Homework 4
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**Exercise 1**
Suppose the Central Bank (CB) minimizes the following loss function $L$:

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + \varpi(x - x^*)^2\right]$$

where $\pi$ and $x$ stand for actual inflation and output gap, respectively, while $\pi^*$ and $x^*$ are inflation and output gap targets. The economy is described by a Phillips curve (PC) of the form:

$$\pi = \pi^e + \lambda x + \upsilon$$

where $\pi^e$ denotes expected inflation and where initially the shock $\upsilon = 0$.

1. Explain the intuition behind the CB's objective function.

2. State the CB's reduced optimization problem by indicating which terms are exogenous from the CB's point of view.

3. Derive the CB's monetary policy rule (MPR) by solving the CB's optimization problem and interpret the first order condition.

4. For simplicity assume $x^* = 0$. Discuss the monetary policy implications of the MPR and the factors determining it.

5. Suppose now the economy is hit by a positive temporary supply shock $\upsilon$. How does the realization of the supply shocks affect the MPR?


**Exercise 2**
Consider two economies, economy A and economy B. Suppose that the central bank (CB) of economy A assigns a higher weight to output gap stabilization relative to inflation stabilization than the CB of economy B. Other than that, the two economies are identical. Both CBs minimize a loss function of the form

$$L = \frac{1}{2}\left[(\pi - \pi^*)^2 + \varpi(x - x^*)^2\right]$$

where $\pi$ and $x$ stand for actual inflation and output gap, respectively, while $\pi^*$ and $x^*$ are inflation and output gap targets.

The economy is described by a Phillips curve (PC) of the form

$$\pi = \pi^e + \lambda x$$

where $\pi^e$ denotes expected inflation.

1. Derive the monetary policy rule (MPR) of the two CBs by solving their optimization problem. Then assume that target output gap and target inflation are both equal to zero, $x^* = \pi^* = 0$.

2. Represent graphically the long run equilibrium for both economies in the same graph and discuss its properties in terms of equilibrium inflation and

output gap. Is the long run equilibrium different across the two economies? Explain.

3. Assume now that suddenly agents expect higher inflation. Represent graphically the new equilibrium in the short run for both economies and discuss how equilibrium inflation and output gap change. Is the short run equilibrium different across the two economies? Explain.

4. Use the MPR equation together with the PC equation to solve analytically for inflation and output gap in the short run equilibrium as a function of expected inflation, $\pi^e$, and the model parameters. Explain and discuss differences across the two economies.