ALM - Interest Rate Risk Management

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Course Program

- Basic interest rate theory
- `Interest rate risk management`
- Stochastic term structure models
- Risk measurement
- Reinsurance and insurance-linked securities
- Mean-variance analysis for ALM

Contents of the chapter

- Matching (`cash flow matching`).
- Immunisation(`duration/convexity matching`).

Asset-Liability Matching Assumptions

- Assume that the insurance company has an expected `liability cash flow` of $\{L(t) : t > 0\}$ and
- they valued it in its balance sheet by discounting using the `zero-coupon yield curve` $\{y(t) : t > 0\}$:

$$PV_L = \int_0^\infty e^{-y(t)t} dL(t)$$

- Assume also that the insurance company has invested in assets which provide a `future cash flow` of $\{A(t) : t > 0\}$.

Asset-Liability Matching 1

- The discounted value of the assets is

$$PV_A = \int_o^{\infty} e^{-y(t)t} dA(t).$$

- Let $PV = PV_A - PV_L$ be the surplus or `net asset value`.
- The only way to protect the surplus `against all changes in the yield curve` is by matching the asset and liability cash flows.

Asset-Liability Matching 2

- Assume we have n bonds and that $\mathbf{C}^{n \times n}$ is the `payoff matrix` of n bonds. In theory, a matching portfolio of bonds can be found by solving the system of equations

$$\begin{aligned} \left(w_1, \ldots, w_n\right) \cdot \mathbf{C} &\overset{!}{=} \left(L_1, \ldots, L_n\right) \\ \Rightarrow \left(w_1, \ldots, w_n\right) \quad &= \left(L_1, \ldots, L_n\right) \cdot \mathbf{C}^{-1} \end{aligned}$$

- The market value of this matching portfolio is then

$$\left(w_1, \ldots, w_n\right) \cdot \mathbf{B} \overset{!}{=} \left(L_1, \ldots, L_n\right) \cdot \mathbf{C}^{-1}\mathbf{B},$$

a.k.a. the `discounted value of the liabilities`.

Example 1

- In the beginning of script 5, we present an example where:
- we consider a liability cash flow of 1M per year during 15 years;
- we find a portfolio of 15 bonds (the same as in our `market assumptions of script 3`) that matches it.

Practical problems

- Not enough bonds available for a long-tailed liability cashflow.
- Insufficient market liquidity at some maturities.
- The matching portfolio may have some $w_i < 0$ (inadmissible).
- Investment manager may consider matching portfolio sub-optimal.
- Liabilities are random and change all the time (need for rebalancing).

Cash flow immunization

- Proposed by `Frank Redington` in 1952.
- Given that full asset-liability matching is not practical, immunisation attempts to give approximately the `same interest rate sensitivity` to the assets and liabilities.
- Let $PV = PV_A - PV_L$ be the `surplus` or net asset value to be protected from changes in the yield curve.
- The dollar duration and dollar convexity of the surplus are:

$$\begin{aligned} DD &= PV_A \cdot D_A - PV_L \cdot D_L \\ DC &= PV_A \cdot C_A - PV_L \cdot C_L \end{aligned}$$

Immunization - first order matching 1

- A parallel shift of $\Delta\bar{y}$ in the yield curve will change the net asset value by approximately

$$\Delta PV \approx -DD.\,\Delta\bar{y} = \left(-PV_A \cdot D_A + PV_L \cdot D_L\right)\Delta\bar{y}.$$

- Here we use the `first term` of the `Taylor expansion`.
- To protect the value of its surplus against a `shift` in the yield curve, we could select its assets in such a way that

$$DD_A = PV_A \cdot D_A = PV_L \cdot D_L = DD_L.$$

- This is known as `immunization` or `duration matching`.

Immunization - first order matching 2

- Summing up, `immunization` means that the dollar duration of assets equals the dollar duration of liabilities.
- If

$$PV_A = PV_L$$

then we must make sure that the durations are also equal,

$$D_A = D_L,$$

in order to achieve first order immunization.

Immunization - second order matching

- We can add a term to the Taylor expansion and write

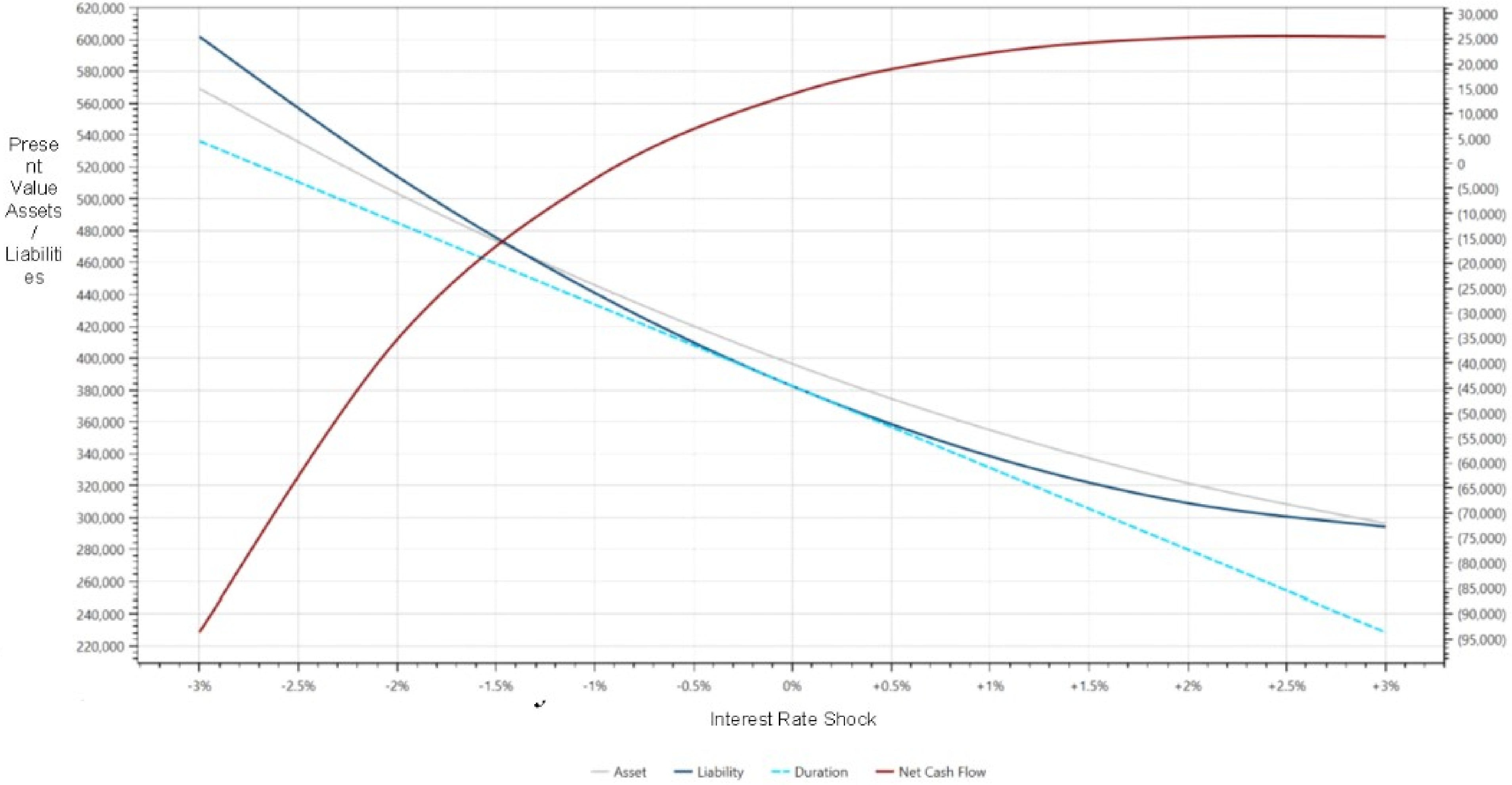
$$\Delta PV \approx \; - DD \cdot \bar{y} + \frac{1}{2} DC \cdot \left(\Delta \bar{y}\right)^2$$

- If $DD = 0$, one could select assets in such a way that $DC \geq 0$, i.e.

$$PV_A \cdot C_A \geq PV_L \cdot C_L$$

- The asset cash flow should have the `same or a higher dollar convexity` than the liability cash flow

Convexity Exposure



Convexity measures the `sensitivity of the duration`.

Some Linear Algebra

- Any liability cash flow can be `duration`-immunised with `two` asset cash-flows by solving the set of linear equations that equate

  1. the dollar present value
  2. the dollar duration

- Any liability cash flow can be `duration and convexity` immunised with `three` asset cash flows, by solving the set of linear equations that equate

  1. the dollar present value
  2. the dollar duration `and` the dollar convexity.

Example 2

- In the remainder of script 5, we present an example where:
- we consider the same liability cash flow of `1M` per year during 15 years;
- we find a portfolio of `3 given bonds` that achieves immunization.
- we check its resilience against `perturbations` in the yield curve (homework).

Alternatives to immunization

- Alternatives between the extremes of complete matching and total independence, of asset and liability values:
  - Immunising with "maturity buckets" of bonds gives a `less spiky` asset cash flow.
  - Immunising separate `maturity sections` of the liability cash flow gives better protection.
  - Not immunising but `limiting` the `dollar duration` of the surplus: $PV_A D_A - PV_L D_L \leq \epsilon \cdot PV_A$
  - Make sure the assets have a `larger dollar convexity` than the liabilities: $PV_A C_A > PV_L C_L$