival (*p'(s)*)** - probability of survival between any times *t* and $s \geq t$ as seen today ⇔ difference between the cumulative probability of survival until *s* and the same probability until *t*:

  $p'(s)=[P(s)-P(t)]$

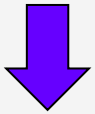
- **Unconditional marginal probability of default between *t* and *s* (*d'(s)*)** - difference between the cumulative probability of default until *s* and the same probability until *t*:

  $d'(s) = [1-P(s)]-[1-P(t)] = P(t) - P(s) = D(s) - D(t)$

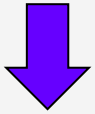
- **Cumulative default frequencies are the sum of unconditional marginal default frequencies.**

PDs

- **Cumulative probability of surviving to time *s*** ($P(s)$) - probability of surviving until *t* ($P(t)$) x probability of surviving between *t* and *s*, given that it has survived until *t* ($p(s|t)$):
  $P(s) = P(t) \times p(s|t)$



- **Conditional marginal probability of surviving to time *s*, given survival to time *t*:**
  $p(s|t) = P(s)/P(t)$



- **Conditional marginal probability of default at time *s*, given survival to time *t*** (or forward default probability):
  $d(s|t)=1-p(s|t) = 1-P(s)/P(t) = [P(t)-P(s)]/P(t) = d'(s)/P(t) \Leftrightarrow d'(s) = P(t) \times d(s|t)$

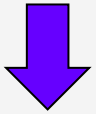
PDs

Average cumulative issuer-weighted global default rates by letter rating, 1920-2022*

Rating\Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Aaa	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.3%	0.4%	0.5%	0.7%	0.8%	0.9%	1.0%	1.0%	1.0%	1.1%	1.1%	1.2%	1.2%	1.3%
Aa	0.1%	0.2%	0.3%	0.4%	0.6%	0.9%	1.1%	1.4%	1.6%	1.8%	2.1%	2.4%	2.7%	2.9%	3.1%	3.3%	3.4%	3.6%	3.8%	4.0%
A	0.1%	0.2%	0.5%	0.7%	1.0%	1.3%	1.7%	2.0%	2.3%	2.7%	3.1%	3.4%	3.8%	4.1%	4.5%	4.8%	5.1%	5.4%	5.7%	5.9%
Baa	0.2%	0.7%	1.2%	1.7%	2.3%	2.9%	3.4%	4.0%	4.6%	5.2%	5.8%	6.4%	7.1%	7.6%	8.2%	8.7%	9.2%	9.7%	10.1%	10.5%
Ba	1.2%	2.8%	4.6%	6.5%	8.3%	10.1%	11.7%	13.3%	14.8%	16.4%	17.9%	19.3%	20.6%	21.8%	23.0%	24.1%	25.2%	26.2%	27.1%	27.8%
B	3.3%	7.5%	11.8%	15.7%	19.3%	22.4%	25.2%	27.6%	29.9%	31.9%	33.8%	35.6%	37.4%	39.1%	40.7%	42.3%	43.7%	44.9%	45.9%	46.8%
Caa-C	9.1%	16.2%	22.1%	27.2%	31.5%	35.2%	38.4%	41.2%	43.8%	46.0%	48.1%	49.9%	51.4%	52.9%	54.5%	56.0%	57.5%	58.9%	60.2%	61.5%
IG	0.1%	0.4%	0.7%	1.0%	1.4%	1.8%	2.1%	2.5%	2.9%	3.3%	3.8%	4.2%	4.6%	4.9%	5.3%	5.7%	6.0%	6.3%	6.5%	6.8%
SG	3.7%	7.5%	11.0%	14.2%	17.1%	19.6%	21.8%	23.8%	25.7%	27.5%	29.1%	30.6%	32.1%	33.4%	34.7%	35.9%	37.1%	38.2%	39.1%	39.9%
All	1.5%	3.1%	4.5%	5.8%	7.0%	8.0%	8.9%	9.7%	10.5%	11.2%	11.9%	12.6%	13.2%	13.8%	14.3%	14.8%	15.3%	15.7%	16.1%	16.5%

Source: Moody's (2023), "Default Trends – Global".

- For the B rating, the **unconditional marginal probability of default** (d') seen today for the 3rd year is equal to the difference between the cumulative probabilities of default for 3 (s) and 2 (t) years: $d'(3) = D(3) - D(2)$ = 11,8% - 7,5% = 4,3%
- **Conditional marginal probability of surviving** at year 3, given survival to year 2: $p(3|2)=P(3)/P(2)$ = (1-0,118)/(1-0,075) =0.9535



- **Conditional marginal probability of default** at year 3, given survival to year 2: $d(3|2)=1-p(3|2)$=1-0.9535=0,0465 or $d(3|2)=d'(s|t)/P(t)$=(0,118-0,075)/(1-0,075)=0,0465

Default Intensity

- **The conditional probability of default between *t* and *s***, given survival until *t*, (*d(s|t)* = *d'(s)/P(t)*), **is also called default intensity or hazard rate**.

- The conditional marginal default probability of the rating B previously calculated (4,65%) was for a 1-year period (between years 2 and 3).

- If one considers a very short period of time Δt, denoting the hazard rate at *t* by λ*(t)*, the **conditional marginal probability of default between *t* and *t* + *Δt*** is λ(*t*) *x Δt* (assuming that the λ is equal for all moments, λ(*t*) is denoted just by λ).

Default Intensity

- Many models of PDs are based on the notion of the **arrival intensity of default**.
- The simplest version of such a model defines default as the 1st arrival time τ of a Poisson process with a constant mean arrival rate = hazard rate (λ):

$P(t) = e^{-\lambda t}$ - cumulative probability of survival for t years

$1/\lambda$ - expected time to default

$\lambda\Delta t$ – default intensity in t over a small period of length Δ (between t and $t+\Delta t$), given survival until t.

- Example: default intensity (λ) = 0,04 =>

=> 1-year PD = $D(1) = (1-P(1)) = 1-e^{-0.04x1}$ =3,9% => expected time to default ($1/\lambda$) = 1/0,04 = 25 (years).

Default Intensity

- As it was shown before, $d(s|t) = d'(s) / P(t)$.
- For a very short period of time Δt, this result becomes:

  $d(t+\Delta t|t) = d'(t+\Delta t)/P(t)$
- As $d'(t+\Delta t)$ is the unconditional probability of default between t and $t+\Delta t$, it is the **difference between the cumulative probabilities of default for $t+\Delta t$ and t:**

$d'(t+\Delta t) = [1– P(t+\Delta t)]-[1– P(t)] = P(t) - P(t+\Delta t) \Rightarrow$ the previous equation becomes (from the equation at the top of the slide):

$d(t+\Delta t|t) = d'(t+\Delta t)/P(t) =$ (from the previous equation) $[P(t) – P(t+\Delta t)]/P(t)$

- As the **conditional marginal probability of default (or default intensity) for a very short period of time is $\lambda\Delta t$**, we have:

  $d(t+\Delta t|t)=[P(t) – P(t+\Delta t)]/P(t) = \lambda\Delta t \Leftrightarrow [P(t+\Delta t) - P(t)] = -\lambda P(t)\ \Delta t \Leftrightarrow$

  $\Leftrightarrow [P(t+\Delta t) - P(t)]/\ \Delta t=-\lambda P(t)$ *or* $dP(t)/dt = -\lambda P(t)$ (when $\Delta t \rightarrow 0$)

Default Intensity

- **If default intensity varies along time**, default intensity becomes $\lambda(t)\Delta t$ and the probability of survival for *t* years becomes:

  $P(t) = e^{-[\lambda(1)+\lambda(2)+\ldots+\lambda(t)]}$

- In continuous time, we get $P(t) = e^{-\int_0^t \lambda(t)dt}$ => $D(t) = 1 - P(t) = 1 - e^{-\int_0^t \lambda(t)dt}$
- However, in reality, as time passes, one should have new information, beyond simply survival, that would bear on the credit quality of an issuer.
- Accordingly, the default intensity will generally vary at random as this additional information arrives.
- For example, one may assume that the intensity varies with an underlying state variable (driver), such as the credit rating, the equity price, or the business cycle.

Default Intensity

- If intensities are updated with new information at the beginning of each year and are constant during the year => Probability of survival to time t given survival to $t-1$, and given all other information available at time $t-1$:

$$P(t-1,t) = e^{-\lambda(t)}$$

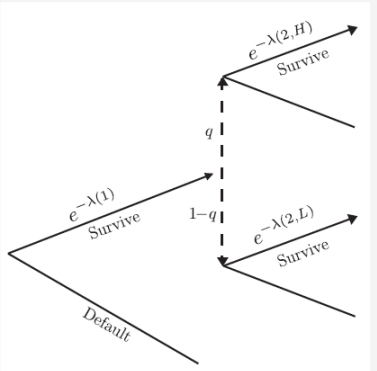
$P(t-1,t)$ is unknown before *t*-1, as $\lambda(t)$ is based on information that is revealed only at time *t*-1.

- At time *t*, we have **2 sources of uncertainty**:

(i) the behaviour in the following period (survival or default);

(ii) new information that will become available during the next period that will be relevant to calculate probabilities of survival and default in the following period.

- Example – 2 periods:



Default Intensity

- Default intensity in the 2nd year ($\lambda(2)$), assuming the firm survives the 1st, is uncertain and takes 2 possible levels, $\lambda(2,H)$ and $\lambda(2, L)$, with conditional probabilities q and $1 - q$, respectively ($p(2|1)$):

$$p(2|1) = qe^{-\lambda(2,H)} + (1-q)e^{-\lambda(2,L)} = E\left[e^{-\lambda(2)}\right]$$

- 2-year survival probability ($\mathrm{P}(2)$):

$$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-\lambda(1)} \cdot E\left[e^{-\lambda(2)}\right] = E\left[e^{-[\lambda(1)+\lambda(2)]}\right]$$

When there was no new information on the hazard rate there was no uncertainty about the λ's:

$$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-[\lambda(1)+\lambda(2)]}$$