

Problem Set - Game Theory

October 26, 2025

[1, JR 7.17-18] Consider the strategic form game depicted below. Each of two countries must simultaneously decide on a course of action. Country 1 must decide whether to keep its weapons or to destroy them. Country 2 must decide whether to spy on country 1 or not. It would be an international scandal for country 1 if country 2 could prove that country 1 was keeping its weapons. The payoff matrix is as follows:

	Spy	Don't Spy
Keep	$(-1, 1)$	$(1, -1)$
Destroy	$(0, 2)$	$(0, 2)$

- Does either player have a strictly dominant strategy?
- Does either player have a weakly dominant strategy?
- Find a Nash equilibrium in which neither player employs a weakly dominant strategy.

Now suppose that country 1 can be one of two types, 'aggressive' or 'non-aggressive'. Country 1 knows its own type. Country 2 does not know country 1's type but believes that country 1 is aggressive with probability $\varepsilon > 0$. The aggressive type places great importance on keeping its weapons. If it does so and country 2 spies on the aggressive type, this leads to war, which the aggressive type wins and justifies because of the spying, but which is very costly for country 2. When country 1 is non-aggressive, the payoffs are as before (i.e., as in the previous exercise). The payoff matrices associated with each of the two possible types of country 1 are given below.

Country 1 is 'aggressive' (Probability ε)

	Spy	Don't Spy
Keep	$(10, -9)$	$(5, -1)$
Destroy	$(0, 2)$	$(0, 2)$

Country 1 is 'non-aggressive' (Probability $1 - \varepsilon$)

	Spy	Don't Spy
Keep	$(-1, 1)$	$(1, -1)$
Destroy	$(0, 2)$	$(0, 2)$

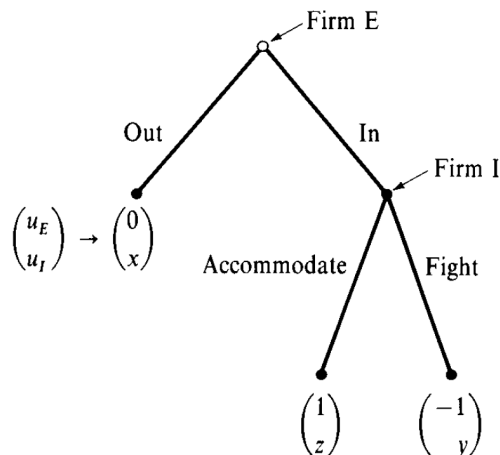
- (c) What action must the aggressive type of country 1 take in any Bayesian-Nash equilibrium?
- (d) Assuming that $\varepsilon < \frac{1}{5}$, find the unique Bayes-Nash equilibrium.

[2, MWG 8.D.5] Consumers are uniformly distributed along a boardwalk that is 1 mile long. Ice-cream prices are regulated, so consumers go to the nearest vendor because they dislike walking (assume that at the regulated prices all consumers will purchase an ice cream even if they have to walk a full mile). If more than one vendor is at the same location, they split the business evenly.

- (a) Consider a game in which two ice-cream vendors pick their locations simultaneously. Show that there exists a unique pure strategy Nash equilibrium and that it involves both vendors locating at the midpoint of the boardwalk.
- (b) Show that with three vendors, no pure strategy Nash equilibrium exists.

[3, MWG 9.B.14] At time 0, an incumbent firm (firm I) is already in the widget market, and a potential entrant (firm E) is considering entry. In order to enter, firm E must incur a cost of $K > 0$. Firm E's only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game in the figure is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight (the upper payoff is for firm E). Once firm E plays "out," it is out of the game.

Assume that:



- (A.1) $x > z > y$,
- (A.2) $y + \delta x > (1 + \delta)z$,
- (A.3) $1 + \delta > K$.

(a) What is the (unique) subgame perfect Nash equilibrium of this game?

(b) Suppose now that firm E faces a financial constraint. In particular, if firm I fights *once* against firm E (in any period), firm E will be forced out of the market from that point on. Now what is the (unique) subgame perfect Nash equilibrium of this game? (If the answer depends on the values of parameters beyond the three assumptions, indicate how.)

[4, Collusion in a Cournot Duopoly] Consider a Cournot duopoly where two firms produce a homogeneous product. Let q_1 and q_2 denote the quantities produced by Firm 1 and Firm 2, respectively. The market price $P(Q)$ is determined by the total quantity produced, given by the formula:

$$P(Q) = a - Q = a - q_1 - q_2 \quad \text{for } Q < a,$$

where Q is the total quantity in the market. If the total quantity produced by both firms is greater than or equal to a , the market price becomes zero, i.e., $P(Q) = 0$ for $Q \geq a$.

Each firm incurs a production cost. The total cost of producing q_i units for Firm i is:

$$C_i(q_i) = cq_i,$$

where c is the marginal cost of production, assumed to be less than a to ensure positive profits for both firms.

In this setup, both firms choose their production quantities q_1 and q_2 simultaneously, aiming to maximize their respective profits.

(a) Compute the unique Nash Equilibrium of the game.

Now consider an infinitely repeated game based on the Cournot stage game. Both firms have the discount factor δ

(b) Compute the values of δ , δ^* , for which the following trigger strategy is a subgame-perfect Nash Equilibrium:

Produce half the monopoly quantity, $qm/2$ in the first period. In the t^{th} period, produce $qm/2$ if both firms have produced $qm/2$ in each of the $t - 1$ previous periods; otherwise produce the Cournot quantity, qc (i.e., what you found on (a)).

(c) Show, for a given value of $\delta < \delta^*$, the most-profitable quantity, q^* , that can be produced if both play trigger strategies that switch to Cournot after any deviation. You might consider the following strategy:

Produce q^ in the first period. In the t^{th} period, produce q^* , if both firms have produced q^* in each of the $t-1$ previous periods; otherwise, produce the Cournot quantity, q_c .*

(d) Consider the following carrot-and-stick strategy to show that the monopoly outcome can be achieved when $\delta = 1/2$:

Produce half the monopoly quantity, $q_m/2$, in the first period. In the t^{th} period, produce $q_m/2$ if both firms produced $q_m/2$ in period $t-1$, produce $q_m/2$ if both firms produced x in period $t-1$, and otherwise produce x .