Controling inflation

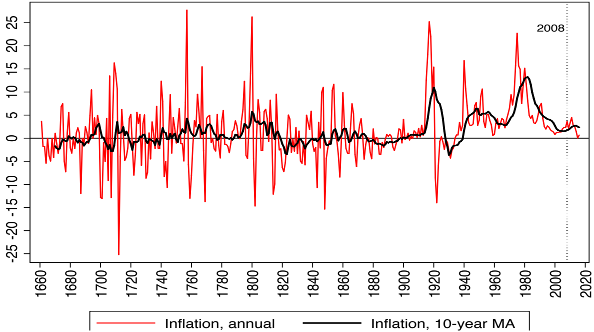
Lecture 14

THE 5 STAGES IN INFLATION

History of inflation

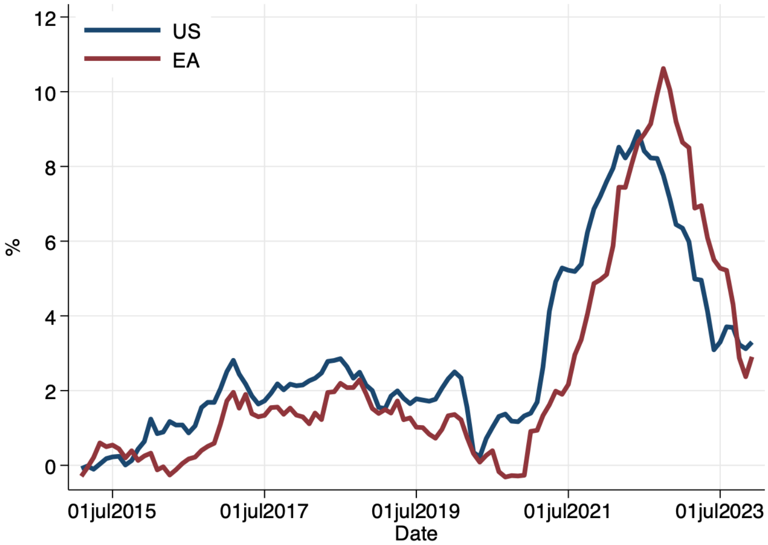
UK: 1660-2016
Average: 1.5%, Std. Dev: 6.5%

Gold Standard (1717-1913)	WW1 & WW2 (1914-1945)	Bretton Woods (1946-1973)	Up to EMS crisis (1974-1993)	Great Mod. (1994-2008)	Post GFC (2008-2016)
μ=0.5%	μ=3.6%	μ=4.8%	μ=8.7%	μ=1.9%	μ=2.2%
σ=6%	σ=8.8%	σ=2.7%	σ=5.6%	σ=0.7%	σ=1.4%



Gold standard; interwar high volatile; Bretton Woods; volatile 70s-80s; the Great stability.

The recent past (CPI-US, HICP-EZ): bad luck or policy?



Questions

- Determinacy: can policy deliver a unique prive level?
- Effectiveness: can policy minimize deviations between actual and target inflation?
- Explain how inflation was controlled for twenty years.

Reference: Laura Castillo-Martinez and Ricardo Reis, 2023, "How do central banks control inflation? A guide for the perplexed", mimeo

Classical economy: savers and investment

$$\mathbb{E}_t\left[M_{t+1}(1+R_t)\right]=1$$

- From micro: M_{t+1} is the MRS, how many units of a good the private agents would require next period in exchange for one unit of good now; $1+R_t$: the opportunity cost of consuming one more unit today in terms of foregone consumption tomorrow.

- From macro: Euler equation. Smooth out marginal utility of consumption over time. But tilt them according to the interest rate.

- From finance: a no-arbitrage condition, risk and time adjusted net return on any investment is zero. SDF is adjustment factor (with risk neutrality: $M_{t+1}=\beta$)

Classical economy: consumers and goods market

$$\Re_t(i) = \frac{P_t(i)}{P_t(0)} \quad \text{for } i = 1, ..., I$$

- Households equate static marginal rates of substitution and relative prices across goods within the same period.

- $\Re(i)$ be how many units of good i consumers would trade for one unit of good 0,

- $P(i)$ is the nominal price of good i.

Firms, workers, and the labor market

$$\tilde{P}_t(i) = Z_t(i) P_t F(Y_t(i), Q_t)$$

- Together maximize surplus from production
- Desired price: $\tilde{P}_t(i)$
- Markup $Z_t(i)$
- Real marginal cost of production are function $F(Y_t(i), Q_t)$ that depends on how much is produced $Y_t(i)$ and on the real cost of inputs, Q_t:

Price index and inflation

- Denominate prices in a unit of account, say EUR.
  . Common unit of account across goods: since care about relative prices only, easier.

- A price: value of good in terms of a unit of account.

- The price level: how much must you must give to get the overall set of goods in the economy.

$$P_t = \mathbb{P}\left(\{P_t(i)\}_{i=0,\ldots I}\right)$$

Linearly homogeneous so that it doubles when all prices double.

- Inflation is the loss of real value of the unit of account.

Market clearing conditions

Stick to closed economy with no savings

$$C_t(i) = Y_t(i)$$

- Consumption CAPM

$$M_{t+1} = \beta \mathbb{U}'(Y_{t+1}) / \mathbb{U}'(Y_t)$$

- Marginal rates of substitution:

$$\Re(i) \equiv \frac{\partial \mathbb{C}(.) / \partial C_t(i)}{\partial \mathbb{C}(.) / \partial C_t(0)}$$

- Disutility from supplying labor

$$Q_t = \mathbb{V}'(Y_t) / \mathbb{U}'(Y_t)$$

Indeterminacy

- Goal is to study $\{P_t\}_{t=0}^{\infty}$ and inflation: $\pi_t = \log(P_t) - \log(P_{t+1})$ or $\Pi_t = P_t/P_{t-1}$, so $\pi_t = \log(\Pi_t)$

- If actual and desired prices are the same, $\tilde{P}_t(i) = P_t(i)$, then $\Re_t(i)$ and M_t are both exogenous with respect to P_t. The real quantities and relative prices are pinned down.

- Nothing in classical supply-demand economics pins down the price level or inflation, like nothing determines whether measurements should be in inches or centimeters.

- Marshallian economics pins down relative prices by marginal rate of substitution and marginal rates of transformation. Not absolute prices. Classical dichotomy.

Nominal rigidities

- There are many ways to break the classical dichotomy. For instance, nominal rigidities that drive a wedge between desired and actual prices.

New Keynesian model, log linearizing the economy around the classical equilibrium, and with firms having Calvo-prices.

$$y_t = \mathbb{E}_t(y_{t+1}) - \theta r_t$$
$$\pi_t = \beta \mathbb{E}_t(\pi_{t+1}) + \kappa\alpha(y_t - y_t^n) + z_t$$

- There are now two equations in three unknowns, r_t, y_t and π_t.

- Price stickiness of firms and workers and aggregate demand makes inflation indeterminacy be a real indeterminacy as well.

Modern monetary system

- Make payments electronically: call to subtract from my cell in the bank's spreadsheet and add to your cell. Since many payments are to buy goods, again same unit of account in the bank spreadsheet.

- When multiple banks, need upper layer spreadsheet. A bank for the banks to perform clearing or settlement.

- The owner of the top spreadsheet is the central bank. The units in the spreadsheet are called the EUR. It determines the unit of account everywhere else.

Central banks and inflation

- Reserves or bank deposits: are the balance in the cell of each bank at the central banks (so are liabilities of central bank). So reserves are the unit of account, price is how many units of reserves must give away to obtain the good.

- Minimal central bank: clearing house / spreadsheet where payments take place using reserves as a digital mean of payment.

- Central bank controls spreadsheet: amount of reserves, V_t, and a rate of remuneration, I_t^v, by which multiply every entry overnight.

Nominal bonds and no arbitrage

- Say that there is a piece of paper that promises to give you 1 nominal unit, that is a +1 entry in your cell.

- This piece of paper costs Q_t nominal units today.

- The return on the bond is $1 + I_t = 1/Q_t$.

- Principle of no arbitrage: I could freely buy bonds with reserves, and vice-versa, and if the return was different, I could make infinite profits by going long on the high-interest rate one, and short in the low-interest rate one.

- By no arbitrage between nominal bonds and reserves delivers $I_t = I_t^v$.

- Central bank has enormous power in affecting nominal interest rates in the economy.

STILL INDETERMINACY OF THE UNIT OF ACCOUNT

- By definition of the price level, it costs Q_t/P_t in real goods to buy a bond. In turn, its real payoff, in units of the consumption good then is $(1 + I_t)/P_{t+1}$.

- Pricing equation for this bond:

$$\mathbb{E}_t \left[M_{t+1} \left(\frac{P_t}{Q_t P_{t+1}} \right) \right] = \mathbb{E}_t \left[M_{t+1} \left(\frac{1 + I_t}{\Pi_{t+1}} \right) \right] = 1.$$

  Reserves promise a nominal interest rate I_t. Real return depends on inflation. Indifference towards holding them must result from equating this expected return times the MRS between consumption today and tomorrow to one.

- Still, for every I_t (or Q_t) there is a different Π_{t+1}. Indeterminacy of equilibrium $\{P_t\}_{t=0}^{\infty}$

WHAT DOES CENTRAL BANK WANT TO DO?

- Policy aim: keep $\{P_t\}_{t=0}^{\infty}$ close to target $\{P_t^*\}_{t=0}^{\infty}$. Target is exogenous w.r.t P_t.

- Policy rules: choose policy tool $I_t^v = f(P_t, X_t^i)$, where X_t^i is an exogenous component.

- Log-linearize around steady state point where the real interest rate and inflation are equal to constants, β and $\bar{P}_t = P_0\bar{\Pi}$ to get $p_t = \log P_t - \log \bar{P}_t$. Notation: $\mathbb{E}_t(p_{t+j})$ is the public's expectation at t of what the price level will be at date $t+j$, while $\hat{p}_{t+j}$ is the central bank's expectation at t

- The effectiveness of a policy is assessed by how small the sequence of deviations between the log price level and its target is:

$$\varepsilon_t \equiv p_t - p_t^*.$$

- The most effective rule, X_t^*: so that errors expected by central bank are $\hat{\varepsilon}_t = 0$.

The Fisher equation

- Combine the two Euler equations to get the Fisher equation

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1 + I_t}{\Pi_{t+1}} \right) \right] = 0.$$

- Economic force: banks can choose to hold reserves or real investments. Say P_t was too low, relative to future fixed P_{t+1}, so higher Π_{t+1}.

- Real returns on nominal reserves is lower. Banks would want to hold zero reserves and invest all of their resources in real terms

- Values of reserves must fall. Because reserves are unit of account, real value is $1/P_t$

- As P_t rises back into equilibrium, lower Π_{t+1}, more demand for reserves, market for reserves clears, banks indifferent between real investment and reserves.

Interest rate peg

Central bank chooses: $I_t = I_t^v = X^i{}_t$.

- From Fisher equation:

$$\mathbb{E}_t\left(\frac{M_{t+1}}{\Pi_{t+1}}\right) = \frac{1}{1+X_t^i}$$

- If there is no uncertainty, choosing X_t^i pins down a single Π_{t+1} at each date. Central bank can pin long run inflation.

- But no other condition to pin down P_0. Units indeterminacy.

- And with uncertainty, only expected time-risk adjusted inflation is pinned down. Actual inflation itself is not determinate.

Payment on reserves

- Central bank promises to remunerate reserve holders with a payment in real goods. The nominal return on reserves in euros would then be $1 + I^{v}_{t,t+1} = (1 + X^{i}_{t})P_{t+1}$.

- Rearrange Fisher equation:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1 + I^v_{t,t+1}}{\Pi_{t+1}} \right) \right] = 0 \Rightarrow \mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{(1 + X^i_t) P_{t+1} P_t}{P_{t+1}} \right) \right] = 0$$

$$\Rightarrow 1 - \mathbb{E}_t \left[M_{t+1} (1 + X^i_t) P_t \right] = 0 \Rightarrow 1 - (1 + X^i_t) P_t / (1 + R_t) = 0$$

$$\Rightarrow P_t = \frac{1 + R_t}{1 + X^i_t}$$

- Since X^i_t is exogenously chosen by policy, and R_t is exogenously pinned down by real forces, then the above equation delivers a determinate price level.

Intuition for payment on reserves

- No central bank does this, but instructive to understand intuition. If the central bank promises a real payment on reserves, then arbitrage pins down how many goods reserves are worth today.

- Since real bonds and reserves both deliver the same payment tomorrow, they must be worth the same today. Since reserves are denominated in euros, not goods, this pins down the price level.

- Rule: $1 + X_t^{i*} = (1 + \hat{R}_t) / \hat{P}_t^*$ is most effective:

$$\varepsilon_t = r_t - \hat{r}_t + \hat{p}_t^* - p_t^*$$

Only current estimation errors matter.

Interest rate feedback rules

- Feedback rules: $I_t = f(P^t, X_t^i)$. Most famous log-linear rules rule is the Taylor rule.

$$i_t = x_t^i + \phi \pi_t.$$

- Combine with log-linearized Fisher equation $i_t = r_t + \mathbb{E}_t(\pi_{t+1})$ to get:

$$\phi(\pi_t - \pi_t^*) = r_t + \mathbb{E}_t(\pi_{t+1}^*) - \phi \pi_t^* - x_t^i + \mathbb{E}_t(\pi_{t+1} - \pi_{t+1}^*)$$

- Iterate forward, Taylor principle sets $\phi > 1$ needed for sums to be well defined.

$$\pi_t = \pi_t^* + \sum_{j=0}^{T-t} \phi^{-j-1} \mathbb{E}_t \left[r_{t+j} + \pi_{t+1+j}^* - \phi \pi_{t+j}^* - x_{t+j} \right] + \phi^{-T+t} \mathbb{E}_t \left(\pi_{T+1} - \pi_{T+1}^* \right).$$

- Impose: $\lim_{T \to \infty} \phi^{-T} \mathbb{E}_t \left(\pi_{t+T} - \pi_{t+T}^* \right) = 0$, argue can't expect inflation to explode

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left(r_{t+j} + \pi_{t+1+j}^* - \phi \pi_{t+j}^* - x_{t+j}^i \right)$$

Most effective Taylor rule

- Respond to the central bank's forecast of real interest rates and the inflation target:

$$x_t^{i*} = \hat{r}_t + \hat{\pi}_{t+1}^* - \phi \hat{\pi}_t^*$$

- Effectiveness is:

$$\varepsilon_t = \varepsilon_{t-1} + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left[r_{t+j} - \hat{r}_{t+j} + \pi_{t+1+j}^* - \hat{\pi}_{t+1+j}^* - \phi(\pi_{t+j}^* - \hat{\pi}_{t+j}^*) \right].$$

- Job of economists in central bank: measure state of the economy $\hat{r}_t$, and solve for optimal inflation target as trade-off with other goal $\hat{\pi}_{t+1}^*$.

- Other job: communication. Say what it thinks the future states of the economy will be. Crucial part of Taylor-style policymaking.

Transparency and management of expectations

Selected Changes in Fed Communication Practices, 1993–2004

November 1993	Decided to release lightly edited transcripts of FOMC meetings for all prior meetings for which a tape existed.
February 1994	For the first time, Chairman Greenspan announced a decision to raise federal funds rate at the conclusion of the policy meeting.
February 1995	Made official the informal policy of announcing decisions of change in policy stance immediately after a policy meeting.
	Agreed to continue to release lightly edited transcripts of meetings after a lag of five years.
August 1997	Included a numerical target for federal funds rate in the policy directive.
May 1999	Began issuing a statement at the conclusion of every meeting, not just after meetings at which policy was changed.
	Began announcing bias in the policy directive (an indicator of future policy) at the conclusion of meeting (accelerating the release of this information by about six weeks).
February 2000	Bias in the policy directive was replaced with a statement about the balance of risks with respect to long-run goals for price stability and economic growth in the foreseeable future.
May 2002	Began releasing roll call vote on the federal funds rate target and the preferred policy for dissenters at the conclusion of the meeting (accelerating the release of this information by about six weeks).
March 2003	Deliberately refrained from the "balance of risks" language. Instead, encouraged "heightened surveillance."
May 2003	Modified language in balance-of-risks statement. Began issuing separate statements about upside and downside risks to inflation and growth.
August 2003	"Balance of risks" was replaced with "considerable period" language.
December 2004	Began publishing meeting minutes three weeks after each meeting (accelerating their release by about three weeks).

Policy discussions

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1}\, \mathbb{E}_t \left(r_{t+j} + \pi^*_{t+1+j} - \phi \pi^*_{t+j} - x^i_{t+j} \right)$$

$$x_t^{i*} = \hat{r}_t + \hat{\pi}^*_{t+1} - \phi \hat{\pi}^*_t$$

- Does raising interest rates raise or lower inflation? If permanent then π^*, raise. If transitory, then x_t^i, lower it.

- Did ECB follow Taylor rule in 2021? Maybe x_t^i was high before because of ZLB, really just reduction in it. And extra cut in it because evaluated weak economy and desired higher inflation target to help recovery.

- Is the central bank credible? Is it succeeding to keep private expectations near its own forecasts and choices of policy?

Other feedback rules: Wicksellian

Table 1: Determinacy conditions

Rule	*Condition*
Benchmark: $x_t^i + \phi\pi_t$	$\phi > 1$
Inertial: $x_t^i + \phi\pi_t + \chi i_{t\ \ 1}$	$\phi + \chi > 1$
Forecast targeting: $x_t^i + \phi\pi_t + \chi\,\mathbb{E}_t(\pi_{t+1})$	$\phi + \chi > 1$
Core inflation: $x_t^i + \phi(1-\chi)\sum_{j=0}^{\infty}\chi^j\pi_{t-j}$	$\phi > 1$
Wicksellian: $x_t^i + \phi p_t$	$\phi > 0$

Mathematics and economic logic of all these cases are similar to the ones in the analysis of the Taylor rule.

- Example with Wicksellian rule

$$\phi p_t + x_t^i = i_t = r_t + \mathbb{E}_t(\pi_{t+1})$$

- Assume $r_t = p_t^* = 0$, difference equation with $\phi > 0$:

$$(1+\phi)p_t = -x_t^i + \mathbb{E}_t(p_{t+1})$$

- Iterate forward and impose terminal condition $\lim_{T\to\infty}(1+\phi)^{-T}\,\mathbb{E}_t\left(p_{t+T}\right) = 0$ to get:

$$p_t = -\sum_{j=0}^{\infty}(1+\phi)^{-j-1}\,\mathbb{E}_t\left(x_{t+j}^i\right)$$

Nominal Rigidities

- Defining the output gap as $\tilde{y}_t \equiv y_t - y_t^n$, there are three relevant equations:

$$\pi_t = \beta \, \mathbb{E}_t(\pi_{t+1}) + \kappa\alpha\tilde{y}_t + z_t$$
$$\tilde{y}_t = \mathbb{E}_t(\tilde{y}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{t+1}) - r_t^n)$$
$$i_t = x_t^i + \phi\pi_t + \phi_y\tilde{y}_t.$$

- Can solve system to get generalized Taylor principle for determinacy

$$\phi > 1 - \frac{\phi_y(1-\beta)}{\kappa\alpha}$$

- Real indeterminacy has an aggregate demand channel as well: changes in the return of financial assets affect households' desire to save, while nominal rigidities make output demand determined. Therefore changes in the interest rate now also affect inflation through changes in consumption.

THE TAYLOR RULE HISTORICALLY

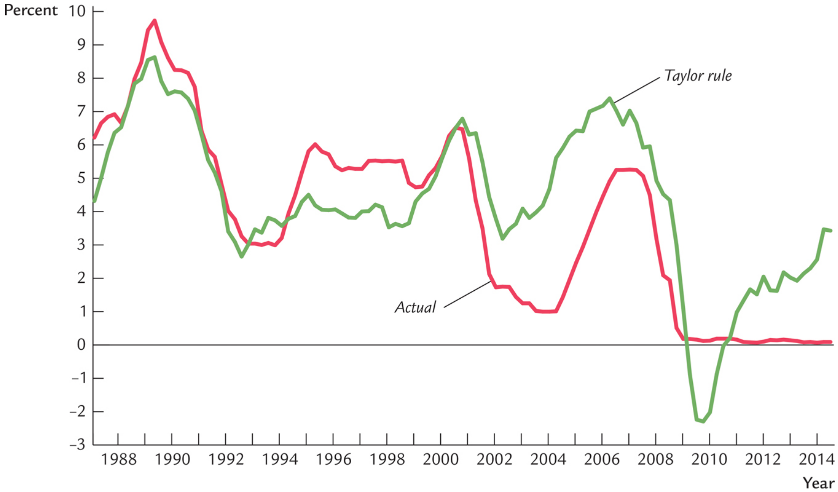
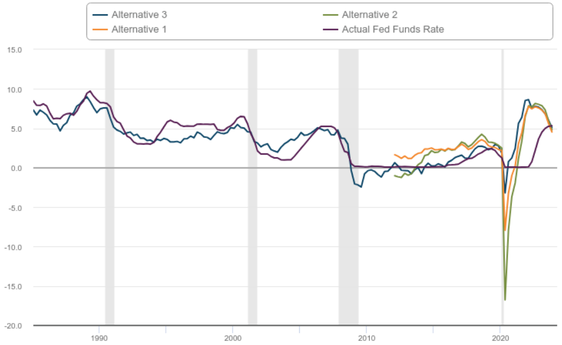


Figure 15.1 The Federal Funds Rate: Actual and Suggested
Mankiw: Macroeconomics, Ninth Edition

THE BIG 2021-22 DEVIATION

Actual Fed Funds Rate and Taylor Rule Prescriptions



	Alternative 1	Alternative 2	Alternative 3
Inflation Target Measures:	2PercentInflation	2PercentInflation	2PercentInflation
Natural Real Interest Rate Measures:	RstarFOMCMedian	RstarFOMCMedian	LWRstar1side
Resource Gap Measures:	U3gapFOMC	U3gapFOMC	CBOGDPgap
Inflation Measures:	CorePCEInflation	CorePCEInflation	CorePCEInflation
Weight on Gap:	0.5	1	0.5
Interest Rate Smoothing:	0	0	0

Source: Atlanta Fed

Federal Reserve Bank *of* Atlanta

Exported on: Sunday, January 21, 2024

IS THE TAYLOR RULE TESTABLE? -

First case: if $\varepsilon_t = 0$, then $\pi_t = \pi_t{}^*$. But then observe:

$$i_t = x_t^i + \phi \pi_t^* = r_t + \pi_{t+1}^*$$

Very effective Taylor rule is observationally equivalent to the peg. Can't estimate ϕ.

- Second case: assume $r_t = \pi_t^* = 0$ and that $x_t^i = \rho x_{t-1}^i + \epsilon_t$ where ϵ_t is iid mean zero.

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left(r_{t+j} + \pi_{t+1+j}^* - \phi \pi_{t+j}^* - x_{t+j}^i \right) = -\sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left(x_{t+j}^i \right) = -\frac{x_t^i}{\phi - \rho}$$

Since inflation is proportional to x_t^i it follows an AR(1) as well. Then:

$$i_t = x_t^i + \phi \pi_t = -(\phi - \rho)\pi_t + \phi \pi_t = \rho \pi_t$$

A regression of i_t on π_t would deliver ρ, not ϕ.

More generally

- More general conclusion: no data on (i_t, π_t) can estimate ϕ unless you can measure (or instrument) changes in $\pi_t - \pi_t^*$ that are uncorrelated with x_t^i.

- Example: assume that $x_t^i = \pi_t^* = 0$ and that $r_t = \rho r_t + \epsilon_t$ where ϵ_t is iid mean zero.

$$\pi_t = \frac{r_t}{\phi - \rho} \Rightarrow \quad i_t = \phi \pi_t$$

  Perfectly identify ϕ, now can't identify ρ.

- But if monetary policy shocks x_t^i are all responses to the state of the economy (including r_t), then there are no such instruments.

- General challenge with estimating macro policy rules: policy affects outcomes and outcomes affect policy. No instruments unless policymakers ignore something.

INTUITION BEHIND DETERMINACY -

The mere presence of $\phi > 1$ solves indeterminacy. How?

- Imagine that inflation is higher at date t by one log unit. Taylor rule raises the nominal interest rate by ϕ.

- Fisher equation increases expected inflation between t and $t+1$ by ϕ.

- But this in turn leads the central bank to raise i_{t+1} by ϕ^2, which raises expected inflation between $t+1$ and $t+2$ by that amount.

- The process continues so inflation keeps on rising exponentially. Inflation in T periods is larger by ϕ^T. Terminal condition rules these deviations out.

- But where does the terminal condition come from in the first place?

The elusive terminal condition

$$\lim_{T\to\infty} \phi^{-T}\,\mathbb{E}_t\left(\pi_{t+T} - \pi^*_{t+T}\right) = 0$$

- Equivalently, the random variable $\mathbb{E}_t\left(\pi_{t+T} - \pi^*_{t+T}\right)$ belongs to $O(\ln(\phi))$.

- This is not an optimality condition. The unit of account may be exploding, but agents don't care as real outcomes continue to be finite.

- Behavioral argument: People would never believe explosive paths for inflation, $\mathbb{E}_t\left(\pi_{t+T} - \pi^*\right)$ is $O(0)$. Limited planning horizons, limited GE understanding, ...

- Economy blows up: With nominal rigidities, explosion in consumption, violate TVC. But with explosion, prices would not stay sticky, so subtle and unclear.

- Coherence bad argument: The derivations relied on log-linearization, bounded ...

Escape clauses

- The central bank commits to a feedback rule only while inflation does not go on an explosive path. If inflation exceeds a pre-announced threshold, the central bank would switch to a different policy approach.

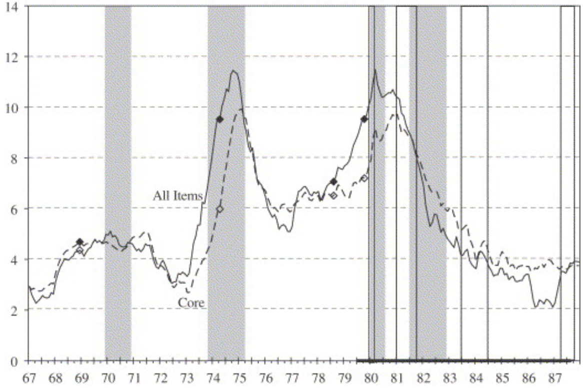
- Recall solution for inflation with a Taylor rule:

$$\pi_t = \pi^* - \sum_{j=0}^{T-t} \phi^{-j-1} \mathbb{E}_t \left[\hat{x}_{t+j}\right] + (1+\phi)^{-T+t} \mathbb{E}_t \left(\hat{\pi}_{T+1} - \pi^*\right)$$

  Switch pins down last term. Inflation is uniquely pinned down as well.

- Realistically, if inflation was rising without bound, no central bank would stick to following blindly a Taylor rule that tells it to raise interest rates more and more, even as it sees inflation rising faster and faster. ECB's monetary pillar perhaps.

ESCAPE CLAUSE: DO A PAUL VOLCKER?



Escape clause: January 2022?

‘No more Mr Nice Guy’: Fed chair signals tougher stance on inflation

Jay Powell refuses to rule out string of aggressive rate rises to bring US prices under control

Jay Powell repeatedly dodged questions about the central bank’s thinking now that inflation appears to be persistent © Financial Times

Colby Smith in Washington JANUARY 27 2022

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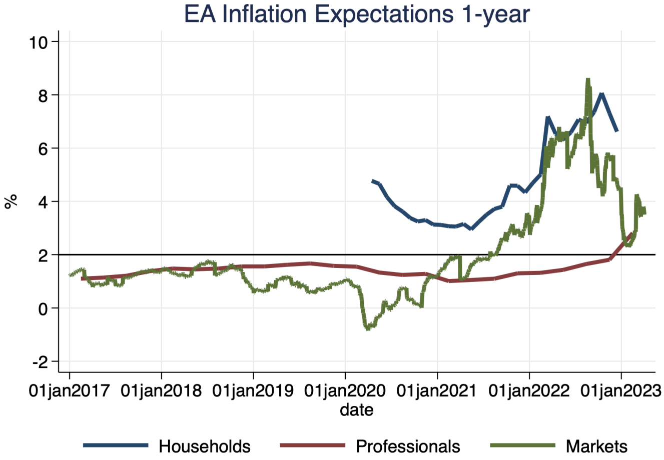


GLOBAL ANALYSIS: NON-LINEAR TAYLOR RULE -

Simplify: no uncertainty, $M_{t+1} = \beta$ and inflation target ${\Pi_t}^*$ is deterministic.
Therefore, there is possibly indeterminacy only with respect to the initial price level.

- Fisher equation:

$$\frac{\beta(1 + I_t)}{\Pi_{t+1}} = 1$$

- Taylor rule:

$$1 + I_t = \Pi_t^{\phi} X_t^{i*}$$

- Most effective rule:

$$X_t^{i*} = \left(\frac{\Pi_{t+1}^*}{\beta \Pi_t^{*\phi}} \right)$$

Global analysis adds nothing

- Combining three equations:

$$\frac{\Pi_{t+1}}{\beta} = \Pi_t^{\phi} \left(\frac{\Pi_{t+1}^*}{\beta \Pi_t^{*\phi}} \right) \Rightarrow \quad \frac{\Pi_{t+1}}{\Pi_{t+1}^*} = \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\phi}$$

- Taking logs gives precisely the same dynamics as in the log-linearized case. Nothing changes.

- Lays to waste defense of terminal condition based on coherence with log-linearizations...

- To carry fewer terms, set target $\Pi_t^* = 1$ from now on, $X_t^{i*} = 1/\beta$

Banknotes and the ELB

- Central banks issue physical banknotes together with reserves. Stand ready to exchange reserves for banknotes one-to-one at all times, so they control only the sum, the monetary base. People can freely substitute between the two components.

- Banknotes have the property that they pay no interest, and given storage costs and risk of theft, they have some gross nominal return of $\xi < 1$.

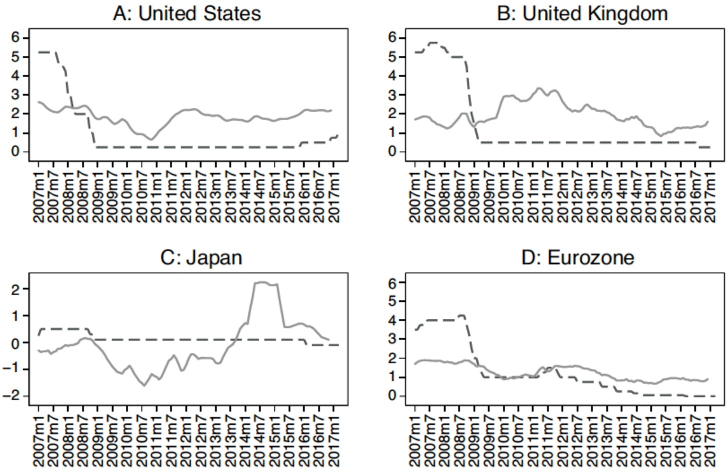
- If inflation explodes downwards, it goes below the real interest rate so the nominal interest rate must be negative as well.

- This puts a constraint on the interest on reserves. Banks would want to substitute all of their reserves for banknotes if interest rates went below ξ. Banknotes imply an effective lower bound (ELB):

$$1 + I_t = \max\{\Pi_{t+1}^{\phi}/\beta, \xi\}$$

The pre-pandemic decade and the ELB

Figure 1.7 Inflation and policy interest rates, 2007-2016



Notes: The solid grey lines plot core inflation and the black dashed lines the policy rate (overnight interbank rate when the policy rate is not available).

The peril of Taylor rules

- Combine Fisher equation on the left with policy rule on right to get

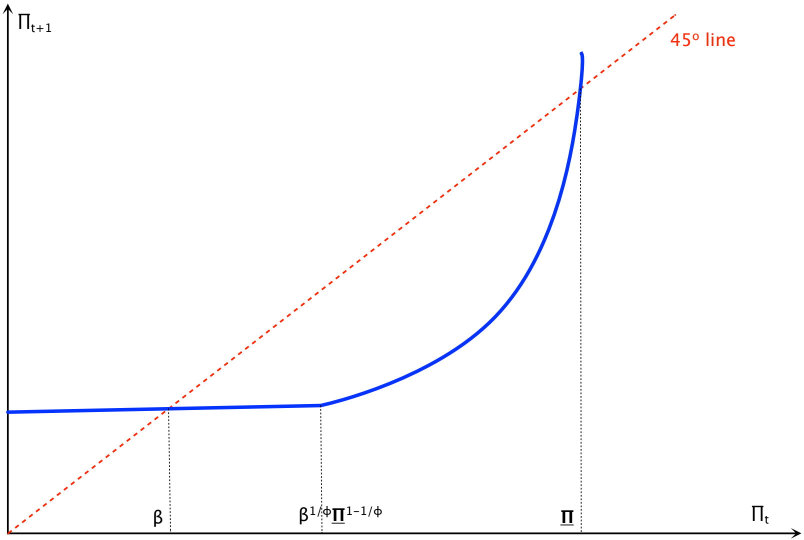
$$\frac{\Pi_{t+1}}{\beta} = \max\left\{\frac{\Pi_t^{\phi}}{\beta\xi}, 1\right\}$$

- Can draw this in phase diagram (next slide but ignore labels in axis). Horizontal line until $\Pi_t^{\phi} = \beta\xi$. After, upward-sloping exponential that crosses 45o line at $\Pi_t = \Pi_{t+1} = 1$ so inflation on target. But to left, horizontal line also crosses 45o line at $\Pi_t = \beta\xi$ with permanent deflation.

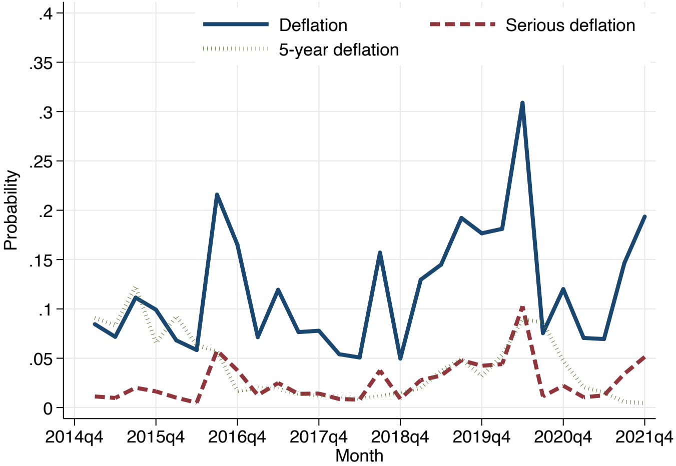
- If start at any $\Pi_0 < 1$, converge to $\beta\xi$, hit it in finite time, stay there forever.

- Price level is again indeterminate: any initial inflation (relative to target) between $\beta\xi$ and 1 is not ruled out by excluding explosive solutions..

Perils of Taylor rules



Lagarde's challenge



How to rule out deflation equilibrium?

1) Use escape clauses whenever the economy threatens to converge to deflation, together with escape clauses if inflation explodes. ECB mission review, state escape clauses.

2) Eliminate or relax the ELB constraint by lowering ξ, perhaps all the way to zero. By: (i) eliminate banknotes, (ii) charge a tax on them, (iii) default on the commitment to exchange currency and reserves one-for-one.

FORWARD GUIDANCE -

Price rigidities interacting with the effective lower bound

- Consider an economy that is at the effective lower bound from period 0 to T. Say pin down inflation from date T onwards.

- With the classical dichotomy: $P_t = (\beta\xi)^{t-T} P_T$ for $t < T$. With nominal rigidities, assuming away shocks for simplicity ($y_t^n = z_t = 0$):

$$\pi_t = (1 + \beta + \kappa\alpha)\pi_{t+1} - \beta\pi_{t+2} - \kappa\alpha(i_t - r_t^n)$$

- When the ELB binds $i_t = \ln(\xi)$. Since π_T and π_{T+1} are determined, there are two terminal conditions for this equation to give the whole path of inflation from 0 to $T - 1$. Just as before, the central bank has no power to affect this path for inflation, which may be very far from the target inflation rate. During this path, deflation comes with output below its natural level (a recession).

Forward guidance puzzle

- The larger is T, the lower is inflation and output at date 0. In the limit, an interest rate peg that lasts forever has an unboundedly large effect on inflation and output today.

- Say that for a fixed number of periods $T^Z < T$, we have $r_t^n = \underline{r} < \ln \xi$, making it impossible to achieve a $\pi_t^* = 0$ target, but that between T^Z and T the central bank chooses to keep the nominal interest rate at $\ln \xi$ even though $r_t^n = 0$.

- The period between T^Z and T is a period of strict *forward guidance*: nominal interest rate at the ELB by choice, it was not constrained.

- The second-order difference equation above has a startling property: the larger is T keeping T^Z fixed (that is the larger is the period of forward guidance), the higher are inflation and output at date Puzzle because patently false in the data

Alternative: going long

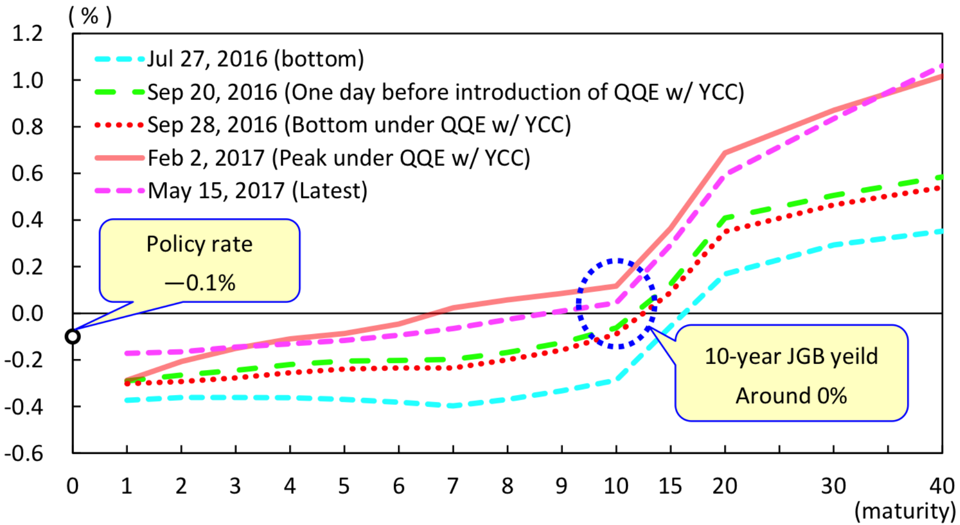
- Focus monetary policy on long-term interest rates.

- By announcing the path for future short-term interest rates (Odyssean forward guidance), and purchases of long-term bonds funded by issuing reserves (quantitative easing).

- Most radical version: yield curve control. Target a 10-year bond rate. How?

- In theory, if the central bank issued bonds of a fixed maturity that were later paid off with reserves, it could choose how to remunerate these bonds just as it does with reserves.

Yield curve control in Japan



Long-term Taylor rules

- If the central bank issues a j period bond and pays I_t^j interest rate on it, then the Euler equation that applies to this new form of investment is:

$$\mathbb{E}_t \left[\frac{M_{t,t+j}(1+I_t^j)}{\Pi_{t+1}\Pi_{t+2}...\Pi_{t+j}} \right] = 1$$

- The stochastic discount factor between two non-successive dates is:

$$M_{t,t+j} = M_{t+1}M_{t+2}...M_{t+j}$$

- By choosing a feedback rule for I_t^j in much the same way as it did for one-period reserves, the central bank can control the price level. The condition for determinacy still requires ϕ to be larger than some threshold.

Setting both short-term and long-term rates

- Simple case: only uncertainty about M_{t+1}, which follows a two-state stationary Markov chain with values M_H and M_L and transition matrix with non-negative probabilities satisfying $f_{HH} + f_{HL} = 1$ and $f_{LH} + f_{LL} = 1$.

- Controlling inflation is determining the two values of inflation, Π_H and Π_L, uniquely.

- The Euler equations for the one-period reserves and the two-period bonds:

$$(1 + I_s^1)\left(f_{sH}\frac{M_H}{\Pi_H} + f_{sL}\frac{M_L}{\Pi_L}\right) = 1,$$

$$(1 + I_s^2)\left(f_{sH}\frac{M_H}{\Pi_H(1 + I_H^1)} + f_{sL}\frac{M_L}{\Pi_L(1 + I_L^1)}\right) = 1.$$

  Two equations in two unknowns. As long as $I_H^1 \neq I_L^1$, unique solution.

- This approach does not pin down P_0.

Defining currency

- Real-life central banks do more than manage their spreadsheet. For one, they issue banknotes and commit to exchange them for reserves one for one at all times.

- Banknotes, or currency, are distinct from reserves in five ways.
  1) They can be freely held by anyone in the economy, not just banks.
  2) They are physical
  3) They are anonymous as people do not have to declare to the government how much currency they have or from whom they got it.
  4) For some payments it may be easier to use banknotes than electronic means backed by reserves (and for others the opposite).
  5) Banknotes pay no interest.

- The first four properties create a demand for the services provided by banknotes separate to the demand for reserves. The fifth property implies that the opportunity cost of using banknotes is the interest rate paid on reserves.

DEMAND AND SUPPLY FOR CURRENCY

- Capture preferences with a utility function: $\mathbb{H}(H_t/P_t)$ where $H_t \geq 0$ are the banknotes held in nominal units. At an optimum the marginal rate of substitution between banknotes and consumption must equal this opportunity cost:

$$\frac{\mathbb{H}'(H_t/P_t)}{\mathbb{U}'(C_t)} = \frac{I_t}{1+I_t}$$

- A log-linearized version of the Fisher equation together with the demand for currency, assuming log utility, gives:

$$h_t - p_t = c_t - \eta(r_t + \mathbb{E}_t\, \pi_{t+1}) + u_t$$

- The u_t represents a shock, disconnect between the banknotes the central bank prints and the money that people find useful given the existence of close substitutes to currency produced by the private market. Large and volatile

Economic forces

1) All else equal, a higher price level today lowers real currency balances supplied by the central bank.
2) Also, it lowers expected inflation between the present and the next period, which lowers the nominal interest rate and raises the demand for banknotes.
3) With lower supply and higher demand for banknotes, the price level must fall to raise the real value of banknotes.
4) This re-equilibrates the market by both increasing the supply, and by lowering demand through a higher nominal interest rate.
5) The logic is soothingly familiar because it reintroduces Marshallian partial-equilibrium supply and demand. But p_t is not the price of the banknotes. Changes in p_t bring the market to equilibrium by affecting both the actual cost of currency i_t *and* also by directly changing the quantity of real currency that is held.

Terminal condition

- Extra optimality condition from the household: at infinity the utility value of the wealth held by the consumer must be zero, otherwise she would be better off consuming more and saving less.

- This is the transversality condition:

$$\lim_{T \to \infty} M_{t,T} \left(\frac{H_T + V_T}{P_T} \right) = 0$$

- Without reserves, log-linearized

$$\lim_{T \to \infty} \beta^T \left(h_T - p_T \right) = 0$$

CLASSIC K-RULE -

Classic rule (Friedman k% rule):

$$h_t = \bar{x}^h t$$

- Replace into the key equation:

$$(1+\eta)(p_t - \bar{x}^h t) = \eta(\mathbb{E}_t(p_{t+1}) - \bar{x}^h(t+1)) + \bar{x}^h + \eta r_t - c_t - u_t$$

  A difference equation for the price level

- Now the transversality condition ensures that the limit term is zero.

- The price level is thus determinate and given by

$$p_t = \bar{x}^h t + \eta \bar{x}^h + \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^j \mathbb{E}_t[\eta r_{t+j} - c_{t+j} - u^d_{t+j} + u^s_{t+j}].$$

Link to interest rate rules

- We can rewrite the equilibrium in the currency market as:

$$i_t = \frac{p_t}{\eta} + \frac{c_t + u_t - h_t}{\eta}.$$

- This is mathematically equivalent to a Wicksellian interest rate feedback rule.

- Since $1/\eta > 0$, it satisfies the determinacy condition. But not a policy rule, rather an equilibrium condition.

- Why? Because the nominal interest rate i_t adjusts endogenously so that the market for currency clears.

EFFECTIVENESS -

The most effective rule for currency supply chooses:

$$h_t = p_t^* + \hat{c}_t - \eta(\hat{r}_t + \hat{p}_{t+1}^* - p_t^*) + \hat{u}_t$$

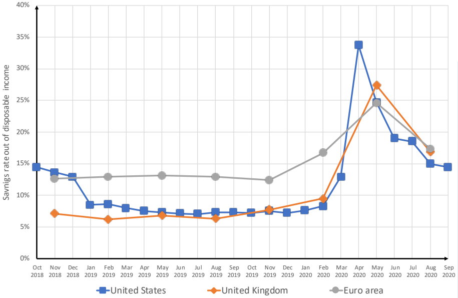
accommodate business cycle and anticipated demand and supply for currency.

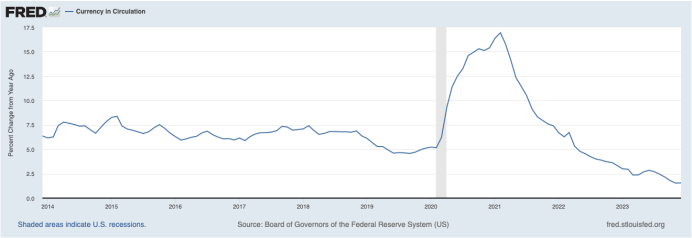
- The effectiveness of this policy is given by

$$\varepsilon_t = \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^j \mathbb{E}_t [\hat{c}_{t+j} - c_{t+j} - \eta(\hat{r}_{t+j} - r_{t+j}) + \eta(p_{t+1+j}^* - \hat{p}_{t+j+1}^*) + (\hat{u}_{t+j} - u_{t+j})].$$

- When tried, usually failed, led to very volatile inflation. But works great as a terminal condition, especially since it rarely needs to activate escape clause.

The pandemic dash for cash and extra savings





Seigniorage

- When the central bank prints currency, it can get goods from agents in return. This gives rise to a resource flow called seignorage.

- Since it costs close to nothing to produce currency and there is a downward-sloping demand for it, currency is not a liability of the central bank, but rather a durable good that it produces and sells for its value $1/P_t$.

- Seignorage is

$$S_t^H = (H_t - H_{t-1})/P_t$$

- The central bank could rebate it right away to the government as a dividend D_t. Danger of the escape clause: monetarism comes with fiscal consequences.

SEIGNIORAGE RULES

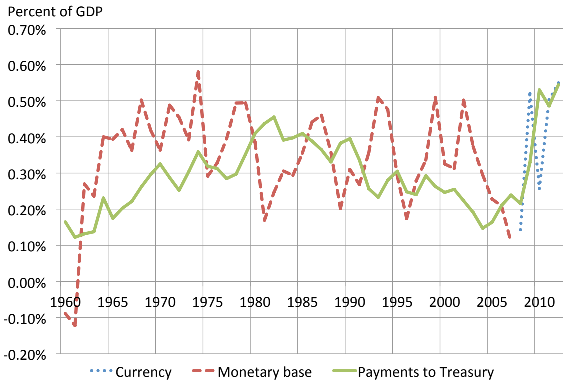
- Seignorage, as a ratio of consumption, is:

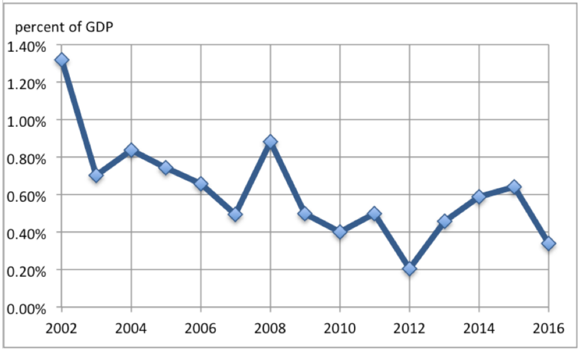
$$\frac{S_t^H}{C_t} = \frac{H_t}{P_t C_t} - \left(\frac{H_{t-1}}{P_{t-1} C_{t-1}}\right)\left(\frac{C_{t-1}}{\Pi_t C_t}\right) \tag{1}$$

- Higher expected inflation comes with higher nominal interest rates, which lowers the demand for currency and lowers seignorage. Also, higher unexpected inflation implies more goods can be bought with newly printed banknotes, raises seignorage.

- Central bank committed to generating some revenues, just like a government fiscal agency that has a target for tax revenues, or a State-owned company providing a public service with a target for profits.

Seigniorage in US and EZ





Policy rules for seignorage

- Fiscal authority says I want you to pay me as dividend some revenue. Central bank must generate it through revenue. Unpleasant monetarist arithmetics.

- Given an exogenous target for seignorage s_t^h, the central bank prints more or fewer banknotes to reach this target. Could solve difference equation, determines inflation. Perfeclty valid way to pin down inflation.

- Large shocks to u_t lead to volatile inflation. In the long run, this approach has often led to hyper-inflation. The reason is that in steady state, $S \leq C$. If try to raise revenue beyond this limit, then inflation is again indeterminate.

- Very common historically. Why monetarism is so widely taught. The dark side of using it as an escape clause.

END OF HYPERINFLATIONS COMES WITH FISCAL REFORMS

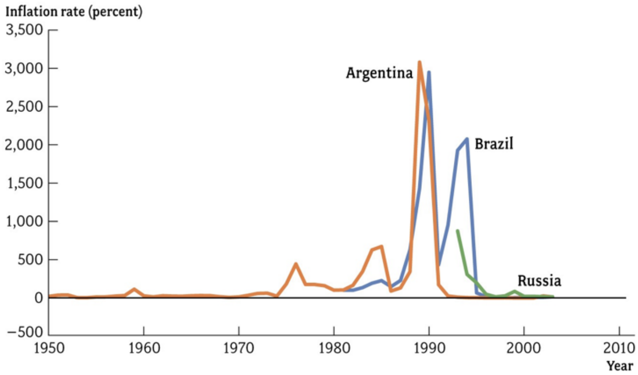


FIGURE 8.5 **Hyperinflations in Argentina, Brazil, and Russia, 1950–2005**

Macroeconomics, Charles I. Jones

Relative prices and price index

- Combining the equality of the marginal rate of substitution across goods with the definition of the price index, we get the log-linearized equation:

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) = p_t(0) + \sum_{i=1}^{I} \omega_i \rho_t(i). \tag{2}$$

- The parameters ω_i, non-negative and summing to one, reflect the weights of each good in the price index, while $\rho(i)$ is the marginal rate of substitution between good i and an arbitrary good 0.

Commodity pegs

- The central bank chooses an arbitrary good, good 0, to denominate its reserves.

- This can happen by decree: the central bank simply announces that 100 pounds will be able to buy one gram of gold. It can then issue reserves (which recall it can do in unlimited amounts) to buy and sell gold to keep this exchange rate fixed forever.

- This uniquely determines the price level.

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) = p_t(0) + \sum_{i=1}^{I} \omega_i \rho_t(i)$$

- Since $p_t(0) = 1$, the price level p_t is determined. Relative price movements would however lead the price level to deviate from target.

EFFECTIVENESS

- Effective rule $p_t(0) = p_t^* + \sum_{i=1}^{I} \omega_i \hat{\rho}_t(i)$ leads to effectiveness:

$$\varepsilon_t = \sum_{i=1}^{I} \omega_i (\rho_t(i) - \hat{\rho}_t(i))$$

- Changes in the supply of good 0, or in the public's taste for it, become sources of deviations of inflation from target.

- If good 0 is a complement with others in consumption, then the impact on relative prices across all goods can be large. The ideal commodity to peg the price level to has a stable supply and is not complementary or substitutable with many other goods. Gold or other precious metals meet these two criteria.

- Pegging to a foreign currency similar. Now $\sum \omega_i(\rho_t(i))$ is a real exchange rate. But effective alternative escape clause.

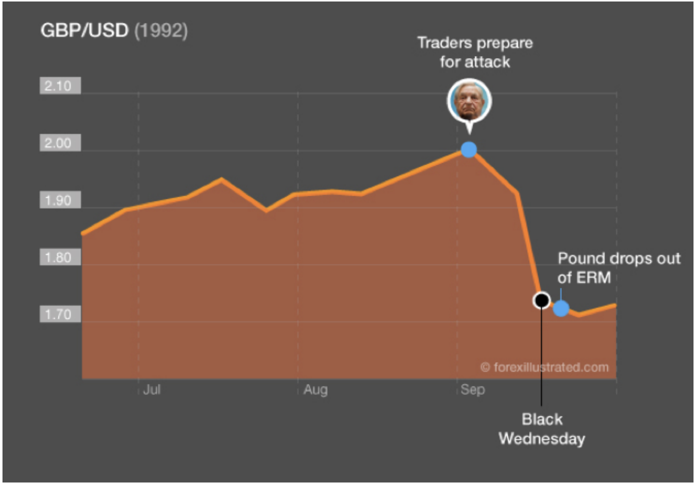
Why do pegs fail?

- Commodity pegs:
  1) Relative-price movements are large enough, so large ε_t.

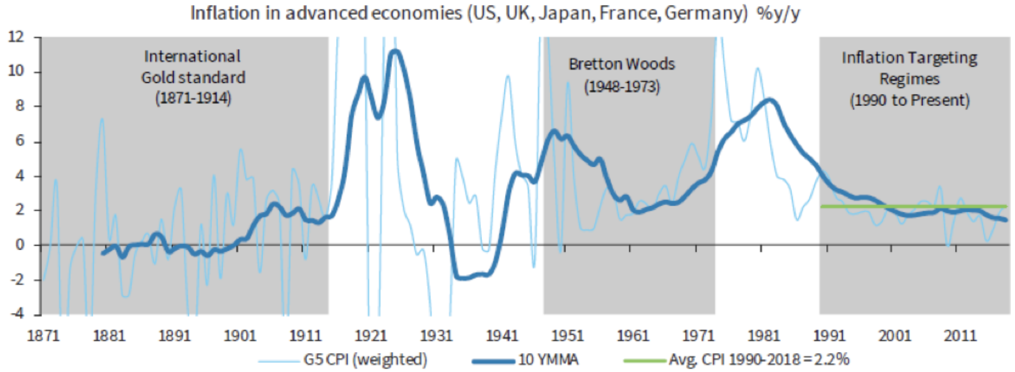
- Currency pegs :
  1) The choice of the q_t target is not made with the goal of delivering a target price level. Changes in $\rho_t(i, j)$ and consequently in the real exchange rate then lead to wide fluctuations in p_t.

  2) Central banks announce a target for q_t but do not commit to it, and deviate when the implications for real activity or foreign trade become unpleasant.

PEGS LEAD TO FLOATS



SUCCESS: CONQUEST OF INFLATION



Source: Jordà-Schularick-Taylor Macrohistory Database, Haver Analytics, Barclays Research

Conclusion

- Inflation: not like other topics in economics, not about supply and demand. Unit of account and determinacy

- Central bank and power of arbitrage: offer a return on the unit of account it controls, pins down loss of value of that unit of account relative to real investments

- Systematic policy rules that distinguish long-run and short-run, measure state of the economy, are transparent, aggressive to control “animal spirits”

- When interest rates near zero, avoid deflation, go long

- Need an escape clause: promises may not be enough, but money pillar is.

- Pegs: when arbitrage not there, alternative to monetarism.