Waiting-Line Exercises

WL1: During busy periods, a new customer walks into StyleHair salon every 20 minutes. A typical service time at StyleHair salon lasts 12 minutes. Assume that service time follows a negative exponential distribution and arrivals follow a Poisson distribution. Currently, the salon has one hairdresser.

a) What is the average time a customer has to wait to get a haircut?
b) What is the average time a customer is in the StyleHair salon?
c) What is the average number of customers waiting to get a haircut?
d) What is the average number of customers in the StyleHair salon?
e) What is the probability that the hairdresser is busy?
f) What is the probability of arriving to "StyleHair" and find 1 customer in the queue?

WL2: Paul has been collecting data at the "Stars" Snack Bar. He has found that, between 1:00 P.M and 2:00 P.M, students arrive at the Snack Bar at a rate of 25 per hour (Poisson distributed) and service time takes an average of 2 minutes (exponential distribution). There is only one server, who can work on only 1 order at a time.

a) What is the probability that a client does not need to wait to be served?
b) What is the average number of clients in line?
c) What is the average time a client is in the Snack Bar area?
d) Suppose that a second server can be added to team up with the first (and, in effect act as one faster server). This will reduce the average service time to 90 seconds. How would this affect the average time a client is in the Snack Bar area?
e) Suppose that a second server is added and the two servers work independently, with each taking an average of 2 minutes. What would be the average time a student is in the system?

WL_3: Consider a M/M/1 model and determine the average number of clients in system when:

a) ρ =0,5; b) ρ =0,8; c) ρ =0,9; and d) ρ =0,95.

WL_4: At a loading and unloading dock, trucks arrive at the rate of 3 per day. The operations of loading and unloading are performed by a team of two men who, on average, serve four trucks per day (8 hours). It is estimated that for every additional man on the team, up to a maximum of six men, an extra truck is served. The hourly cost of a truck stopped (and its driver) is 40 €. The hourly cost of a man in the operations of loading and unloading is 12 €. Should we maintain the team of two men or change it? Justify your answer.

WL_5: At the LOGIST Company there are 4 employees responsible for receiving the orders that arrive in the trucks. The LOGIST Director is planning to change the number of employees in order to reduce some costs. The trucks arrive to the LOGIST according to a Poisson rate of 1 per hour. The necessary average time to receive the orders that arrive in the trucks follows an exponential distribution that depends on the number of employees doing the job (suppose that 1 μ =1 n , where n is the number of employees). The cost of each employee is €20,00 per hour. The trucks' waiting time is worth €0,50 per minute.

a) What is the optimal number of employees?

b) After unloading all the orders, each truck is immediately washed by the employees. The necessary time to wash the trucks follows an exponential distribution with an average of 20 minutes. What is the average time that each truck spends in the LOGIST? (Consider that the optimal number of employees is 2 and they always work together)

WL_6: At a hospital emergency room, patients arrive at the rate of 8 per hour. The Manager of the hospital wants that a patient does not wait for the doctor, on average, more than 10 minutes.

a) Determine the average service time of medical staff per patient and the other measures of system performance;

b) The manager decided that if the expected waiting time for a patient exceed one hour it will be transferred to the nearest support hospital. What is the expected number of patients to be transferred per year? (assume 365 days per year and an emergency operation of 24 hours per day).

WL_7:

Relax is a company that offers massage and beauty services in several Airports. The company has just opened a new store, RELAXFARO, at Faro Airport. Every hour, on average 4 passengers arrive to RELAXFARO to receive a massage, according to a Poisson distribution. Currently, RELAXFARO has only one masseur, that takes 12 minutes to do a massage. After finishing the massage, about 50% of the passengers also use the beauty service. The beauty service has one employee that takes, on average, 20 minutes to execute the beauty service. The service time of the beauty service follows a negative exponential distribution.

a) On average, how long does a passenger wait in the massage service?

b) What is the probability of, at any given moment, having 2 or more passengers waiting in the beauty service?

c) What is the probability that, at any given moment, there are four passengers in the beauty service?

c) On average, how many passengers are in RELAXFARO?

(Adapted from exam June 23, 2017).

WL_8:

The insurance company INSURANCE+ will open a new branch, ALMAINSURANCE+, in the iconic R. Flores in Almada. From experience, the operations manager observed service time is variable, approximately following a negative exponential distribution with an average of 20 minutes. A market research study further revealed that the arrival of clients proceeds at a rate of 5 per hour, according to a Poisson distribution. The operations manager will hire three employees to work independently, in two counters that serve a single waiting line. The daily wages will be of 30€ per worker. The estimated hourly waiting cost is 1,5€ per client. The branch will be operating 8 hours per day.

a) What is the average time a client waits before being serviced?
b) How many hours per week an employee expects to spend with customers?
c) What is the total daily cost of the queuing system implemented at the ALMAINSURANCE+ branch?
d) If the arrival rate increases to 14 clients per hour, what is the minimum number of employees that the operations manager needs to hire?

(Adapted from exam June 26, 2013).

WL_9: The manager of the Smile Hospital faces the problem of providing treatment for patients who arrive at different rates during the day. There are only 4 doctors available to treat patients when needed. If not needed, they can be assigned other responsibilities or rescheduled to work at other hours. Patients are treated on a first-come, first-served basis and see the first available doctor. Treatment times follow the exponential pattern and takes, on average, 12 minutes. The arrival pattern for a typical day follows a Poisson distribution and is as follows:

Time	Arrival Rate
9 am-3 pm (09:00-15:00)	6 patients/hour
3 pm–8 pm (15:00-20:00)	4 patients/hour
8 pm–midnight (20:00-24:00)	12 patients/hour

Management feels that, on average, patients should not have to sit in the waiting area for more than 5 minutes before being seen by a doctor. It is known that for the third period indicated the probability of 0 units in the system is 5/89 when S=3 and 25/301 when S=4.

How many doctors should be on duty during each period to maintain the level of patient care expected?

Adapted from Heizer, J. & Render, B. (2017). Operations Management, Global Edition, 12/E, Prentice Hall. (page 796, exercise D.19)

WL_10: The wheat harvesting season in the American Midwest is short, and farmers deliver their truckloads of wheat to a giant central storage bin within a 2-week span. Because of this, wheat-filled trucks waiting to unload and return the fields have been known to back up for a block at the receiving bin. The central bin is owned cooperatively, and it is to every farmer's benefit to make the unloading/storage process as efficient as possible. The cost of grain deterioration cause by unloading delays and the cost of truck rental and idle driver time are significant concerns to the cooperative members. Although farmers have difficulty quantifying crop damage, it is easy to assign a waiting unloading cost for truck and driver of $18 per hour. During the 2-week harvest season, the storage bin is open and operated 16 hours per day, 7 days per week, and can unload 35 trucks per hour according to an exponential negative distribution. Full trucks arrive all day long (during the hours the bin is open) at a rate of about 30 per hour, following a Poisson pattern.

To help the cooperative get handle on the problem of lost time while trucks are waiting n line or unloading at the bin, find the following:

a) The average number of trucks in the unloading system.
b) The average time per truck in the system.
c) The utilization rate for the bin area.
d) The probability that there are three or more trucks in the system at any given time.
e) The total daily cost to the farmers of having their trucks tied up in the unloading process.
f) As mentioned, the cooperative uses the storage bin heavily only two weeks per year. Farmers estimate that enlarging the bin would cut unloading costs by 50% next year. It will cost $ 9,000 to do so during the off-season. Would it be worth the expense to enlarge the storage area?

Adapted from Heizer, J. & Render, B. (2017). Operations Management, Global Edition, 11/E, Prentice Hall. (page 795, exercise D.13)

WL_11:

An average of one client arrives every three minutes to the CARIMBADO print shop. Arrivals follow a *Poisson* process. CARIMBADO has two counters for the print jobs: an automatic self-service counter and a personalised service counter, with one employee. It is estimated that ¾ of the clients use the self-service counter, where they wait an average of 5 minutes before using the machine. The employee on the personalised service counter takes an average of 0.1 hours to serve a client. Service time follows a negative exponential distribution.

a) What is the probability of a client having to wait to be served in the personalised counter?

b) What is the average number of clients waiting on the queue for the self-service counter?

c) What is the average time a personalised service client spends on the print shop store?

WL_12:

BLOODLAB is a laboratory specializing in two types of clinical analysis: XPT analyzes, and COVID 19 analyzes. XPT analyzes are performed during the morning (from 7 a.m. to 1 p.m.), and COVID 19 analyzes during the afternoon (from 2 p.m. to 6 p.m.). In both cases, customers arrive at the clinic according to a Poisson process. In the morning, an average of 7 customers arrive per hour. At the clinic there are two nurses, who work independently, to carry out XPT analyzes. It takes an average of 12 minutes for each nurse to assist a customer who comes to perform XPT type analyzes. Customers who perform XPT analyzes spend an average of 23.5 minutes on BLOODLAB. In the afternoon, an average of 11 customers arrive per hour to perform COVID 19 analyzes. BLOODLAB has only one nurse specialized in this type of analysis that takes, on average, 5 minutes to serve a client. The service time, in both types of analysis, follows a negative exponential distribution.

a) (1.0 point) On average, how long does a customer wait to perform a COVID 19 analysis?
b) (1.0 point) What is the probability of, at a given moment, having more than 5 customers waiting to perform the COVID 19 analysis?
c) (1.0 point) How many hours a day does a nurse spend with customers who are going to perform the XPT analysis?
d) (1.0 point) What is the average number of customers waiting to perform the XPT analysis?

(Adapted from the exam held on 19 June 2020).

MULTIPLE CHOICE QUESTIONS

1. Clients arrive at the ticket office of MetroLx at an average rate of 27 per hour (according to a Poisson distribution). The ticket office employs one worker whom, on average, takes 2 minutes to serve a client. Servicing time follows a negative exponential distribution.

What is the average number of clients waiting in line to purchase a ticket?		
1		30 clients
2		9 clients
3		8.1 clients
4		7 clients

What is the probability of having, at any given moment in time, more than 3 clients on the ticket office?		
1		65.61%
2		72.90%
3		53.14%
4		59.05%

What is the likelihood of a client having to wait to be served?		
1		90.0%
2		10.0%
3		50.0%
4		81.0%

(Adapted from quiz 2 2012/2013)

2. Six students arrive, on average, every hour to the reception desk of S. Bernard's high school administrative office in accordance with a *Poisson* process. Only one clerk works at administrative office, which led to complaints that the average time students spend in the office, which is 5 minutes, is excessive. Please assume that service time follows a negative exponential distribution.

How long does it take, on average, a student being served by the office clerk?		
1		5 minutes
2		18 minutes
3	x	3.33 minutes
4		1,67 minutes

Assuming the average number of students served in one hour is μ=8 students, what is the probability of finding more than 2 students in line at any given moment?		
1		10.55%
2		31.64%
3		14.06%
4		42.19%

(Adapted from quiz 2 2013/2014)

3. A new customer arrives Mrs Bina Grocery store every 3 minutes. Clients wait for their turn in a single line. Two employees work at the grocery store helping each other: one as a cashier (register the purchases) and other in the packaging of the groceries. This process allows for a service of 25 customers per hour. Assume arrivals follow a *Poisson* distribution and that service follows a negative exponential distribution.

What is the average number of customers waiting in queue to pay for their groceries?		
1		4 clients
2		0.15 clients
3		3.2 clients
4		0.016 clients

(Adapted from quiz 2 2013/2014)

4. The pharmacy GOODHEALTH employs two workers. As clients arrive, they wait on a single queue for the first available clerk to serve them. Average hourly arrivals are 18 and follow a Poisson distribution. Service time is, on average, of 6 minutes and follows a negative exponential distribution. It is known that the probability of the system being empty is equal to 5.26%.

What is the utilisation rate of the system?		
1		77.15%
2		94.74%
3		83.8%
4		90%

What is the average time a client needs to wait before being served?		
1		1.85 hours
2		0.52 hours
3		25.56 minutes
4		7.67 minutes

(Adapted from quiz 2 2012/2013)