STATISTICAL METHODS

Master in Industrial Management,

Operations and Sustainability (MIMOS)

2nd year/1st Semester

2025/2026

CONTACT

Professor: Elisabete Fernandes
E-mail: efernandes@iseg.ulisboa.pt

https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM

1. Fundamental Concepts of Statistics
2. Descriptive Data Analysis
3. Introduction to Inferential Analysis
4. Parametric Hypothesis Testing
5. Non-Parametric Hypothesis Testing
6. Linear Regression Analysis

LECTURE 6 HOMEWORK: QUESTIONS AND SOLUTIONS

EXERCISE 5.22

5.22 It is known that amounts of money spent on clothing in a year by students on a particular campus follow a normal distribution with a mean of \$380 and a standard deviation of \$50.

a. What is the probability that a randomly chosen student will spend less than \$400 on clothing in a year?

b. What is the probability that a randomly chosen student will spend more than \$360 on clothing in a year?

c. Draw a graph to illustrate why the answers to parts (a) and (b) are the same.

d. What is the probability that a randomly chosen student will spend between \$300 and \$400 on clothing in a year?

e. Compute a range of yearly clothing expenditures—measured in dollars—that includes 80% of all students on this campus? Explain why any number of such ranges could be found, and find the shortest one.

Newbold et al (2013)

EXERCISE 5.22 A): SOLUTION

✅ Answer:

$X \sim$ Normal$(380, 50^2)$

a. Probability that $X < 400$

Step 1: Compute the z-score:

$$Z = \frac{X - \mu}{\sigma} = \frac{400 - 380}{50} = \frac{20}{50} = 0.4$$

Step 2: Find $P(Z < 0.4)$ using the standard normal table:

Solution:
$P(Z < 0.4) = \Phi(0.4) = 0.6554$

✅ Probability ≈ **0.655** (65.5%)

Standard Normal Table

EXERCISE 5.22 B): SOLUTION

Answer:

$X \sim \text{Normal}(380, 50^2)$

b. Probability that $X > 360$

Step 1: Compute z-score:

$$Z = \frac{360 - 380}{50} = \frac{-20}{50} = -0.4$$

Solution:

P(X > 360) = P(Z > -0.4) = 1-P(Z < -0.4) = 1- $\Phi(-0.4)$ =
1-[1- $\Phi(0.4)$] = $\Phi(0.4)$ = 0.6554

Standard Normal Table

Probability ≈ **0.655** (65.5%)

Observation: Answers to (a) and (b) are the same because of **symmetry of the normal distribution**:

$$P(X < 400) = P(X > 360)$$

since 400 is 20 above the mean and 360 is 20 below the mean.

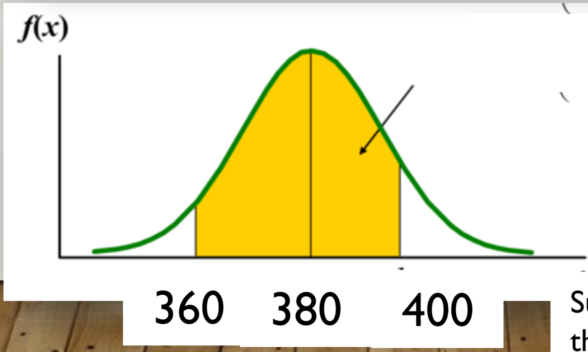
EXERCISE 5.22 C): SOLUTION

Answer:

$X \sim \text{Normal}(380, 50^2)$

c. Graph illustrating symmetry

- Draw a normal curve with mean at 380.
- Shade area to the **left of 400** (part a) and to the **right of 360** (part b).
- The two shaded areas are **mirror images across the mean**, explaining why probabilities are equal.



$$P(X < 400) = P(X > 360)$$

Suppose the area between 360 and the mean of X is the same as the area between the mean of X and 400.

EXERCISE 5.22 D): SOLUTION

✅ Answer:

$X \sim$ Normal(380, 50^2)

d. Probability that $300 \leq X \leq 400$

Step 1: Compute z-scores:

$$Z_1 = \frac{300-380}{50} = \frac{-80}{50} = -1.6$$

$$Z_2 = \frac{400-380}{50} = \frac{20}{50} = 0.4$$

Solution:

P(300 ≤ X ≤ 400) = P(-1.6 ≤ Z ≤ 0.4) = Φ (0.4) - Φ (−1.6) = Φ (0.4) - [1- Φ (1.6)] = 0.6554 – [1-0.9452] = 0.6554 – 0.0548 = 0.6006

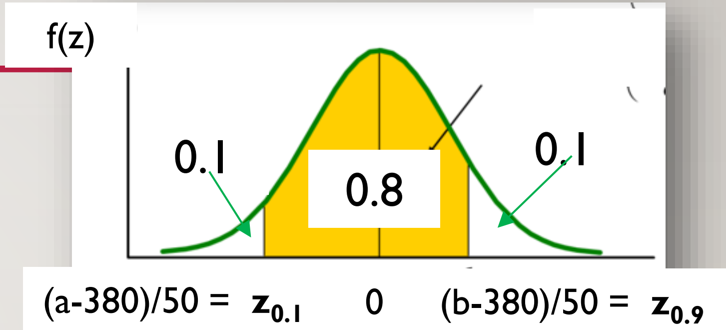
Standard Normal Table

✅ Probability ≈ **0.601** (60.1%)

EXERCISE 5.22 E): SOLUTION

Answer:

$X \sim$ Normal(380, 50^2)



Solution:

$P(a \leq X \leq b) = 0.8 \Leftrightarrow P\left(\frac{a-380}{50} \leq \frac{X-380}{50} \leq \frac{b-380}{50}\right) = 0.8 \Leftrightarrow$

$P\left(\frac{a-380}{50} \leq Z \leq \frac{b-380}{50}\right) = 0.8 \Leftrightarrow \Phi\left(\frac{b-380}{50}\right) - \Phi\left(\frac{a-380}{50}\right) = 0.8$

$\Phi^{-1}(0.9) = z_{0.9}$ is the quantile of the standard normal distribution corresponding to a probability of 0.9 (area on the left).

Then,

$\frac{b-380}{50} = \Phi^{-1}(0.9) = z_{0.9} = 1.28 \Leftrightarrow b = 50 \times 1.28 + 380 = 444$

$\frac{a-380}{50} = \Phi^{-1}(0.1) = -z_{0.9} = -1.28 \Leftrightarrow b = 50 \times (-1.28) + 380 = 316$

Standard Normal Table

✅ Shortest range ≈ **\$316 to \$444**

Note: Any other range covering 80% probability is possible (not necessarily symmetric), but the **shortest range** is always **symmetric around the mean**.

EXERCISE 5.27

5.27 A contractor has concluded from his experience that the cost of building a luxury home is a normally distributed random variable with a mean of \$500,000 and a standard deviation of \$50,000.

a. What is the probability that the cost of building a home will be between \$460,000 and \$540,000?
b. The probability is 0.2 that the cost of building will be less than what amount?
c. Find the shortest range such that the probability is 0.95 that the cost of a luxury home will fall in this range.

Newbold et al (2013)

EXERCISE 5.27 A): SOLUTION

Answer:

$X \sim \text{Normal}(500000, 50000^2)$

a) $P(460{,}000 \le X \le 540{,}000)$.

Standardize:

$$Z_1 = \frac{460{,}000 - 500{,}000}{50{,}000} = -0.8, \qquad Z_2 = \frac{540{,}000 - 500{,}000}{50{,}000} = 0.8.$$

From the standard normal,

$$P(-0.8 \le Z \le 0.8) = \Phi(0.8) - \Phi(-0.8) = 0.78814 - 0.21186 = 0.57628.$$

Answer: ≈ 0.5763 (57.63%).

EXERCISE 5.27 B): SOLUTION

Answer:

X ~ Normal(500000, 50000²)

b) Find x such that $P(X < x) = 0.20$.

The 20th percentile of the standard normal is $z_{0.20} \approx -0.841621$. Convert:

$$x = \mu + z\sigma = 500{,}000 + (-0.841621)(50{,}000) = 500{,}000 - 42{,}081.05 \approx 457{,}919.$$

Round sensibly: **$457,920** (approximately).

Solution:

$$P(X < x) = 0.2 \Leftrightarrow P\left(\frac{X - 500000}{50000} < \frac{x - 500000}{50000}\right) = 0.2 \Leftrightarrow$$

$$P\left(Z \leq \frac{x - 5000000}{50000}\right) = 0.2 \Leftrightarrow \Phi\left(\frac{x - 500000}{50000}\right) = 0.2 \Leftrightarrow$$

$$\frac{x - 500000}{50000} = \Phi^{-1}(0.2) = -z_{0.8} = -0.841621 \Leftrightarrow$$

$$x = 500000 + (-0.841621) \times 50000 = 457.919$$

EXERCISE 5.27 C): SOLUTION

Answer: **$X \sim$ Normal(500000, 50000^2)**

c) Shortest range containing 95% of the distribution.

For a normal distribution the shortest (and symmetric) 95% interval about the mean uses $z_{0.975} \approx 1.96$:

$$\mu \pm 1.96\sigma = 500{,}000 \pm 1.96(50{,}000) = 500{,}000 \pm 98{,}000.$$

Answer: $[\$402{,}000,\ \$598{,}000]$.

Solution:

P(a ≤ X ≤ b) = 0.95 $\Leftrightarrow \Phi\left(\frac{b-500000}{50000}\right) - \Phi\left(\frac{a-500000}{50000}\right) = 0.95$

Then,

$\frac{b-500000}{50000} = \Phi^{-1}(0.975) = z_{0.975} = 1.96 \Leftrightarrow b = 50000 \times 1.96 + 50000 = 598000$

$\frac{a-500000}{50000} = \Phi^{-1}(0.025) = -z_{0.975} = -1.28 \Leftrightarrow b = 50000 \times (-1.96) + 500000 = 402000$

EXERCISE 5.47

5.47 A hospital finds that 25% of its accounts are at least 1 month in arrears. A random sample of 450 accounts was taken.

a. What is the probability that fewer than 100 accounts in the sample were at least 1 month in arrears?

b. What is the probability that the number of accounts in the sample at least 1 month in arrears was between 120 and 150 (inclusive)?

Newbold et al (2013)

EXERCISE 5.47 A): SOLUTION

Answer:

Parameters:

$$X \sim \text{Binomial}(n = 450,\ p = 0.25), \qquad \mu = np = 112.5, \qquad \sigma = \sqrt{np(1-p)} = \sqrt{450 \cdot 0.25 \cdot 0.75} \approx 9.1856.$$

a) $P(\text{fewer than } 100) = P(X < 100) = P(X \leq 99).$

With continuity correction approximate by

$$P(Y \leq 99.5), \qquad Y \sim N(112.5,\ 9.1856^2).$$

Standardize:

$$z = \frac{99.5 - 112.5}{9.1856} \approx -1.4153.$$

Using the standard normal CDF,

$$P(X < 100) \approx \Phi(-1.4153) \approx 0.07850.$$

Answer (a): ≈ 0.0785 (7.85%).

When the normal distribution is used to approximate a binomial distribution, a **continuity correction of 0.5** can be applied to improve accuracy — although in some cases it may be omitted.

Solution (no continuity correction):
P(X < 100) = P(X ≤ 99) ~ P(Z ≤ -1.4697) = $\Phi\,(-1.4697)$
= 1- $\Phi\,(1.4697)$ ~ 1 - 0.9292 = 0.0708

EXERCISE 5.47 B): SOLUTION

✅ Answer:

Parameters:

$$X \sim \text{Binomial}(n = 450,\ p = 0.25), \qquad \mu = np = 112.5, \qquad \sigma = \sqrt{np(1-p)} = \sqrt{450 \cdot 0.25 \cdot 0.75} \approx 9.1856.$$

b) $P(120 \le X \le 150)$ (inclusive). With continuity correction:

$$P(119.5 \le Y \le 150.5).$$

Standardize:

$$z_1 = \frac{119.5 - 112.5}{9.1856} \approx 0.7621, \qquad z_2 = \frac{150.5 - 112.5}{9.1856} \approx 4.1369.$$

So

$$P(120 \le X \le 150) \approx \Phi(4.1369) - \Phi(0.7621) \approx 0.99998 - 0.7770 \approx 0.22299.$$

Answer (b): ≈ 0.2230 (22.30%).

Solution (no continuity correction):
P(120 ≤ X ≤ 150) ~ P(0.8165 ≤ Z ≤ 4.0825) =
Φ (4.0825) - Φ (0.8165) ~1 - 0.7939 = 0.2061

LECTURE 7: EXPONENTIAL DISTRIBUTION

APPLICATIONS OF THE EXPONENTIAL DISTRIBUTION

- Used to model the length of time between two occurrences of an event (the time between arrivals)
  - Examples:
    - Time between trucks arriving at an unloading dock
    - Time between transactions at an ATM Machine
    - Time between phone calls to the main operator

Newbold et al (2013)

PDF OF THE EXPONENTIAL DISTRIBUTION

Probability Density Function (PDF) of a Exponential Distribution

- The exponential random variable *T* (*t* > 0) has a probability density function

$$\boxed{f(t) = \lambda e^{-\lambda t} \quad \text{for} \quad t > 0}$$

The random variable X follows a Exponential distribution with parameter λ > 0: **X ~ Exponential(λ)**.

- Where
  - λ is the mean number of occurrences per unit time
  - *t* is the number of time units until the next occurrence
  - *e* = 2.71828
- *T* is said to follow an exponential probability distribution

Newbold et al (2013)

- Right-skewed distribution
- E(X) = 1/λ
- Var(X) = $1/\lambda^2$
- Possible values: [0, +∞[

CDF OF THE EXPONENTIAL DISTRIBUTION

- Defined by a single parameter, its mean λ (lambda)
- The cumulative distribution function (the probability that an arrival time is less than some specified time *t*) is

Cumulative Distribution Function (CDF) of a Exponential Distribution

$$\boxed{F(t) = P(T \leq t) = 1 - e^{-\lambda t}}$$

X ~ Exponential(λ)

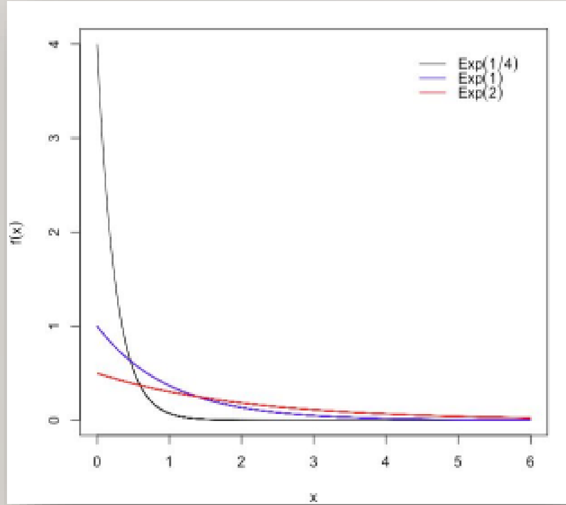
where e = mathematical constant approximated by 2.71828

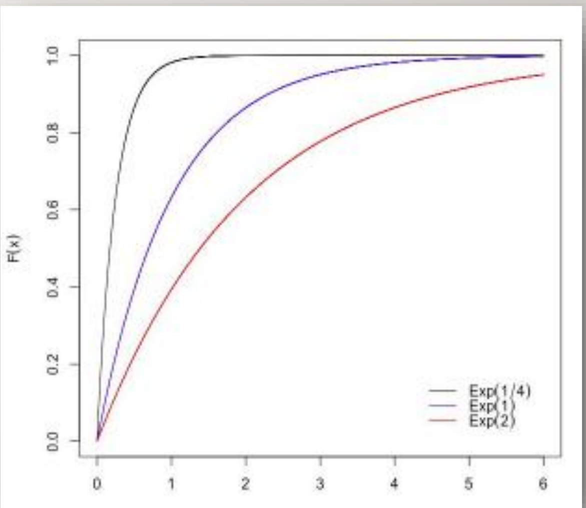
λ = the population mean number of arrivals per unit

t = any value of the continuous variable where $t > 0$

Newbold et al (2013)

PDF AND CDF FOR EXPONENTIAL DISTRIBUTION





Graphical representation of the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable with an Exponential distribution for various values of the parameter λ.

EXPONENTIAL DISTRIBUTION EXAMPLE

Example: Customers arrive at the service counter at the rate of 15 per hour. What is the probability that the arrival time between consecutive customers is less than three minutes?

- The mean number of arrivals per hour is 15, so $\lambda = 15$

X ~ Exponential(15)

- Three minutes is .05 hours

$P(X < 0.05) = F_X(0.05)$

Exponential CDF

$F(t) = P(T \leq t) = 1 - e^{-\lambda t}$

- $P(\text{arrival time} < .05) = 1 - e^{-\lambda x} = 1 - e^{-(15)(.05)} = 0.5276$

- So there is a 52.76% probability that the arrival time between successive customers is less than three minutes

Newbold et al (2013)

In the Exponential distribution, **λ is the rate parameter**, representing the mean number of events per unit of time, while **1/λ is the mean time between events**.

EXERCISE 5.55

5.55 A professor sees students during regular office hours. Time spent with students follows an exponential distribution with a mean of 10 minutes.

a. Find the probability that a given student spends fewer than 20 minutes with the professor.
b. Find the probability that a given student spends more than 5 minutes with the professor.
c. Find the probability that a given student spends between 10 and 15 minutes with the professor.

Newbold et al (2013)

EXERCISE 5.55 A): SOLUTION

Answer:

If $X \sim \text{Exponential}(\lambda)$, then:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

Exponential CDF F(t) = P(T ≤ t) = $1 - e^{-\lambda t}$

We're told the **mean = 10 minutes**, so:

$$\text{Mean} = \frac{1}{\lambda} = 10 \quad \Rightarrow \quad \lambda = 0.1$$

X ~ Exponential(0.1)

(a) Probability that a student spends fewer than 20 minutes:

$$P(X < 20) = F(20) = 1 - e^{-0.1 \cdot 20} = 1 - e^{-2}$$

$$e^{-2} \approx 0.1353 \quad \Rightarrow \quad P(X < 20) \approx 1 - 0.1353 = 0.8647$$

Answer (a): 0.865

EXERCISE 5.55 B): SOLUTION

Answer:

If $X \sim \text{Exponential}(\lambda)$, then:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

Exponential CDF $F(t) = P(T \leq t) = 1 - e^{-\lambda t}$

We're told the **mean = 10 minutes**, so:

$$\text{Mean} = \frac{1}{\lambda} = 10 \quad \Rightarrow \quad \lambda = 0.1$$

$X \sim$ Exponential(0.1)

(b) Probability that a student spends more than 5 minutes:

$$P(X > 5) = 1 - P(X \leq 5) = e^{-\lambda \cdot 5} = e^{-0.5}$$

$$e^{-0.5} \approx 0.6065$$

Answer (b): 0.607

EXERCISE 5.55 C): SOLUTION

Answer:

If $X \sim \text{Exponential}(\lambda)$, then:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

Exponential CDF $F(t) = P(T \leq t) = 1 - e^{-\lambda t}$

We're told the **mean = 10 minutes**, so:

$$\text{Mean} = \frac{1}{\lambda} = 10 \quad \Rightarrow \quad \lambda = 0.1$$

X ~ Exponential(0.1)

(c) Probability that a student spends between 10 and 15 minutes:

$$P(10 \leq X \leq 15) = F(15) - F(10)$$

$$= \left(1 - e^{-0.1 \cdot 15}\right) - \left(1 - e^{-0.1 \cdot 10}\right)$$

$$= e^{-1} - e^{-1.5}$$

$$e^{-1} \approx 0.3679, \quad e^{-1.5} \approx 0.2231$$

$$P(10 \leq X \leq 15) \approx 0.3679 - 0.2231 = 0.1448$$

Answer (c): 0.145

MEMORYLESS PROPERTY OF THE EXPONENTIAL DISTRIBUTION

- **Definition:**

  The Exponential distribution is **memoryless**, meaning:
  $$\boxed{P(X > s + t \mid X > s) = P(X > t)}$$
  for any s, t ≥ 0.

- **Explanation:**

  The probability that the event occurs **after an additional time t** does not depend on how much time has already passed.

- **Example:**

  Suppose the lifetime of a light bulb is Exponential with $\lambda = 0.1$ (per hour).
  If a bulb has already lasted 5 hours, the probability it lasts **at least 3 more hours** is:
  $$P(X > 8 \mid X > 5) = P(X > 3) = e^{-0.1 \cdot 3} \approx 0.741$$

X ~ Exponential(0.1)

Exponential CDF

$F(t) = P(T \leq t) = 1 - e^{-\lambda t}$

RELATIONSHIP BETWEEN THE EXPONENTIAL AND POISSON DISTRIBUTIONS

- Let events occur according to a **Poisson process** with rate $\lambda > 0$.
- **Poisson distribution:** counts the number of events in a fixed interval:

$$\boxed{X \sim \mathrm{Poisson}(\lambda), \quad P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \; k = 0, 1, 2, \ldots}$$

- **Interpretation of λ:** average number of events per unit time (or per unit interval)
- **Exponential distribution:** models the **time between consecutive events**:

$$\boxed{T \sim \mathrm{Exponential}(\lambda), \quad f_T(t) = \lambda e^{-\lambda t}, \; t \geq 0}$$

- **Interpretation of $1/\lambda$:** average waiting time until the first event occurs
- **Key relationship:**

$$\boxed{P(T > t) = P(\text{no events in } [0, t]) = P(X = 0) = e^{-\lambda t}}$$

- **Usage:**
  1. Poisson → number of events in an interval
  2. Exponential → waiting time until next event

EXERCISE ON THE POISSON AND EXPONENTIAL DISTRIBUTIONS

A call center receives calls at an average rate of **5 calls per hour** ($\lambda = 5$). Assume that calls arrive according to a **Poisson process**.

1. What is the probability that **no calls** are received in the next **30 minutes**?
2. What is the probability that the **first call** occurs **after 30 minutes**?
3. What is the probability that **at least one call** is received in the next **30 minutes**?

EXERCISE ON THE POISSON AND EXPONENTIAL DISTRIBUTIONS: SOLUTION

Answer:

- Call center: average 5 calls per hour, λ = 5
- Interval: 30 minutes = 0.5 hours

Step 0: Define variables

- $X \sim \text{Poisson}(5)$ → number of calls per **1 hour**
- $Y \sim \text{Poisson}(2.5)$ → number of calls in **0.5 hours** (Y is scaled from X)

1 Probability of no calls in the next 30 minutes

$$P(Y=0) = \frac{2.5^0 e^{-2.5}}{0!} = e^{-2.5} \approx 0.0821$$

2 Probability the first call occurs after 30 minutes (Exponential)

- $T \sim \text{Exponential}(\lambda = 5)$ per hour

$$P(T > 0.5) = e^{-5 \cdot 0.5} = e^{-2.5} \approx 0.0821$$

Same as $P(Y=0)$, because P(Y = 0) $= P(T > t)$

3 Probability at least one call in 30 minutes

$$P(Y \geq 1) = 1 - P(Y=0) = 1 - 0.0821 \approx 0.9179$$

LECTURE 7: CHI-SQUARE DISTRIBUTION

CHI-SQUARE DISTRIBUTION

- If the population distribution is normal then

$$Q = \frac{(n-1)s^2}{\sigma^2}$$

$Q \sim \chi^2_{(n-1)}$

has a chi-square (χ^2) distribution with $n - 1$ degrees of freedom

Newbold et al (2013)

RELATIONSHIP BETWEEN NORMAL AND CHI-SQUARE DISTRIBUTIONS

- Let $Z_1, Z_2, \dots, Z_n \sim N(0,1)$ be independent standard normal variables.
- Then the sum of their squares follows a Chi-Square distribution:

$$\boxed{Q = \sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)}$$

$Q \sim \chi^2_{(n)}$

- **Key properties:**
  1. **Support:** $Q > 0$
  2. **Shape:** Right-skewed, becomes more symmetric as n increases
  3. **Additivity:** If $Q_1 \sim \chi^2(n_1)$ and $Q_2 \sim \chi^2(n_2)$ are independent, then

$$\boxed{Q_1 + Q_2 \sim \chi^2(n_1 + n_2)}$$

- **Usage:** Widely used in **goodness-of-fit tests**, **tests of independence**, and **variance estimation**.

DERIVATION OF THE CHI-SQUARE DISTRIBUTION

Let

$$X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$$

be a random sample from a normal population.

The **sample variance** is

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Then, the following result holds:

$$\boxed{Q = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}}$$

That is, the statistic Q follows a **chi-square distribution with $n-1$ degrees of freedom.**

Q ~ $\chi^2_{(n-1)}$

WHY Q HAS $n - 1$DEGREES OF FREEDOM (DF), NOT n

When we compute the **sample variance**

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

we use $\bar{X}$, the **sample mean**, which is itself estimated from the data.

Because $\bar{X}$ is calculated from the n observations, only $n-1$ deviations $(X_i - \bar{X})$ are **free to vary independently**.

Hence, the sum of squared deviations $\sum(X_i - \bar{X})^2$, and the statistic

$$\boxed{Q = \frac{(n-1)S^2}{\sigma^2}}$$

Q ~ $\chi^2_{(n-1)}$

follows a **chi-square distribution with $n-1$ degrees of freedom**.

💡 Key idea: **subtract 1 because 1 parameter (the mean) is estimated from the data.**

WHY Q HAS $n-1$DEGREES OF FREEDOM (DF), NOT n: EXAMPLE

Idea: Number of observations that are free to vary after sample mean has been calculated

Example: Suppose the mean of 3 numbers is 8.0

Let $X_1 = 7$
Let $X_2 = 8$
What is $\boldsymbol{X_3}$?

$\Longrightarrow$

If the mean of these three values is 8.0, then X_3 must be 9
(i.e., X_3 is not free to vary)

Here, n = 3, so degrees of freedom = n − 1 = 3 − 1 = 2

(2 values can be any numbers, but the third is not free to vary for a given mean)

Newbold et al (2013)

MEAN AND VARIANCE OF THE CHI-SQUARE DISTRIBUTION

- Let $\chi^2 \sim \chi^2(n)$, where n = degrees of freedom.
- **Mean:**

$$\boxed{E[\chi^2] = n}$$

$Q \sim \chi^2_{(n)}$

- **Variance:**

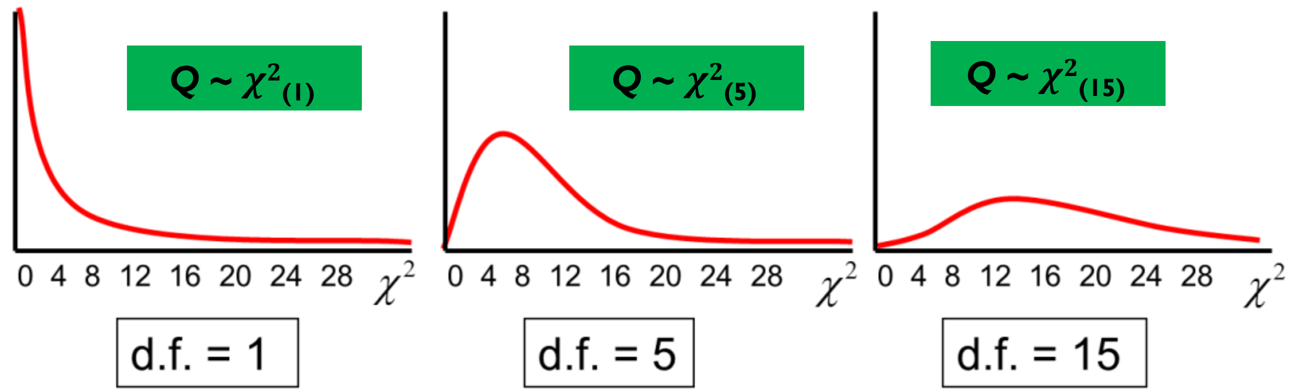
$$\boxed{\mathrm{Var}(\chi^2) = 2n}$$

Notes:

- The Chi-Square distribution is widely used in goodness-of-fit tests, tests of independence, and variance estimation.

CHI-SQUARE DISTRIBUTIONS WITH DIFFERENT DEGREES OF FREEDOM

- The chi-square distribution is a family of distributions, depending on degrees of freedom:
- d.f. = $n - 1$



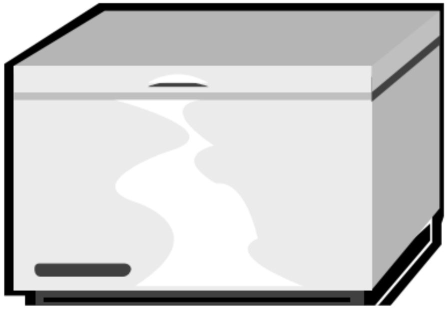
- Text Appendix Table 7 contains chi-square probabilities

Newbold et al (2013)

The chi-square distribution becomes more symmetric as the degrees of freedom increase.

CHI-SQUARE EXAMPLE

- A commercial freezer must hold a selected temperature with little variation. Specifications call for a standard deviation of no more than 4 degrees (a variance of 16 degrees^2).
- A sample of 14 freezers is to be tested
- What is the upper limit (K) for the sample variance such that the probability of exceeding this limit, given that the population standard deviation is 4, is less than 0.05?



Newbold et al (2013)

EXAMPLE: FINDING A CRITICAL VALUE/QUANTILE FOR THE SAMPLE VARIANCE

We have $\sigma^2 = 16$ (so $\sigma = 4$), sample size $n = 14$. For a normal population

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}.$$

Here $n - 1 = 13$. We want k such that

$$P(S^2 > k) < 0.05.$$

Set the chi-square cutoff at the 95th percentile:

$$P(S^2 > k) = 0.05$$

n = 14 (sample size)
σ^2 = 16 (population variance)

Solution:

k?: $P(S^2 > k) = 0.05 = P\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)k}{\sigma^2}\right) = 0.05 \Leftrightarrow 1 - P\left(Q \leq \frac{(14-1)k}{16}\right) = 0.05$

$\Leftrightarrow F\left(\frac{(14-1)k}{16}\right) = 0.95 \Leftrightarrow \frac{(14-1)k}{16} = F^{-1}(0.95) \Leftrightarrow k = F^{-1}(0.95) \times 16/13 \Leftrightarrow$

$\Leftrightarrow k = 22.362 \times 16/13 = 27.522$

$F^{-1}(0.95) = \chi^2_{(0.95, 13)}$ is the **quantile of probability 0.95** of the **chi-square distribution** with **13 degrees of freedom**. The method to find this quantile using the **chi-square table** is explained on the next slide.

That means $P(S^2 > 27.522) = 0.05$, so choosing this k ensures the probability is ≤ 0.05 (strictly speaking equal to 0.05; any slightly larger k gives < 0.05).

EXAMPLE: FINDING A CRITICAL VALUE/QUANTILE FOR THE SAMPLE VARIANCE

We want to determine the value **k** such that the probability of the sample variance being greater than **k** is **0.05**:

$$P(S^2 > k) = 0.05$$

Let

$$Q = \frac{(n-1)S^2}{\sigma^2}$$

Then

$$Q \sim \chi^2_{(n-1)}$$

n = 14 (sample size)
σ^2 = 16 (population variance)

$Q \sim \chi^2_{(13)}$

Solution:

k?: $P(S^2 > k) = 0.05 = P\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)k}{\sigma^2}\right) = 0.05 \Leftrightarrow 1 - P\left(Q \le \frac{(14-1)k}{16}\right) = 0.05$

$\Leftrightarrow F\left(\frac{(14-1)k}{16}\right) = 0.95 \Leftrightarrow \frac{(14-1)k}{16} = F^{-1}(0.95) \Leftrightarrow k = F^{-1}(0.95) \times 16/13 \Leftrightarrow$

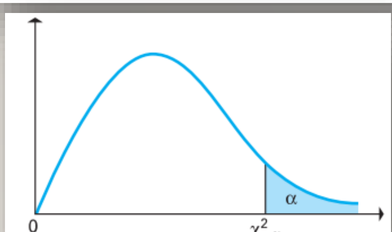
$\Leftrightarrow k = 22.362 \times 16/13 = 27.522$

$F^{-1}(0.95) = \chi^2_{(0.95,\,13)}$ is the **quantile of probability 0.95** of the **chi-square distribution** with **13 degrees of freedom**. The method to find this quantile using the **chi-square table** is explained on the next slide.

FINDING A CHI-SQUARE QUANTILE: EXAMPLE

Table 7a Upper Critical Values of Chi-Square Distribution with ν Degrees of Freedom

	Probability of Exceeding	
ν	0.10	0.05
1	2.706	3.841
2	4.605	5.991
3	6.251	7.815
4	7.779	9.488
5	9.236	11.070
6	10.645	12.592
7	12.017	14.067
8	13.362	15.507
9	14.684	16.919
10	15.987	18.307
11	17.275	19.675
12	18.549	21.026
13	19.812	22.362
14	21.064	23.685

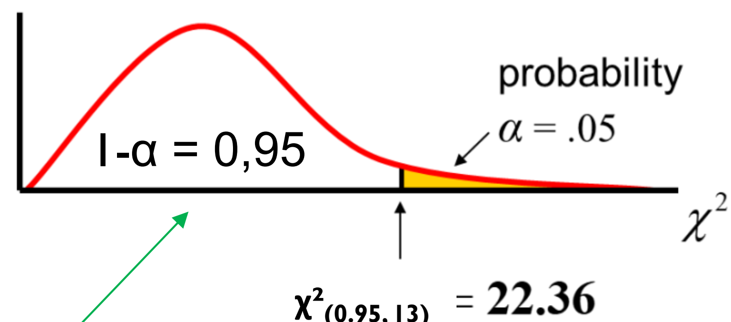


$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Is chi-square distributed with $(n-1) = 13$ degrees of freedom

Q ~ $\chi^2_{(13)}$

- Use the the chi-square distribution with area 0.05 in the upper tail:

$$\chi^2_{(0.95,\,13)} = 22.36 \ (\alpha = .05 \text{ and } 14 - 1 = 13 \text{ d.f.})$$



Newbold et al (2013)

Because the chi-square quantile $\chi^2_{(0.95,\,13)}$ is associated with a probability of 0.95, the **area behind (to the left)** of this point is **1 – α = 0.95**, while the **area ahead (to the right)** is **α = 0.05**.

EXERCISE ON THE CHI-SQUARE DISTRIBUTION

Questions

1. **Determine the following probabilities:**

   a) $P(\chi^2_2 > 5.99)$

   b) $P(\chi^2_5 < 11.07)$

   c) $P(3 < \chi^2_{10} < 18)$

   d) $P(\chi^2_{12} > 21.03)$

   e) $P(\chi^2_3 < 0.584)$

2. **Determine the following chi-square quantiles:**

   a) The 95th percentile $\chi^2_{0.95,\, df=8}$

   b) The 1st percentile $\chi^2_{0.01,\, df=10}$

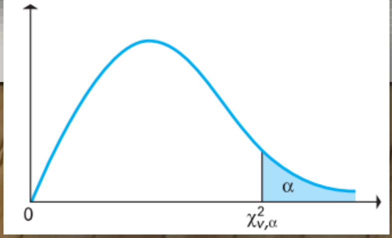
   c) The 97.5th percentile $\chi^2_{0.975,\, df=4}$

EXERCISE ON THE CHI-SQUARE DISTRIBUTION: SOLUTION

Answer:

Table 7a Upper Critical Values of Chi-Square Distribution with ν Degrees of Freedom

	Probability of Exceeding the Critical Value			
ν	0.10	0.05	0.025	0.01
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209
11	17.275	19.675	21.920	24.725
12	18.549	21.026	23.337	26.217
13	19.812	22.362	24.736	27.688



These values cannot be obtained using the table presented.

Questions

1. Determine the following probabilities:
   a) $P(\chi^2_2 > 5.99)$
   b) $P(\chi^2_5 < 11.07)$
   c) $P(3 < \chi^2_{10} < 18)$
   d) $P(\chi^2_{12} > 21.03)$
   e) $P(\chi^2_3 < 0.584)$
2. Determine the following chi-square quantiles:
   a) The 95th percentile $\chi^2_{0.95,\, df=8}$
   b) The 1st percentile $\chi^2_{0.01,\, df=10}$
   c) The 97.5th percentile $\chi^2_{0.975,\, df=4}$

Answers (for checking with the chi-square table)

1a) 0.0500 1b) 0.9500 1c) 0.9265 1d) 0.0499 1e) 0.0999

2a) 15.5073 2b) 2.5582 2c) 11.1433

LECTURE 7: STUDENT'S T DISTRIBUTION

STUDENT'S T DISTRIBUTION

- Consider a random sample of *n* observations
  - with mean $\bar{x}$ and standard deviation *s*
  - from a normally distributed population with mean μ
- Then the variable

$$\boxed{T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}}$$

$T \sim t_{(n-1)}$

follows the Student's *t* distribution with (n − 1) degrees of freedom

Newbold et al (2013)

RELATIONSHIP OF THE STUDENT'S T DISTRIBUTION WITH NORMAL AND CHI-SQUARE

- Let $Z \sim N(0,1)$ (standard normal) and $X \sim \chi^2(n)$ (Chi-Square with n degrees of freedom), independent.
- The **t-Student distribution** with n degrees of freedom is defined as:

$$\boxed{t_n = \frac{Z}{\sqrt{X/n}}, \quad t_n \sim t(n)}$$

$T \sim t_{(n)}$

- **Key properties:**
  1. **Support:** $-\infty < t_n < \infty$
  2. Symmetric and bell-shaped, like the Normal distribution.
  3. As $n \to \infty$, $t_n \to N(0,1)$ (approaches the standard normal).
- **Usage:** Commonly used in **hypothesis testing** and **confidence intervals** for small sample sizes.

DERIVATION OF THE STUDENT'S T DISTRIBUTION

- Let $\bar{X}$ and S^2 be the sample mean and variance from a normal population with mean μ and variance σ^2.
- Define:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$Q = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1), \quad \text{independent of } Z$$

- Then the **t-statistic** is:

$$\boxed{T = \frac{Z}{\sqrt{Q/(n-1)}} = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}}$$

$T \sim t_{(n\text{-}1)}$

Key points:

1. Shows how the t-Student arises from **normal and Chi-Square variables**.
2. Degrees of freedom = $n - 1$ because the sample variance is estimated.
3. Provides the link between the **theoretical t** and the **practical sample t** used in hypothesis testing.

MEAN AND VARIANCE OF THE STUDENT'S T DISTRIBUTION

- Let $t \sim t(n)$, where n = degrees of freedom.
- **Mean:**

$$\boxed{E[t] = 0, \quad \text{for } n > 1}$$

- **Variance:**

$$\boxed{\mathrm{Var}(t) = \frac{n}{n-2}, \quad \text{for } n > 2}$$

$T \sim t_{(n)}$

Notes:

- The t-distribution is symmetric and bell-shaped, used mainly in hypothesis testing for small samples and confidence intervals.
- Variance is only defined for $n > 2$.

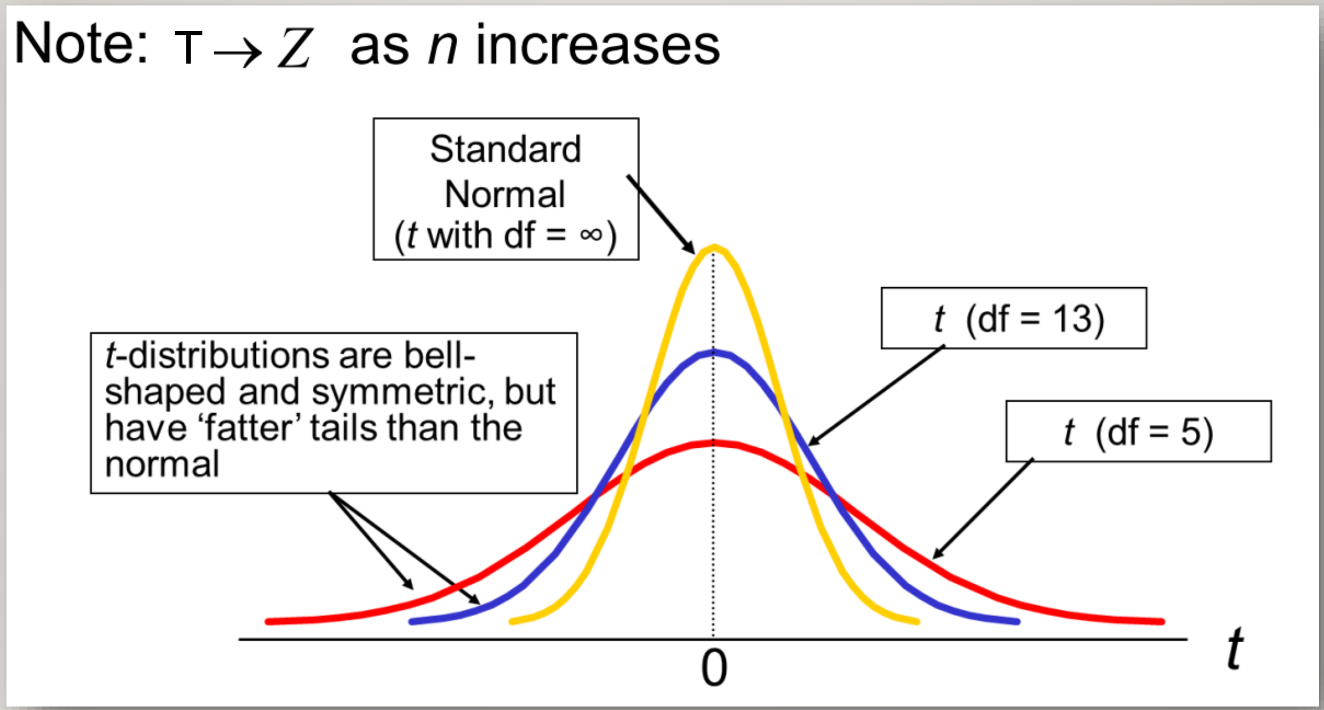
STUDENT'S T DISTRIBUTION

- The T is a family of distributions
- The T value depends on degrees of freedom (d.f.)
  - Number of observations that are free to vary after sample mean has been calculated

$$\text{d.f.} = n - 1$$

Newbold et al (2013)

STUDENT'S T DISTRIBUTIONS WITH DIFFERENT DEGREES OF FREEDOM



Newbold et al (2013)

FINDING A T-STUDENT QUANTILE: EXAMPLE

Table 8 Upper Critical Values of Student's t Distribution with ν Degrees of Freedom

	Probability of Exceeding the Critical Value					
ν	0.10	0.05	0.025	0.01	0.005	0.001
1	3.078	6.314				
2	1.886	2.920				
3	1.638	2.353				
4	1.533	2.132				
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.94				
7	1.415	1.89				
8	1.397	1.86				
9	1.383	1.83				

Note on Notation:
There are two common ways to express t-student quantiles: $t_{(2,0.95)} = t_{0.95,2}$

Objective

To determine the quantile of probability **0.95** for a **t-distribution** with **2 degrees of freedom**, denoted by

$$t(2, 0.95)$$

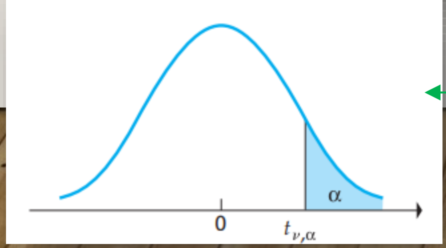
Procedure

1. Identify the parameters:

$$P(T \leq t(2, 0.95)) = 0.95, \quad \text{with } df = 2$$

2. Using the **t-distribution table**, find the row for **2 degrees of freedom** and the column corresponding to **0.95 cumulative probability**.
3. The value obtained is

$$t(2, 0.95) = 2.920$$



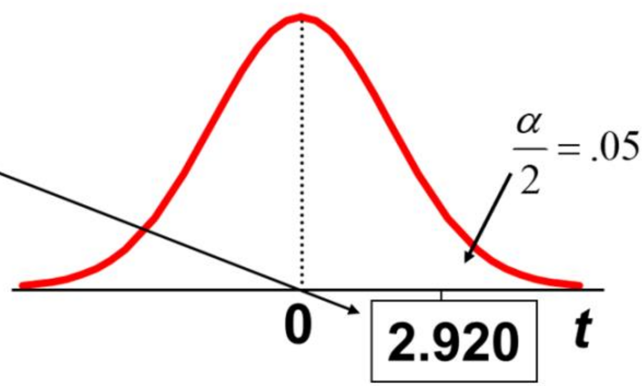
Interpretation:
The value **t(2, 0.95) = 2.920** means that **95% of the area** under the **t-distribution** curve with **2 degrees of freedom** lies **to the left** of 2.920, and the **right tail area** is $\alpha = 0.05$.

STUDENT'S T TABLE

df	Upper Tail Area .10	.05	.025
1	3.078	6.314	12.706
2	1.886	$\boxed{\mathbf{2.920}}$	4.303
3	1.638	2.353	3.182

The body of the table contains *t* values, not probabilities

Let: $n = 3$
df $= n - 1 = 2$
$\alpha = .10$
$\frac{\alpha}{2} = .05$



Newbold et al (2013)

T DISTRIBUTION VALUES

With comparison to the *Z* value

Confidence Level	*t* (10 d.f.)	*t* (20 d.f.)	*t* (30 d.f.)	*z*
.80	1.372	1.325	1.310	1.282
.90	1.812	1.725	1.697	1.645
.95	2.228	2.086	2.042	1.960
.99	3.169	2.845	2.750	2.576

Note: $T \rightarrow Z$ as *n* increases

Newbold et al (2013)

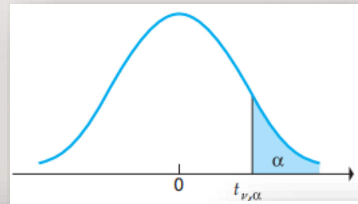
EXERCISE ON THE STUDENT'S T DISTRIBUTION

Questions

1. **Determine the following probabilities:**
   a) $P(t_{10} > 1.812)$
   b) $P(t_{15} < -2.131)$
   c) $P(-1.753 < t_8 < 1.860)$
   d) $P(t_{20} > 2.845)$
   e) $P(t_5 < 0)$
2. **Determine the following t-distribution quantiles:**
   a) The 95th percentile $t_{0.95,\, df=12}$
   b) The 97.5th percentile $t_{0.975,\, df=9}$
   c) The 2.5th percentile $t_{0.025,\, df=30}$

EXERCISE ON THE STUDENT'S T DISTRIBUTION: SOLUTION

✅ Answer:



These values cannot be obtained using the table presented.

Table 8 Upper Critical Values of Student's t Distribution with ν Degrees of Freedom

	Probability of Exceeding the Critical Value					
ν	0.10	0.05	0.025	0.01	0.005	0.0
1	3.078	6.314	12.706	31.821	63.657	318.
2	1.886	2.920	4.303	6.965	9.925	22.
3	1.638	2.353	3.182	4.541	5.841	10.
4	1.533	2.132	2.776	3.747	4.604	7.
5	1.476	2.015	2.571	3.365	4.032	5.
6	1.440	1.943	2.447	3.143	3.707	5.
7	1.415	1.895	2.365	2.998	3.499	4.
8	1.397	1.860	2.306	2.896	3.355	4.
9	1.383	1.833	2.262	2.821	3.250	4.
10	1.372	1.812	2.228	2.764	3.169	4.
11	1.363	1.796	2.201	2.718	3.106	4.
12	1.356	1.782	2.179	2.681	3.055	3.
13	1.350	1.771	2.160	2.650	3.012	3.
14	1.345	1.761	2.145	2.624	2.977	3.
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	
17	1.333	1.740	2.110	2.567	2.898	
18	1.330	1.734	2.101	2.552	2.878	
19	1.328	1.729	2.093	2.539	2.861	
20	1.325	1.725	2.086	2.528	2.845	
21	1.323	1.721	2.080	2.518	2.831	

Questions

1. Determine the following probabilities:
   a) $P(t_{10} > 1.812)$
   b) $P(t_{15} < -2.131)$
   c) $P(-1.753 < t_8 < 1.860)$
   d) $P(t_{20} > 2.845)$
   e) $P(t_5 < 0)$
2. Determine the following t-distribution quantiles:
   a) The 95th percentile $t_{0.95,\, df=12}$
   b) The 97.5th percentile $t_{0.975,\, df=9}$
   c) The 2.5th percentile $t_{0.025,\, df=30}$

Answers (for checking with the t-Student table)

1a) 0.05 1b) 0.025 1c) 0.90 1d) 0.005 1e) 0.50

2a) 1.782 2b) 2.262 2c) −2.042

LECTURE 7: F-SNEDECOR DISTRIBUTION

F-SNEDECOR DISTRIBUTION

- The **F-distribution** with n numerator and m denominator degrees of freedom arises from the ratio of two independent Chi-Square random variables:

$$\boxed{F = \frac{(X/n)}{(Y/m)},} \quad X \sim \chi^2(n),\ Y \sim \chi^2(m)$$

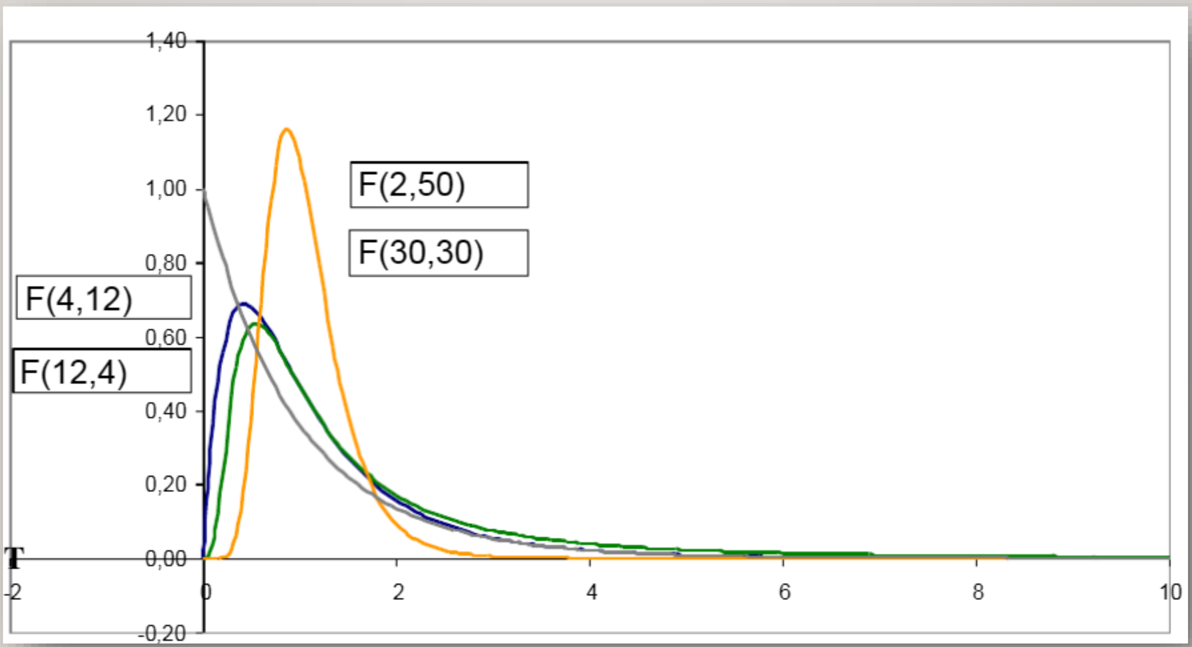
$F \sim F_{(n, m)}$

Properties:

1. **Support:** $F > 0$
2. **Shape:** Right-skewed (positively skewed), becomes more symmetric as n and m increase
3. **Mean and Variance:** Defined for $m > 2$ and $m > 4$ respectively
4. **Limit:** As $n \to \infty$ and $m \to \infty$, $F \to 1$

Usage: Commonly used in **ANOVA**, **regression analysis**, and **tests of equality of variances**.

F-SNEDECOR DISTRIBUTIONS WITH DIFFERENT DEGREES OF FREEDOM



Distribuição Qui-quadrado, Distribuição t-Student e Distribuição F-Snedecor - Distribuições - Studocu

MEAN AND VARIANCE OF THE F-DISTRIBUTION

- Let $F \sim F(n, m)$, where n = numerator degrees of freedom, m = denominator degrees of freedom.
- **Mean:**

$$\boxed{E[F] = \frac{m}{m-2}, \quad \text{for } m > 2}$$

F ~ F $_{(n, m)}$

- **Variance:**

$$\boxed{\mathrm{Var}(F) = \frac{2m^2(n+m-2)}{n(m-2)^2(m-4)}, \quad \text{for } m > 4}$$

Notes:

- Only defined for $m > 2$ (mean) and $m > 4$ (variance).
- Useful in ANOVA and hypothesis testing.

- Test for equality of two variances:

$$F = \frac{S_1^2}{S_2^2}$$

- Analysis of Variance (ANOVA):

$$F = \frac{\text{Mean Square Between Groups}}{\text{Mean Square Within Groups}}$$

- **Regression:** testing significance of multiple coefficients.

MEAN AND VARIANCE OF A F-SNEDECOR DISTRIBUTION: EXAMPLE

If $U_1 \sim \chi^2(4)$ and $U_2 \sim \chi^2(10)$, independent, then

$$F = \frac{(U_1/4)}{(U_2/10)} \sim F(4, 10)$$

Mean $= \frac{10}{8} = 1.25$

Variance $= \frac{2 \times 10^2 (4+10-2)}{4(8)^2(6)} = 0.911$

RELATIONSHIP BETWEEN F, CHI-SQUARE, AND STUDENT'S T DISTRIBUTIONS

- Let $X \sim \chi^2(n)$ and $Y \sim \chi^2(m)$ be independent Chi-Square random variables.
- The **F-distribution** with n numerator and m denominator degrees of freedom is defined as:

$$\boxed{F = \frac{(X/n)}{(Y/m)}, \quad F \sim F(n,m)}$$

- **Connection with t-Student:**

$$\boxed{t_n^2 \sim F(1,n)}$$

- That is, the square of a t-Student variable with n degrees of freedom follows an F-distribution with 1 and n degrees of freedom.
- **Key points:**
  1. F is always **non-negative**.
  2. Chi-Square is a special case of F when $m \to \infty$:

$$\boxed{\frac{X/n}{1} \sim F(n,\infty) \sim \chi^2(n)/n}$$

- **Reciprocal property of F:**

$$\boxed{\text{If } F \sim F(n,m), \quad \frac{1}{F} \sim F(m,n)}$$

FINDING A F-SENEDCOR QUANTILE: EXAMPLE

Table 9a Upper Critical Values of the F Distribution

For ν_1 Numerator Degrees of Freedom and ν_2 Denominator Degrees of Freedom 5% Significance Level $F_{.05}(\nu_1, \nu_2)$

ν_2/ν_1	1	2	3	4	5	6	7	8	9	10
1	161.448	199.500	215.707	224.583	230.					
2	18.513	19.000	19.164	19.247	19.					
3	10.128	9.552	9.277	9.117	9.					
4	7.709	6.944	6.591	6.388	6.					
5	6.608	5.786	5.409	5.192	5.					
6	5.987	5.143	4.757	4.534	4.					
7	5.591	4.737	4.347	4.120	3.					
8	5.318	4.459	4.066	3.838	3.					
9	5.117	4.256	3.863	3.633	3.					
10	4.965	4.103	3.708	3.478	3.					
11	4.844	3.982	3.587	3.357	3.					

Objective

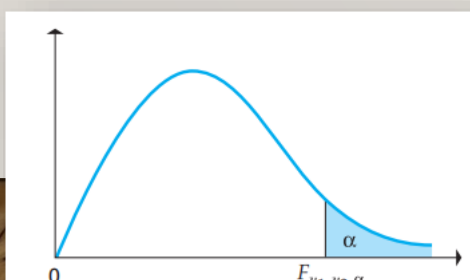
To determine the **quantile of probability 0.95** for an **F-distribution** with **4 numerator and 10 denominator degrees of freedom**, denoted by:

$$F(4, 10, 0.95) \quad \text{or } F_{0.95,4,10}$$

Procedure

1. Identify the parameters:
   - Numerator degrees of freedom: $v_1 = 4$
   - Denominator degrees of freedom: $v_2 = 10$
   - Cumulative probability: 0.95
2. Using the **F-distribution table**, locate the **row for** $v_1 = 4$ (numerator) and the **column for** $v_2 = 10$ (denominator) corresponding to **0.95 cumulative probability**.
3. The value obtained is:

$$F_{0.95,4,10} = 3.48$$



Interpretation
Left area: 0.95 (area under the curve to the left of 3.48)
Right tail: α = 0.05 (area to the right of 3.48)

EXERCISE ON THE F-SNEDECOR DISTRIBUTION

Questions

1. Determine the following probabilities:

a) $P(F_{(3,8)} > 2.5)$

b) $P(F_{(4,6)} < 3.1)$

c) $P(1.2 < F_{(2,5)} < 4.8)$

---

2. Determine the following F-quantiles:

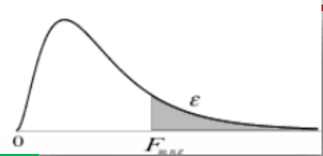
a) The 95th percentile $F_{0.95(2,5)}$

b) The 97.5th percentile $F_{0.975(3,8)}$

c) The 90th percentile $F_{0.90(4,6)}$

EXERCISE ON THE F-SNEDECOR DISTRIBUTION: SOLUTION

Answer:



DF of the Numerator

DF of the Denominator

n - graus de liberdade do denominador	ε	m - graus de liberdade do numerador: 1	2	3	4	5	6	7	8	9	10	12	15	20
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.90	245.95	248.02
	.025	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28	968.63	976.72	984.87	993.08
	.010	4052.18	4999.34	5403.53	5624.26	5763.96	5858.95	5928.33	5980.95	6022.40	6055.93	6106.68	6156.97	6208.66
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45
	.010	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.42	99.43	99.45
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	27.05	26.87	26.69
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81

This value cannot be obtained using the table presented.

Questions

1. Determine the following probabilities:

a) $P(F_{(3,8)} > 2.5)$

b) $P(F_{(4,6)} < 3.1)$

c) $P(1.2 < F_{(2,5)} < 4.8)$

2. Determine the following F-quantiles:

a) The 95th percentile $F_{0.95(2,5)}$

b) The 97.5th percentile $F_{0.975(3,8)}$

c) The 90th percentile $F_{0.90(4,6)}$

1a) $P(F_{(3,8)} > 2.5) \approx 0.22$

1b) $P(F_{(4,6)} < 3.1) \approx 0.85$

1c) $P(1.2 < F_{(2,5)} < 4.8) \approx 0.35$

2a) $F_{0.95(2,5)} = 5.79$

2b) $F_{0.975(3,8)} = 5.42$

2c) $F_{0.90(4,6)} =$ 3.18

HOMEWORK OF LECTURE 7: QUESTIONS

EXERCISE 5.58

5.58 Suppose that the time between successive occurrences of an event follows an exponential distribution with a mean of $1/\lambda$ minutes. Assume that an event occurs.

a. Show that the probability that more than 3 minutes elapses before the occurrence of the next event is $e^{-3\lambda}$.
b. Show that the probability that more than 6 minutes elapses before the occurrence of the next event is $e^{-6\lambda}$.
c. Using the results of parts (a) and (b), show that if 3 minutes have already elapsed, the probability that a further 3 minutes will elapse before the next occurrence is $e^{-3\lambda}$. Explain your answer in words.

Newbold et al (2013)

EXERCISE 5.60

5.60 Delivery trucks arrive independently at the Floorstore Regional distribution center with various consumer items from the company's suppliers. The mean number of trucks arriving per hour is 20. Given that a truck has just arrived answer the following:

a. What is the probability that the next truck will not arrive for at least 5 minutes?
b. What is the probability that the next truck will arrive within the next 2 minutes?
c. What is the probability that the next truck will arrive between 4 and 10 minutes?

Newbold et al (2013)

THANKS!

Questions?