PART III
CREDIT RISK MODELS

1. INTRODUCTION

“Default risk is the risk that an obligor does not honour his payment obligations.”

Typically,

- Default events are rare.
- They may occur unexpectedly.
- Default events involve significant losses.
- The size of these losses is unknown before default.

All payment obligations represent some sort of default risk.

DETERMINANTS OF CREDIT RISK

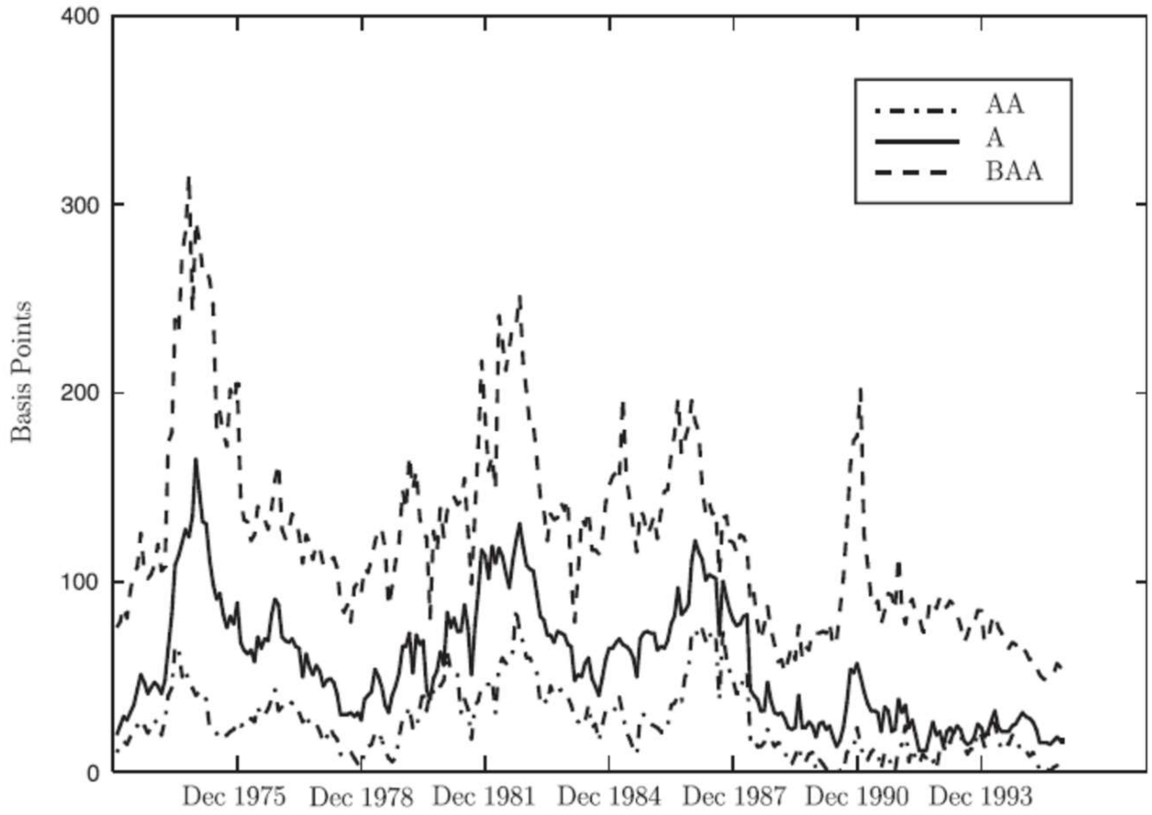
- “Credit risk is the risk of default or of reductions in market value caused by changes in the credit quality of issuers or counterparties”, Duffie, Darrell and Kenneth J. Singleton (2003), “Credit Risk”, Princeton University Press.
- Credit Risk is associated to the PD of the debtor, as well as the LGD.
- Regarding the credit risk of the debtor, it is relevant not only to quantify the PDs but also the rating transition frequencies, which also impact on bond prices.
- Nonetheless, the expected loss is usually calculated taking only default into consideration:

EL = PD x LGD

- Given the diversity of the counterparties, the market usually distinguishes between sovereign, banking, corporate and individual/household credit risk.

DETERMINANTS OF CREDIT RISK

- Bond spreads usually provide relevant information on credit risk.



Source: Duffie, Darrell and Kenneth J. Singleton (2003), “Credit Risk”, Princeton University Press.

COMPONENTS OF CREDIT RISK

Arrival risk is a term for the uncertainty whether a default will occur or not. To enable comparisons, it is specified with respect to a given time horizon, usually one year. The measure of arrival risk is the *probability of default*. The probability of default describes the distribution of the indicator variable *default before the time horizon*.

Timing risk refers to the uncertainty about the precise time of default. Knowledge about the time of default includes knowledge about the arrival risk for all possible time horizons, thus timing risk is more detailed and specific than arrival risk. The underlying unknown quantity (random variable) of timing risk is the *time of default*, and its risk is described by the *probability distribution function of the time of default*. If a default never happens, we set the time of default to infinity.

COMPONENTS OF CREDIT RISK

Recovery risk describes the uncertainty about the severity of the losses if a default has happened. In recovery risk, the uncertain quantity is the actual payoff that a creditor receives after a default. It can be expressed in several ways which will be discussed in a later chapter. Market convention is to express the recovery rate of a bond or loan as the fraction of the notional value of the claim that is actually paid to the creditor. Recovery risk is described by the *probability distribution of the recovery rate*, i.e. the probabilities that the recovery rate is of a given magnitude. This probability distribution is a conditional distribution, conditional upon default.

If we consider the risk of joint defaults of several obligors, an additional risk component is introduced. **Default correlation risk** describes the risk that several obligors default together. Again here we have *joint arrival risk* which is described by the joint default probabilities over a given time horizon, and *joint timing risk* which is described by the joint probability distribution function of the times of default.

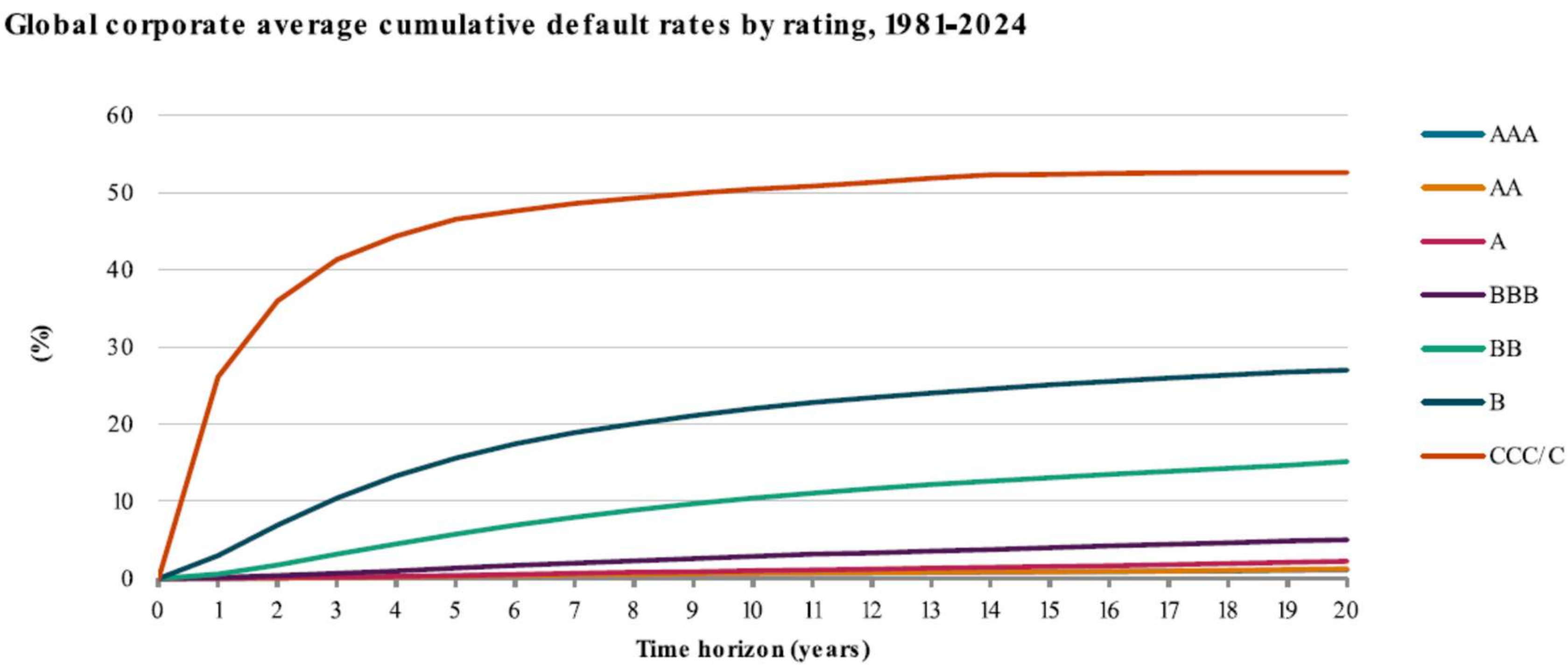
PDs

- Ratings are a ranking of credit risk and do not explicitly provide any PD measure.
- However, one can obtain historical frequencies of default for each rating classification, as well as the historical frequencies of transition between ratings.
- The long-term ratings of the main agencies (S&P and Moody's) split by 9 classes, each of them (excluding AAA) with rating modifiers +/ /- (S&P) or 1/2/3 (Moody's).

	S&P	**Moody's**
Investment Grade	AAA	Aaa
	AA	Aa
	A	A
	BBB	Baa
Speculative Grade	BB	Ba
	B	B
	CCC	Caa
	CC	Ca
	C	C

PDs

- **Simplest measure of credit risk** – default frequencies from rating agencies:



Source: S&P (2025), "Default, Transition, and Recovery: 2024 Annual Global Corporate Default And Rating Transition Study".

PDs

- Transition matrices illustrate the significant stability of rating classifications, being this stability higher for better ratings.

Average one-year letter rating migration rates, 1920-2024

From\To	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	WR	Def
Aaa	87.19%	7.44%	0.77%	0.17%	0.03%	0.00%	0.00%	0.00%	4.40%	0.00%
Aa	0.96%	84.75%	7.48%	0.68%	0.15%	0.04%	0.01%	0.00%	5.87%	0.06%
A	0.06%	2.56%	86.14%	5.07%	0.54%	0.10%	0.03%	0.01%	5.42%	0.07%
Baa	0.03%	0.19%	3.81%	84.44%	3.95%	0.63%	0.10%	0.01%	6.61%	0.22%
Ba	0.01%	0.06%	0.43%	5.93%	74.93%	6.55%	0.67%	0.08%	10.25%	1.08%
B	0.00%	0.04%	0.13%	0.54%	5.52%	72.41%	6.33%	0.44%	11.64%	2.96%
Caa	0.00%	0.01%	0.02%	0.08%	0.34%	5.69%	70.94%	3.00%	13.33%	6.60%
Ca_C	0.00%	0.01%	0.06%	0.07%	0.38%	2.33%	10.01%	45.45%	13.58%	28.10%

Source: Moody’s Ratings (2025), “Annual Default Study”, 28 Feb..

PDs

- Default frequencies also tend to change along time, namely for lower ratings.

Annual issuer-weighted corporate default rates by letter rating, 1920-2024

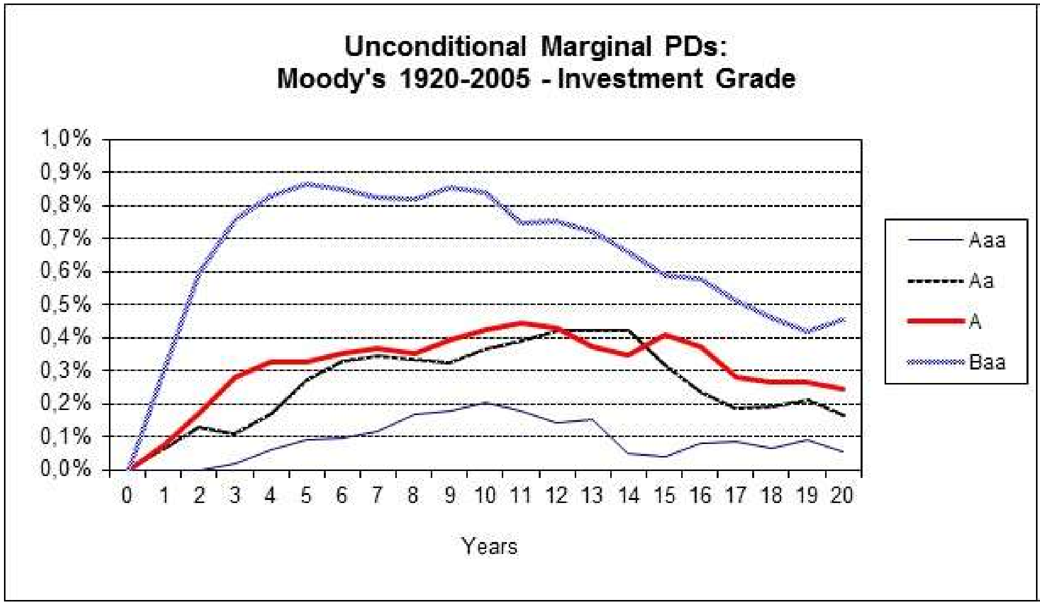
Year	Aaa	Aa	A	Baa	Ba	B	Caa-C	IG	SG	All
1920	0.00%	0.00%	0.32%	0.94%	2.15%	4.38%	0.00%	0.43%	3.01%	1.23%
1921	0.00%	0.19%	0.35%	0.65%	0.44%	2.68%	13.33%	0.39%	2.15%	1.07%
1922	0.00%	0.18%	0.17%	1.10%	1.08%	1.71%	7.63%	0.51%	1.76%	1.01%
1923	0.00%	0.00%	0.00%	0.62%	0.93%	2.27%	5.93%	0.24%	1.70%	0.80%
1924	0.00%	0.37%	0.00%	0.13%	2.06%	2.70%	12.84%	0.14%	2.85%	1.15%
1925	0.00%	0.00%	0.14%	0.71%	1.74%	2.59%	14.40%	0.32%	2.56%	1.17%
1926	0.00%	0.40%	0.15%	0.11%	1.39%	2.90%	3.70%	0.19%	1.91%	0.77%
1927	0.00%	0.00%	0.21%	0.00%	1.30%	1.98%	12.84%	0.07%	1.83%	0.74%
1928	0.00%	0.00%	0.00%	0.00%	0.16%	1.32%	10.48%	0.00%	0.88%	0.36%
1929	0.00%	0.29%	0.00%	0.44%	0.82%	0.92%	9.73%	0.24%	1.40%	0.71%
1930	0.00%	0.00%	0.00%	0.40%	0.92%	3.16%	7.72%	0.15%	2.20%	1.04%
1931	0.00%	0.00%	0.27%	1.08%	3.00%	9.52%	31.67%	0.50%	7.90%	3.80%
1932	0.00%	0.67%	1.10%	0.92%	6.10%	13.98%	24.06%	0.86%	10.99%	5.50%
1933	0.00%	0.00%	0.26%	1.77%	11.71%	16.15%	25.92%	0.79%	15.77%	8.53%
1934	0.00%	0.62%	0.31%	0.86%	2.52%	4.22%	16.50%	0.59%	5.89%	3.40%
1935	0.00%	0.00%	1.43%	1.92%	5.12%	4.27%	13.02%	1.29%	6.25%	3.93%
1936	0.00%	0.85%	0.54%	0.33%	1.23%	2.38%	7.80%	0.48%	2.71%	1.63%
1937	0.00%	0.00%	0.51%	1.04%	0.99%	2.67%	9.07%	0.62%	2.74%	1.72%
1938	0.00%	0.85%	1.64%	1.99%	0.98%	1.47%	12.81%	1.55%	2.59%	2.11%
1939	0.00%	0.00%	0.00%	0.99%	0.62%	1.74%	6.07%	0.41%	1.77%	1.22%
1940	0.00%	0.00%	0.00%	1.37%	0.43%	3.29%	11.83%	0.59%	3.55%	2.47%
1941	0.00%	0.00%	0.00%	0.00%	0.97%	0.81%	5.07%	0.00%	1.71%	1.08%
1942	0.00%	0.00%	0.00%	0.00%	0.00%	0.78%	2.00%	0.00%	0.73%	0.45%
1943	0.00%	0.00%	0.00%	0.00%	0.00%	1.35%	0.00%	0.00%	0.61%	0.37%
1944	0.00%	0.00%	0.00%	0.00%	0.00%	0.49%	2.55%	0.00%	0.66%	0.39%
1945	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.57%	0.00%	0.56%	0.31%
1946	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1947	0.00%	0.00%	0.00%	0.00%	0.00%	0.71%	2.78%	0.00%	0.63%	0.31%
1948	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1949	0.00%	0.00%	0.00%	0.00%	1.36%	1.02%	8.57%	0.00%	1.92%	0.84%
1950	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1951	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%	0.00%	0.43%	0.18%
1952	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1953	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1954	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	7.14%	0.00%	0.47%	0.17%
1955	0.00%	0.00%	0.00%	0.00%	0.00%	1.61%	0.00%	0.00%	0.52%	0.17%
1956	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1957	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%	0.00%	0.00%	0.45%	0.14%
1958	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1959	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1960	0.00%	0.00%	0.00%	0.00%	1.25%	0.00%	0.00%	0.00%	0.75%	0.25%
1961	0.00%	0.00%	0.00%	0.00%	0.60%	0.00%	8.70%	0.00%	1.07%	0.35%
1962	0.00%	0.00%	0.00%	0.00%	1.75%	1.47%	0.00%	0.00%	1.52%	0.47%
1963	0.00%	0.00%	0.00%	0.00%	1.16%	1.47%	0.00%	0.00%	1.15%	0.35%
1964	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1965	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1966	0.00%	0.00%	0.00%	0.00%	0.00%	2.44%	0.00%	0.00%	0.44%	0.12%
1967	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1968	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	5.00%	0.00%	0.37%	0.11%
1969	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1970	0.00%	0.00%	0.00%	0.54%	4.24%	19.44%	50.00%	0.27%	8.68%	2.63%
1971	0.00%	0.00%	0.00%	0.00%	0.89%	0.00%	12.50%	0.00%	1.16%	0.29%
1972	0.00%	0.00%	0.00%	0.00%	0.00%	6.90%	37.50%	0.00%	1.92%	0.45%
1973	0.00%	0.00%	0.00%	0.46%	0.00%	3.85%	37.50%	0.23%	1.28%	0.46%

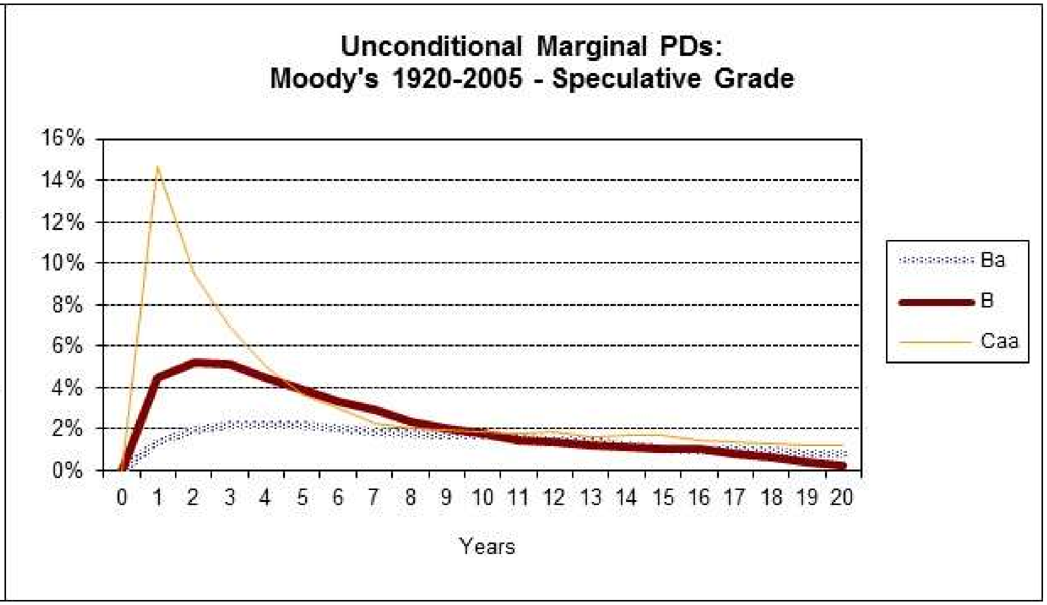
Source: Moody's Ratings (2025), "Annual Default Study", 28 Feb..

Year	Aaa	Aa	A	Baa	Ba	B	Caa-C	IG	SG	All
1974	0.00%	0.00%	0.00%	0.00%	0.51%	7.16%	0.00%	0.00%	1.33%	0.28%
1975	0.00%	0.00%	0.00%	0.00%	1.03%	6.16%	0.00%	0.00%	1.74%	0.36%
1976	0.00%	0.00%	0.00%	0.00%	1.00%	0.00%	0.00%	0.00%	0.87%	0.18%
1977	0.00%	0.00%	0.00%	0.29%	0.54%	3.23%	33.33%	0.11%	1.36%	0.35%
1978	0.00%	0.00%	0.00%	0.00%	1.12%	5.41%	0.00%	0.00%	1.82%	0.35%
1979	0.00%	0.00%	0.00%	0.00%	0.51%	0.00%	0.00%	0.00%	0.43%	0.09%
1980	0.00%	0.00%	0.00%	0.00%	0.00%	5.00%	33.33%	0.00%	1.63%	0.34%
1981	0.00%	0.00%	0.00%	0.00%	0.00%	4.40%	0.00%	0.00%	0.70%	0.16%
1982	0.00%	0.00%	0.26%	0.33%	2.79%	2.22%	21.43%	0.21%	3.54%	1.04%
1983	0.00%	0.00%	0.00%	0.00%	1.16%	2.22%	45.05%	0.00%	4.27%	0.95%
1984	0.00%	0.00%	0.00%	0.61%	0.52%	5.17%	18.18%	0.17%	3.08%	0.85%
1985	0.00%	0.00%	0.00%	0.00%	0.87%	7.11%	6.25%	0.00%	3.72%	0.94%
1986	0.00%	0.00%	0.00%	0.84%	2.37%	10.30%	17.11%	0.21%	6.12%	1.81%
1987	0.00%	0.00%	0.00%	0.00%	3.04%	5.35%	10.90%	0.00%	4.30%	1.41%
1988	0.00%	0.00%	0.00%	0.00%	1.36%	5.61%	13.16%	0.00%	3.74%	1.34%
1989	0.00%	0.50%	0.00%	0.51%	2.97%	7.53%	21.37%	0.25%	5.91%	2.21%
1990	0.00%	0.00%	0.00%	0.26%	3.53%	13.68%	47.53%	0.06%	10.45%	3.51%
1991	0.00%	0.00%	0.00%	0.25%	3.84%	13.12%	18.25%	0.06%	9.14%	2.79%
1992	0.00%	0.00%	0.00%	0.00%	0.34%	7.64%	17.01%	0.00%	4.97%	1.33%
1993	0.00%	0.00%	0.00%	0.00%	0.62%	4.35%	13.43%	0.00%	3.28%	0.86%
1994	0.00%	0.00%	0.00%	0.00%	0.00%	4.18%	4.35%	0.00%	2.23%	0.62%
1995	0.00%	0.00%	0.00%	0.00%	0.27%	3.82%	12.10%	0.00%	3.08%	0.90%
1996	0.00%	0.00%	0.00%	0.00%	0.00%	1.51%	9.88%	0.00%	1.66%	0.50%
1997	0.00%	0.00%	0.00%	0.00%	0.18%	2.01%	9.37%	0.00%	1.90%	0.61%
1998	0.00%	0.00%	0.00%	0.11%	0.76%	3.59%	9.71%	0.03%	2.99%	1.11%
1999	0.00%	0.00%	0.00%	0.09%	1.38%	5.04%	15.41%	0.03%	5.35%	2.09%
2000	0.00%	0.00%	0.00%	0.35%	1.44%	5.64%	18.38%	0.13%	6.19%	2.47%
2001	0.00%	0.00%	0.15%	0.18%	1.17%	8.73%	29.38%	0.12%	9.37%	3.53%
2002	0.00%	0.00%	0.16%	0.84%	1.77%	4.58%	27.30%	0.37%	7.89%	2.94%
2003	0.00%	0.00%	0.00%	0.00%	0.89%	2.57%	21.39%	0.00%	5.41%	1.86%
2004	0.00%	0.00%	0.00%	0.00%	0.38%	0.68%	11.06%	0.00%	2.30%	0.78%
2005	0.00%	0.00%	0.00%	0.16%	0.00%	0.72%	7.35%	0.06%	1.72%	0.64%
2006	0.00%	0.00%	0.00%	0.00%	0.19%	1.09%	5.57%	0.00%	1.67%	0.59%
2007	0.00%	0.00%	0.00%	0.00%	0.00%	0.10%	4.75%	0.00%	0.99%	0.36%
2008	0.00%	0.50%	0.40%	1.01%	2.33%	4.05%	10.65%	0.62%	5.47%	2.50%
2009	0.00%	0.00%	0.24%	0.92%	1.76%	6.82%	26.15%	0.42%	12.06%	4.96%
2010	0.00%	0.00%	0.17%	0.07%	0.00%	0.39%	8.73%	0.09%	3.11%	1.26%
2011	0.00%	0.19%	0.00%	0.36%	0.16%	0.35%	6.05%	0.19%	2.03%	0.92%
2012	0.00%	0.00%	0.00%	0.07%	0.14%	0.54%	8.09%	0.03%	2.85%	1.26%
2013	0.00%	0.00%	0.09%	0.12%	0.60%	1.01%	6.27%	0.09%	2.68%	1.25%
2014	0.00%	0.00%	0.09%	0.06%	0.15%	0.50%	4.76%	0.06%	2.03%	0.97%
2015	0.00%	0.00%	0.00%	0.00%	0.30%	2.41%	6.63%	0.00%	3.71%	1.75%
2016	0.00%	0.00%	0.00%	0.00%	0.14%	1.67%	9.01%	0.00%	4.57%	2.17%
2017	0.00%	0.00%	0.00%	0.00%	0.53%	0.45%	7.55%	0.00%	3.58%	1.71%
2018	0.00%	0.00%	0.00%	0.00%	0.00%	0.68%	5.14%	0.00%	2.45%	1.17%
2019	0.00%	0.00%	0.00%	0.11%	0.00%	1.23%	6.38%	0.06%	3.19%	1.53%
2020	0.00%	0.00%	0.00%	0.11%	0.13%	3.95%	12.52%	0.06%	6.86%	3.20%
2021	0.00%	0.00%	0.00%	0.00%	0.00%	1.36%	2.90%	0.00%	1.75%	0.80%
2022	0.00%	0.00%	0.00%	0.91%	4.17%	5.16%	3.81%	0.48%	4.27%	2.27%
2023	0.00%	0.00%	0.15%	0.11%	0.73%	0.74%	9.27%	0.11%	4.97%	2.35%
2024	0.00%	0.00%	0.00%	0.00%	0.15%	0.52%	9.65%	0.00%	4.82%	2.17%
Mean	0.00%	0.05%	0.09%	0.26%	1.01%	3.06%	10.39%	0.14%	2.90%	1.21%
Median	0.00%	0.00%	0.00%	0.00%	0.52%	1.74%	7.72%	0.00%	1.92%	0.85%
StDev	0.00%	0.17%	0.25%	0.44%	1.59%	3.67%	11.06%	0.26%	2.90%	1.33%
Min	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Max	0.00%	0.85%	1.64%	1.99%	11.71%	19.44%	50.00%	1.55%	15.77%	8.53%

PDs

- Marginal frequencies obtained from the cumulative figures tend to exhibit a very irregular shape.
- Marginal PD curves have different inflection points, depending on the rating class, with the lower inflection points for the higher risk classes.

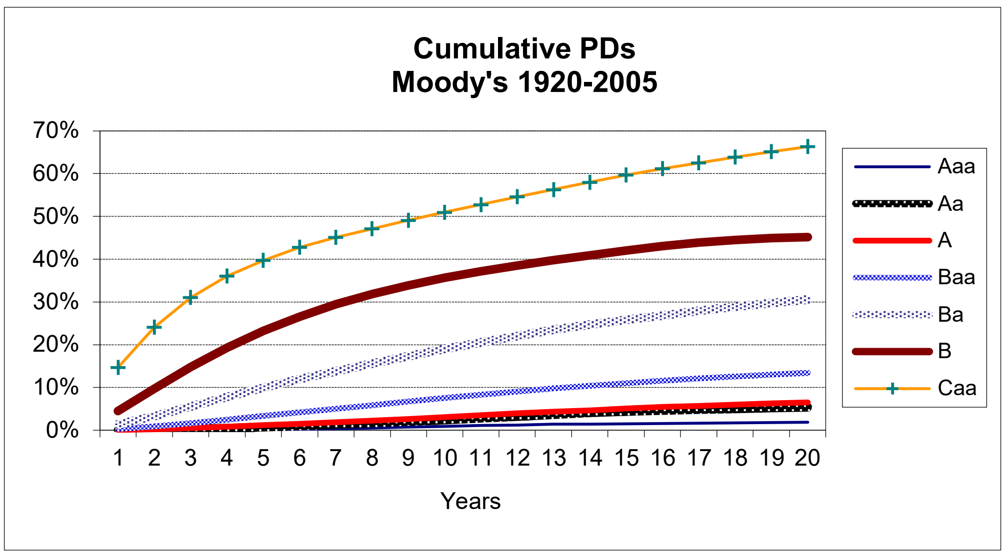




PDs

- **The irregular shape of marginal PD curves occurs even when cumulative PD curves exhibit an apparently smooth behavior.**

- It is recommended to smooth the cumulative PD curves to ensure a smother behavior of marginal PD curves, as the marginal curves are a measure of the 1st derivative of the cumulative curves.
- The cumulative PD curves can be smoothed by methods like the Nelson-Siegel-Svensson, with the cumulative PD curves corresponding to the spot curves and the marginal PD curves to the instantaneous forward curves.



PDs

- ***P(t)* – Cumulative probability of surviving *t* years**

- **Unconditional marginal probability of default between *t* and *s* - probability of default between any times *t* and *s* ≥ *t* as seen today*:*** difference between the cumulative probability of default until *s* and the same probability until *t*:

$$d'(s) = [1-P(s)]-[1-P(t)] = P(t) - P(s) = D(s) - D(t)$$

difference between 2 cumulative probabilities of default (D) seen today (being $D_0=0$)

- **Cumulative default frequencies are the sum of unconditional marginal default frequencies.**

PDs

- **Cumulative probability of surviving to time s (P(s))** = probability of surviving until t (P(t)) x probability of surviving between t and s, given that it has survived until t (p(s|t)):

  P(s) = P(t) x p(s|t)

- **Conditional marginal probability of surviving to time *s*, given survival to time *t***:

  p(s|t) = P(s)/P(t)

- **Conditional marginal probability of default at time *s*, given survival to time *t* (or** forward default probability):

  d(s|t) = 1- p(s|t) = 1-P(s)/P(t) = [P(t)-P(s)]/P(t) = -[P(s)-P(t)]/P(t) = -P'(t)/P(t)
  or = d'(s|t)/P(t) (as d'(s)= P(t) – P(s))

- Cumulative default frequencies can also be calculated as is 1 - the joint (cumulative) probability of surviving until *i-1* and the probability of surviving in *i*:

$$D_i = 1 - (1 - d_i)(1 - D_{i-1})$$

PDs

Table 24.1 Average cumulative default rates (%), 1970–2015 (*Source*: Moody's).

Term (years):	*1*	*2*	*3*	*4*	*5*	*7*	*10*	*15*	*20*
Aaa	0.000	0.011	0.011	0.031	0.087	0.198	0.396	0.725	0.849
Aa	0.022	0.061	0.112	0.196	0.305	0.540	0.807	1.394	2.266
A	0.056	0.170	0.357	0.555	0.794	1.345	2.313	4.050	6.087
Baa	0.185	0.480	0.831	1.252	1.668	2.525	4.033	7.273	10.734
Ba	0.959	2.587	4.501	6.538	8.442	11.788	16.455	23.930	30.164
B	3.632	8.529	13.515	17.999	22.071	29.028	36.298	43.368	48.071
Caa–C	10.671	18.857	25.639	31.075	35.638	41.812	47.843	50.601	51.319

Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

- For the Caa rating, the unconditional marginal probability of default (d') seen today for the 3rd year is equal to the difference between the cumulative probabilities of default for 3 (s) and 2 (t) years: d'(3) = D(3) – D(2) = 25.639% - 18.857% = 6.782%

- Conditional marginal probability of surviving at year 3, given survival to year 2: p(3|2)=P(3)/P(2) = (1-0.25639)/(1-0.18857) =0.91642

- Conditional marginal probability of default at year 3, given survival to year 2: d(3|2) = 1-p(3|2)= 1-0.91642= 0.0836.

PDs

- **The unconditional marginal probability of default between *s* and *t* measured today** is also the product between the cumulative probability of survival until *t* and the probability of default between t and s, given survival until *t*:

conditional marginal probability of default

d'(s) = P(t) x d(*s*|*t*) ⇔ d(*s*|*t*)=d'(s)/P(t) = 0.06782/(1-0.18857)= 0.0836

- Any unconditional probability of default may be measured as:

$$d_i' = d_i \prod_{j=1}^{i} \left(1 - d_{j-1}\right)$$

being d_i = d(s|t), (1-d_{j-1}) = P(t) and with d_0' = 0

DEFAULT INTENSITY

- The **conditional probability of default between *s* and *t*,** given survival until *t* (**d(*s*|*t*) = d'(s) / P(t))**, is also called **default intensity or hazard rate**.

- The conditional marginal default probability to the rating Caa previously calculated (8.36%) was for a 1-year period.

- If one considers a very short period of time Δt, denoting the hazard rate at *t* by $\lambda(t)$, the **probability of default between *t* and *t* + Δt conditional on no previous default (until *t*) is $\lambda(t) \times \Delta t$.**

- Many models of PDs are based on the notion of the arrival intensity of default.

DEFAULT INTENSITY

- The simplest version of such a model defines **default as the 1st arrival time τ** of a Poisson process with some constant mean arrival rate – average default intensity or **hazard rate** (λ):

$P(t) = e^{-\lambda t}$ - probability of survival for *t* years

$1/\lambda$ - expected time to default

$\lambda\Delta t$ – default intensity in t over a small period of length Δ (between *t* and $t+\Delta t$), given survival until *t*.

- Example: default intensity (λ) = 0.04 => $1/\lambda$ (expected time to default) = 25 (years).

=> 1-year PD (1-P(1)) = $1-e^{-0.04\times1}$ =3,9%

DEFAULT INTENSITY

- As it was shown before, $d'(s) = P(t) \times d(s|t) \Leftrightarrow d(s|t) = d'(s) / P(t)$.
- For a very short period of time Δt, this result becomes:

$$d(t+\Delta t|t) = d'(t+\Delta t)/P(t)$$

- As $d'(t+\Delta t)$ is the unconditional probability of default between t and Δt, it is the difference between the cumulative probabilities of default for $t+\Delta t$ and t:

$d'(t+\Delta t) = [1-P(t+\Delta t)]-[1-P(t)] = P(t) - P(t+\Delta t) \Rightarrow$ the previous equation becomes:

$$d(t+\Delta t|t) = d'(t+\Delta t)/P(t) = [P(t) - P(t+\Delta t)]/P(t)$$

DEFAULT INTENSITY

- As the conditional marginal probability of default (or default intensity) for a very short period of time is $\lambda \Delta t$, we have:

$$[P(t) - P(t+\Delta t)]/P(t) = \lambda \Delta t \Leftrightarrow [P(t+\Delta t) - P(t)] = -\lambda P(t)\, \Delta t$$

or

$$dP(t)/dt = -\lambda P(t)$$

DEFAULT INTENSITY

- **If the default intensity varies along time**, default intensity becomes $\lambda(t)\Delta t$ and the probability of survival for t years becomes:

$P(t) = e^{-[\lambda(1)+\lambda(2)+\ldots+\lambda(t)]}$ ⟶ Instead of $\boldsymbol{P(t) = e^{-\lambda t}}$, where λ is constant

- Actually, as P(s) = P(t) x p(s|t), for instance with s = 2 and t = 1 =>

$P(2) = P(1) \times p(2|1) = e^{-[\lambda(1)+\lambda(2)]}$

⬇

- In continuous time, we get $P(t) = e^{-\int_0^t \lambda(t)dt}$

⬇

$$D(t) = 1 - P(t) = 1 - e^{-\int_0^t \lambda(t)dt}$$

⬇

The only relevant information to default risk along time is the survival until then.

Default Intensity

- However, in reality, as time passes, one should have new information, beyond simply survival, that would bear on the credit quality of an issuer.

- The default intensity would generally vary at random as this additional information arrives.

- For example, one may assume that the intensity varies with an underlying state variable (driver), such as the credit rating, distance to default, equity price, or the business cycle.

DEFAULT INTENSITY

- If intensities are updated with new information at the beginning of each year and are constant during the year => Probability of survival to time t given survival to $t-1$, and given all other information available at time $t-1$:

$$P(t|t-1) = e^{-\lambda(t)}$$

$P(t|t-1)$ is unknown before t-1, as $\lambda(t)$ is based on information that is revealed only at time t-1.

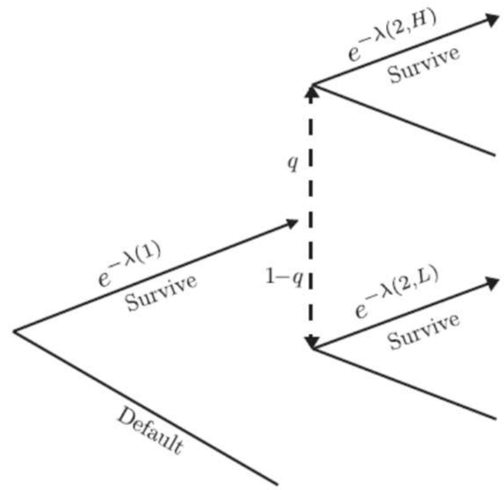
- At time t, we have **2 sources of uncertainty**:

(i) the behaviour in the following period (survival or default);

(ii) new information that will become available during the next period that will be relevant to calculate probabilities of survival and default in the following period.

DEFAULT INTENSITY

- Example – 2 periods:



Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

- Default intensity in the 2nd year (*λ(2)*), assuming the firm survives the 1st, is uncertain and takes 2 possible levels, *λ(2,H)* and *λ(2, L)*, with conditional probabilities *q* and 1 – *q*, respectively ($p(2|1)$):

$$p(2|1) = qe^{-\lambda(2,H)} + (1-q)e^{-\lambda(2,L)} = E\left[e^{-\lambda(2)}\right]$$

- 2-year survival probability (P(2)):

$$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-\lambda(1)} \cdot E\left[e^{-\lambda(2)}\right] = E\left[e^{-[\lambda(1)+\lambda(2)]}\right]$$

When there was no new information on the hazard rate there was no uncertainty about the λ's:

$\mathrm{P}(2) = \mathrm{P}(1) \cdot p(2|1) = e^{-[\lambda(1)+\lambda(2)]}$

DEFAULT INTENSITY

- Default time ⇔ 1st time that a coin toss results in "heads," given independent tosses of coins, one each period, with each toss having a probability λ of heads and $1-\lambda$ of tails ⇔ default is unpredictable ⇔ when default does occur, it is a "surprise." ⇔ default time is inaccessible.

The following assumption describes the way in which default arrival risk is modelled in all intensity-based default risk models:

Assumption 5.1 (intensity model default arrivals) *Let $N(t)$ be a counting process*[1] *with (possibly stochastic) intensity $\lambda(t)$. The time of default τ is the time of the first jump of N, i.e.*

$$\tau = \inf\{t \in \mathbb{R}_+ \mid N(t) > 0\}. \tag{5.1}$$

The survival probabilities in this setup are given by:

$$P(0, T) = \mathbf{P}[N(T) = 0 | \mathcal{F}_0]. \tag{5.2}$$

Poisson Processes

A Poisson process $N(t)$ is an increasing process in the integers 0, 1, 2, 3, ... More important than its unexciting set of values are the *times of the jumps* $\tau_1, \tau_2, \tau_3, \ldots$ and the probability of a jump in the next instant.

We assume that the probability of a jump in the next small time interval Δt is proportional to Δt:

$$\boxed{\mathbf{P}[N(t+\Delta t) - N(t) = 1] = \lambda \Delta t,} \tag{5.3}$$

Probability of default in a small period of time Dt = Probability of 1 jump in the Poisson Process

that jumps by more than 1 do not occur, and that jumps in disjoint time intervals happen independently of each other. This means, conversely, that the probability of the process remaining constant is

$$\boxed{\mathbf{P}[N(t+\Delta t) - N(t) = 0] = 1 - \lambda \Delta t,}$$

There is only 1 default, i.e. the default is an absorbing state.

Probability of survival in a small period of time Δt = Probability of no jumps in the Poisson Process

Poisson Processes

Probability of survival in 2 small periods is the joint probability of survival in each of them (given that the hazard rate is the same for all periods of the same magnitude)

and over the interval $[t, 2\Delta t]$ this probability is

$$\boxed{\begin{aligned}&\mathbf{P}[N(t+2\Delta t)-N(t)=0]\\&\quad=\mathbf{P}[N(t+\Delta t)-N(t)=0]\cdot\mathbf{P}[N(t+2\Delta t)-N(t+\Delta t)=0]=(1-\lambda\Delta t)^2.\end{aligned}}$$

Now we can start to construct a Poisson process. We subdivide the interval $[t, T]$ into n subintervals of length $\boxed{\Delta t = (T-t)/n.}$ In each of these subintervals the process N has a jump with probability $\Delta t\lambda$. We conduct n independent binomial experiments each with a probability of $\Delta t\lambda$ for a "jump" outcome.

The probability of *no* jump at all in $[t, T]$ is given by:

$$\boxed{\mathbf{P}[N(T)=N(t)]=(1-\Delta t\lambda)^n=\left(1-\frac{1}{n}(T-t)\lambda\right)^n}.$$

Δt

Probability of no jumps in *n* periods

POISSON PROCESSES

$$\mathbf{P}[N(T) = N(t)] = (1 - \Delta t\lambda)^n = \left(1 \boxed{- \frac{1}{n}(T - t)\lambda}\right)^n$$

x

Because $(1 + x/n)^n \to e^x$ as $n \to \infty$, this converges to:

$$\boxed{\mathbf{P}[N(T) = N(t)] \to \exp\{-(T - t)\lambda\}}$$

Probability of no jumps with a Poisson process

$$P(t) = e^{-\lambda t}$$

POISSON PROCESSES

Next we look at the probability of exactly *one* jump in $[t, T]$. There are n possibilities of having exactly one jump, giving a total probability of

Probability of a jump

Probability of no jumps in remaining n-1 periods

$$\mathbf{P}[N(T) - N(t) = 1] = n \cdot \Delta t \lambda (1 - \Delta t \lambda)^{n-1}$$

$$= n \cdot \frac{(T-t)}{n} \lambda \left(1 - \frac{1}{n}(T-t)\lambda\right)^n \Big/ \left(1 - \frac{1}{n}(T-t)\lambda\right)$$

Δt

$$= \frac{(T-t)\lambda}{1 - \frac{1}{n}(T-t)\lambda} \left(1 - \frac{1}{n}(T-t)\lambda\right)^n$$

->1 when n->∞

$$\rightarrow \boxed{(T-t)\lambda \exp\{-(T-t)\lambda\} \quad \text{as } n \rightarrow \infty,}$$

$$\mathbf{P}[N(T) = N(t)] = (1 - \Delta t \lambda)^n = \left(1 - \frac{1}{n}(T-t)\lambda\right)^n$$

$$\mathbf{P}[N(T) = N(t)] \rightarrow \exp\{-(T-t)\lambda\}$$

Probability of exactly one jump in [t,T]

$$\mathbf{P}[N(T) = N(t)] = (1 - \Delta t \lambda)^n = \left(1 - \frac{1}{n}(T-t)\lambda\right)^n$$

Probability of no jumps in n periods/Probability of no jump in 1 period (as it is assumed that there is a single **jump during n-1).**

POISSON PROCESSES

- For 2 jumps, there will be n/2 chances => probability of having 2 jumps is the joint probability of each of these jumps:

$$\mathbf{P}[N(T) - N(t) = 2] = \frac{1}{2}(T - t)^2 \lambda^2 \exp\{-(T - t)\lambda\}$$

- Probability of *n* jumps:

$$\mathbf{P}[N(T) - N(t) = n] = \frac{1}{n!}(T - t)^n \lambda^n \exp\{-(T - t)\lambda\}$$

- When a Poisson process with constant intensity λ (homogeneous Poisson process) is used, the hazard rate does not depend on time=> the term structure of spreads will be flat and constant over time, which does not correspond to reality => we need a time-varying λ => **Cox process or inhomogeneous Poisson process.**

POISSON PROCESSES

Roughly speaking, $\boxed{\text{Cox processes are Poisson processes with stochastic intensity.}}$

$$\mathbf{P}[N(t+\Delta t)-N(t)=1]=\lambda(t)dt$$

Now λ is time-varying

$$\mathbf{P}[N(T)-N(t)=0] = \prod_{i=1}^{n}(1-\lambda(t+i\Delta t)\Delta t)$$

The probability of no jumps over the period between t and T is the joint probability of no jumps in each moment during that period

$$\ln \mathbf{P}[N(T)-N(t)=0] = \sum_{i=1}^{n}\ln(1-\lambda(t+i\Delta t)\Delta t) \quad \approx \sum_{i=1}^{n}-\lambda(t+i\Delta t)\Delta t$$

As $ln(1-x)\approx -x$ for small x

$$\rightarrow -\int_t^T \lambda(s)ds \quad \text{as } \Delta t\rightarrow 0$$

As this is the result for the log of the probability of no jumps, this probability will be equal to:

$$\mathbf{P}[N(T)-N(t)=0]\rightarrow \exp\left\{-\int_t^T \lambda(s)ds\right\} \quad \text{as } \Delta t\rightarrow 0$$

$$\mathbf{P}[N(T)-N(t)=n]=\frac{1}{n!}\left(\int_t^T \lambda(s)ds\right)^n \exp\left\{-\int_t^T \lambda(s)ds\right\}$$

inhomogeneous Poisson process

Reminder - with constant l: $\mathbf{P}[N(T)-N(t)=n]=\frac{1}{n!}(T-t)^n\lambda^n \exp\{-(T-t)\lambda\}$

With variable l and t->0, l(T-t) is replaced by the integral of l(s)

DEFAULTABLE ZERO COUPON BONDS

The implied survival probability from t to $T \geq t$ as seen from time t is the ratio of the defaultable to the default-free ZCB prices:

$$\overline{B}(t,T) = B(t,T)e^{-\int_t^T \lambda(s)ds}$$

$$P(t,T) = \frac{\overline{B}(t,T)}{B(t,T)}$$

Zero Coupon Defaultable bond (with recovery rate = 0 and pay-off =1)

Zero Coupon Risk-free bond (with pay-off =1)

$$B(t,T) = \mathbf{E}\left[e^{-\int_t^T r(s)ds} \cdot 1\right]$$

- Probability of Default:

$$P^{\text{def}}(t,T) := 1 - P(t,T)$$

DEFAULTABLE ZERO COUPON BONDS

$$I(t) = \mathbf{1}_{\{\tau > t\}} = \begin{cases} 1 & if \quad \tau > t, \\ 0 & if \quad \tau \leq t. \end{cases}$$

$$\text{Payoff} = \mathbf{1}_{\{\tau > T\}} = \begin{cases} 1 & \text{if default after } T\text{, i.e. } \tau > T, \\ 0 & \text{if default before } T\text{, i.e. } \tau \leq T \end{cases}$$

For the Zero Coupon Defaultable bond, the pay-off will be 1 only if the debtor is still alive at *T*.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

$$\overline{B}(t, T) = \mathbf{E}\left[e^{-\int_t^T r(s)ds} \cdot I(T)\right]$$

$$\overline{B}(t, T) = \mathbf{E}\left[e^{-\int_t^T r(s)ds} \cdot I(T)\right] = \mathbf{E}\left[e^{-\int_t^T r(s)ds}\right]\mathbf{E}\left[I(T)\right]$$
$$= B(t, T)\mathbf{E}[I(T)] = B(t, T)P(t, T),$$

DEFAULTABLE ZERO COUPON BONDS

- If the time of default is the time of the 1st jump of a Poisson process N(t) and is independent from the default-free interest rate, the price of a defaultable bond with zero recovery becomes:

$$\overline{B}(0,T) = \mathbf{E}\left[e^{-\int_0^T r(s)ds}\mathbf{1}_{\{N(T)=0\}}\right]$$

Assuming that the risk-free interest rate is independent from the arrival intensity of default

$$\overline{B}(0,T) = \mathbf{E}\left[e^{-\int_0^T r(s)ds}\right]\mathbf{E}\left[\mathbf{1}_{\{N(T)=0\}}\right],$$

$$\overline{B}(0,T) = B(0,T)e^{-\int_0^T \lambda(s)ds}.$$

The defaultable bond price corresponds to the price of the risk-free bond, discounted by the hazard rate (which corresponds to the expected loss (EL), as EL=PDxLGD and if the recovery rate =0, the LGD=1).

Assuming that the risk-free interest rate is correlated with the arrival intensity of default

$$\overline{B}(0,T) = \mathbf{E}\left[e^{-\int_0^T r(s)+\lambda(s)ds}\right]$$

The defaultable bond price corresponds to the NPV of the expected future cash-flows, using as discount rate the yield of the defaultable bond (the risk-free + hazard rate).

CREDIT DERIVATIVES

Definition:

(a) *A credit derivative is a derivative security that is primarily used to transfer, hedge or manage credit risk.*
(b) *A credit derivative is a derivative security whose payoff is materially affected by credit risk.*

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

Narrower definition:

- A **credit derivative** is a derivative security that has a payoff which is conditioned on the occurrence of a **credit event**.

→ **We need to define what are credit events.**

CREDIT DERIVATIVES

Traditionally, a bank could only manage its credit risks at origination. Once the risk was originated, it remained on the books until the loan was paid off or the obligor defaulted. There was no efficient and standardised way to transfer this risk to another party, to buy or sell protection, or to optimise the risk–return profile of the portfolio. Consequently, the pricing of credit risks was in its infancy, spreads on loans only had to be determined at origination and were often determined by non-credit considerations such as the hope of cross-selling additional business in the corporate finance sector. There was no need to become more efficient because the absence of a transparent market meant that the mode of operation was more like an oligopoly than an efficient competition. Whether a loan was mispriced or not was impossible to determine with certainty, it all depended on the individual subjective assessment of the obligor's default risk. The main "cost" of extending a loan was the cost of the regulatory risk capital as prescribed by the rules of the Basel I capital accord, and this is the point where credit derivatives came in.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DERIVATIVES

Key terms:

Reference entity/reference credit: One (or several) issuer(s) whose defaults trigger the credit event. This can be one or several (a basket structure) defaultable issuers.

Reference obligations/reference credit asset: A set of assets issued by the reference credit. They are needed for the determination of the credit event and for the calculation of the recovery rate (which is used to calculate the default payment). Possible reference credit assets can range from "any financial obligation of the reference entity" to a specific list of some of the bonds issued by the reference entity. Loans and liquidly traded bonds are a common choice. The reference credit assets are clearly identified in the credit derivative's specification.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

Types of credit events:

- bankruptcy
- failure to pay
- obligation default
- obligation acceleration
- repudiation/moratorium
- restructuring
- ratings downgrade below given threshold
- changes in the credit spread

Standardized by ISDA (International Swap Dealers Association), even though they may also be freely negotiated.

- The credit event is defined with respect to a **reference credit**, and the **reference credit asset(s)** issued by the reference credit.

Reference Credit: Firm, institution or person who may default.

Types of reference credit assets:

Reference Credit Assets

- loans
  - floating or fixed rate
  - may include optionality (interest rate caps, credit facilities)
  - not traded, thus recovery rate may be hard to determine
- bonds
  - fixed–coupon or floater
  - zero coupon
  - convertible
- counterparty risk

Market Terminology

- Credit derivatives can be defined on single-name or multi-name.

- Buying a credit derivative typically means **buying credit protection**, which is economically equivalent to **shorting the credit risk**.
- So **selling** credit protection means going **long** the credit risk.
- Alternatively, one may speak of protection buyers/sellers as the payers/receivers of the premium.

- The most popular single-name credit derivative is the CDS.

Table 1.1 Size of the market for credit derivatives according to surveys by the British Bankers' Association and *Risk* (Patel, 2002)

Year	1997	1998	1999	2000	2001
Outstanding notional (USD bn)	170	350	586	893	1398

Table 1.2 Market share by instrument type (rounded numbers)

Instrument	Share (%)
Credit default swaps (including FtDs)	67
Synthetic balance sheet CLOs	12
Tranched portfolio default swaps	9
Credit-linked notes, asset repackaging, asset swaps	7
Credit spread options	2
Managed synthetic CDOs	2
Total return swaps	1
Hybrid credit derivatives	0.2

Source: Risk (Patel, 2002).

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

Credit Default Swaps

The aim is to transfer **ONLY** the default risk from A to B.

The protection seller **B** agrees to pay the default payment

$$\text{notional} \times (1 - \text{recovery rate})$$

to **A** if a default has happened.

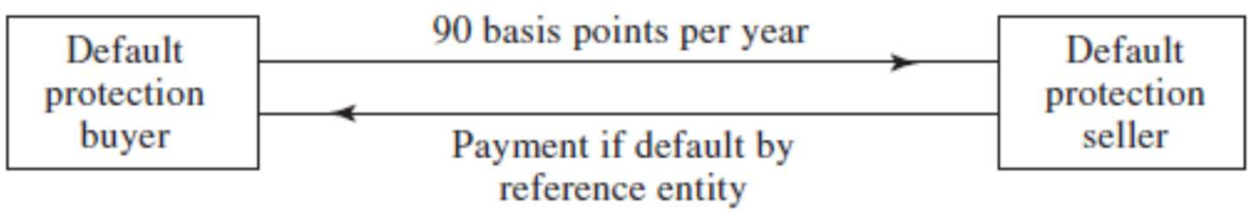
For this, **A** pays a periodic fee $\bar{s}$ to **B** (until maturity of the CDS or until default, whichever comes first)

In a single-name *credit default swap* (**CDS**) (also known as a *credit swap*) **B** agrees to pay the default payment to **A** *if a default has happened*. The default payment is structured to replace the loss that a typical lender would incur upon a credit event of the reference entity. If there is no default of the reference security until the maturity of the default swap, counterparty **B** pays nothing.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DEFAULT SWAPS

- CDS may have different specifications regarding the default payment.



Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

Default swaps can differ in the specification of the default payment. Possible alternatives are:

- Physical delivery of one or several of the reference assets against repayment at par;
- Notional minus post-default market value[3] of the reference asset (cash settlement);
- A pre-agreed fixed payoff, irrespective of the recovery rate (default digital swap).

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DEFAULT SWAPS

- Example of a CDS with a fixed repayment at default:

Example 2.1 *Default digital swap on the United States of Brazil. Counterparty* **B** *(the insurer) agrees to pay USD 1m to counterparty* **A** *if and when Brazil misses a coupon or principal payment on one of its Eurobonds. Here:*

- *The* reference credit *is the United States of Brazil;*
- *The* reference credit assets *are the Eurobonds issued by Brazil (in the credit derivative contract there would be an explicit list of these bonds);*
- *The* credit event *is a missed coupon or principal payment on one of the reference assets;*
- *The* default payment *is USD 1m.*

In return for this, counterparty **A** *pays a fee to* **B**.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DEFAULT SWAPS

- Most CDS have a physical delivery.

To identify a credit default swap, the following information has to be provided:

1. The reference obligor and his reference assets;
2. The definition of the credit event that is to be insured (default definition);
3. The notional of the CDS;
4. The start of the CDS, the start of the protection;
5. The maturity date;
6. The credit default swap spread;
7. The frequency and day count convention for the spread payments;
8. The payment at the credit event and its settlement.

Source: Schonbucher, Philipp J. (2003), “Credit Derivatives Pricing Models – Models, Pricing and Implementation”, Wiley.

CREDIT DEFAULT SWAPS

- The definition of default is key:

The event that is to be insured against is a default of the reference obligor, but because of the large payments involved the definition of what constitutes a default has to be made more precise, and a mechanism for the determination of the default event must be given. The standard definition of default includes:

- bankruptcy, filing for protection,
- failure to pay,
- obligation default, obligation acceleration,
- repudiation/moratorium,
- restructuring.[4]

There is a debate whether restructuring should be included as a default event in the specification or not, and some market makers even quote different prices for CDSs with and without restructuring in the default definition. Sometimes (in particular in default definitions for CDOs), a slightly different default definition is used which is based upon rating agencies' definitions of default. Despite the growing standardisation of the default definition, one advantage of a CDS is that both parties can agree to an event definition that can be completely different from the standard ISDA specification.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DEFAULT SWAPS

- CDS payments before default:

Example 2.3 *Credit default swap on Daimler Chrysler.*

The trade

*At time $t = 0$, **A** and **B** enter a credit default swap on Daimler Chrysler, **A** as protection buyer and **B** as protection seller. They have agreed on:*

(i) *The reference credit: Daimler Chrysler AG.*
(ii) *The term of the credit default swap: 5 years.*
(iii) *The notional of the credit default swap: 20m USD.*
(iv) *The credit default swap fee: $\bar{s} = 116$ bp.*

Semi-annual amount to be paid by the protection buyer

*The credit default swap fee $\bar{s} = 116$ bp is quoted per annum as a fraction of the notional. **A** pays the fee in regular intervals, semi-annually. To make our life easier, we simplify the day count fractions to 1/2 such that **A** pays to **B**:*

$$116\ bp \times 20m/2 = \boxed{116\,000\ USD} \quad at \quad T_1 = 0.5,\ T_2 = 1, \ldots, T_{10} = 5$$

These payments are stopped and the CDS is unwound as soon as a default of Daimler Chrysler occurs.

Source: Schonbucher, Philipp J. (2003), “Credit Derivatives Pricing Models – Models, Pricing and Implementation”, Wiley.

CREDIT DEFAULT SWAPS

- CDS payments after default – physical settlement:

Because the payments are done each semester

The default payment

First, ***A*** *pays the remaining accrued fee. If the default occurred two months after the last fee payment,* ***A*** *will pay* 116 000 × 2/6. *The next step is the determination of the default payment. If physical settlement has been agreed upon,* ***A*** *will deliver Daimler Chrysler bonds to* ***B*** *with a total notional of USD 20m (the notional of the CDS). The set of deliverable obligations has been specified in the documentation of the CDS. As liquidity in defaulted securities can be very low, this set usually contains more than one bond issue by the reference credit. Naturally* ***A*** *will choose to deliver the bond with the lowest market value, unless he has an underlying position of his own that he needs to unwind. (Even then he may prefer to sell his position in the market and buy the cheaper bonds to deliver them to* ***B****.) This* delivery option *enhances the value of his default protection.* ***B*** *must pay the full notional for these bonds, i.e. USD 20m in our example.*

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

CREDIT DEFAULT SWAPS

- CDS payments after default – cash settlement:

If cash settlement has been agreed upon, a robust procedure is necessary to determine the market value of the bonds after default. If there were no liquidity problems, it would be sufficient to ask a dealer to give a price for these bonds, and use that price, but liquidity and manipulation are a very real concern in the market for distressed securities. Therefore not one, but several, dealers are asked to provide quotes, and an average is taken after eliminating the highest and lowest quotes. This is repeated, sometimes several times, in order to eliminate the influence of temporary liquidity holes. Thus the price of the defaulted bonds is determined, e.g. 430 USD for a bond of 1000 USD notional. Now, the protection seller pays the difference between this price and the par value for a notional of 20m USD, i.e.

$$(1000 - 430)/1000 \times 20m\ USD = 11.4m\ USD$$

Because the price determination in cash settlement is so involved, most credit default swaps specify physical delivery in default. Cash settlement is only chosen when there may not be any physical assets to deliver (i.e. the reference entity has not issued enough bonds) or if the CDS is embedded in another structure where physical delivery would be inconvenient, e.g. a credit-linked note.

Source: Schonbucher, Philipp J. (2003), “Credit Derivatives Pricing Models – Models, Pricing and Implementation”, Wiley.

VALUATION

- In the previous example, the CDS fee or spread was given.
- However, in reality, this spread has to be calculated.

Example:

- Maturity = 5 years
- Notional amount = $1
- CDS fee = *s*%
- Frequency of swap payments = yearly
- Recovery Rate = 40%
- Defaults assumed to occur at mid-year
- Risk-free interest rate = 5% (continuously compounded, flat)
- Hazard rate = 2%

VALUATION

- Unconditional probability of default:

$$P(t) = e^{-\lambda t}$$

Year	*Probability of surviving to year end*	*Probability of default during year*
1	0.9802	0.0198
2	0.9608	0.0194
3	0.9418	0.0190
4	0.9231	0.0186
5	0.9048	0.0183

0.9802-0.9608

Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

VALUATION

- Present value of expected payments:

= probability of survival x CDS fee

Time (years)	*Probability of survival*	*Expected payment*	*Discount factor*	*PV of expected payment*
1	0.9802	0.9802*s*	0.9512	0.9324*s*
2	0.9608	0.9608*s*	0.9048	0.8694*s*
3	0.9418	0.9418*s*	0.8607	0.8106*s*
4	0.9231	0.9231*s*	0.8187	0.7558*s*
5	0.9048	0.9048*s*	0.7788	0.7047*s*
Total				4.0728*s*

Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

VALUATION

- Present value of expected payoffs:

Time (years)	*Probability of default*	*Recovery rate*	*Expected payoff ($)*	*Discount factor*	*PV of expected payoff ($)*
0.5	0.0198	0.4	0.0119	0.9753	0.0116
1.5	0.0194	0.4	0.0116	0.9277	0.0108
2.5	0.0190	0.4	0.0114	0.8825	0.0101
3.5	0.0186	0.4	0.0112	0.8395	0.0094
4.5	0.0183	0.4	0.0110	0.7985	0.0088
Total					0.0506

Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

- As the default occurs in mid-year, an accrual payment is owed, due to the period between the last payment and the default date.

VALUATION

- Due to the difference between the default time (that occurs halfway through a year) and the previous payment, an accrual payment is owed.
- This will be the sum of the present value of the expected cash-flows: $\sum_{i=1}^{5} d'_i \cdot \tau_i \cdot s$, being τ_i = the accrual time (0.5, as it is assumed that the default occurs halfway through a year).

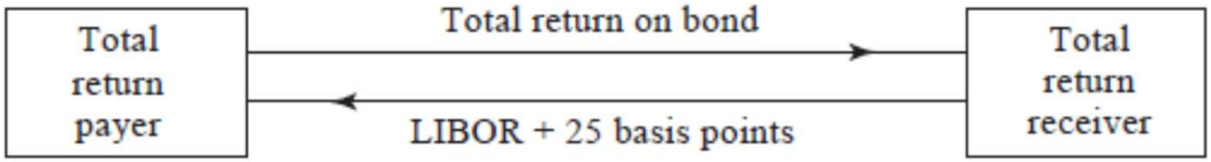
Time (years)	*Probability of default*	*Expected accrual payment*	*Discount factor*	*PV of expected accrual payment*
0.5	0.0198	0.0099*s*	0.9753	0.0097*s*
1.5	0.0194	0.0097*s*	0.9277	0.0090*s*
2.5	0.0190	0.0095*s*	0.8825	0.0084*s*
3.5	0.0186	0.0093*s*	0.8395	0.0078*s*
4.5	0.0183	0.0091*s*	0.7985	0.0073*s*
Total				0.0422*s*

Source: Hull, John (2018), "Options, futures and other derivatives", 10th Edition, Pearson.

- s will be calculated from the identity between the present value of the expected payments and the present value of the expected pay-off:

$$4.0728s + 0.0422s = 0.0506 \leftrightarrow s = 1.23\%$$

TOTAL RETURN SWAPS



In a *total return swap* (**TRS**) (or *total rate of return swap*) **A** and **B** agree to *exchange all cash flows* that arise from two different investments. Usually one of these two investments is a defaultable investment, and the other is a default-free Libor investment. This structure allows an exchange of the assets' payoff profiles without legally transferring ownership of the assets.

The payoffs of a total rate of return swap are as follows. Counterparty **A** pays to counterparty **B** at regular payment dates T_i, $i \leq N$

- The coupon $\bar{c}$ of the bond issued by **C** (if there was one since the last payment date T_{i-1});
- The price appreciation $(\overline{C}(T_i) - \overline{C}(T_{i-1}))^+$ of bond **C** since the last payment;
- The principal repayment of bond **C** (at the final payment date);
- The recovery value of the bond (if there was a default).

B pays at the same intervals:

- A regular fee of Libor $+ s^{TRS}$;
- The price depreciation $(\overline{C}(T_{i-1}) - \overline{C}(T_i))^+$ of bond **C** since the last payment (if there was any);
- The par value of the bond (if there was a default in the meantime).

The aim is to swap the actual return of a defaultable bond into a cash-flow of LIBOR plus a spread

A pays while the bond price increases (like selling a futures contract)

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

TOTAL RETURN SWAPS

Advantages:

- Counterparty **B** is long the reference asset without having to fund the investment up front. This allows counterparty **B** to leverage his position much higher than he would otherwise be able to. Usually, depending on his credit quality, **B** will have to post collateral, though.
- If the reference asset is a loan and **B** is not a bank then this may be the only way in which **B** can invest in the reference asset.
- Counterparty **A** has hedged his exposure to the reference credit if he owns the reference asset (but he still retains some counterparty risk).
- The transaction can be effected without the consent or knowledge of the reference credit **C**. **A** is still the lender to **C** and keeps the bank–customer relationship.
- If **A** does not own the reference asset he has created a short position in the asset. Because of its long maturity, a short position with a TRS is less vulnerable to short squeezes than a short repo position. Furthermore, directly shorting defaultable bonds or loans is often impossible.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

First to Default Swaps

A first-to-default swap (FtD) is the extension of a credit default swap to portfolio credit risk. Its key characteristics are the following:

- Instead of referencing just a single reference credit, an FtD is specified with respect to a **basket of reference credits $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_k$**.
- The set of reference credit assets (the assets that can trigger default events) contains assets by all reference credits.
- The protection buyer **A** pays a regular fee of $\bar{s}^{\mathrm{FtD}}$ to the protection seller **B** until the default event occurs or the FtD matures.
- The default event is the **first default** of any of the reference credits.
- The FtD is terminated after the first default event.
- The default payment is "1 − recovery" on the defaulted obligor. If physical delivery is specified, the set of deliverable obligations contains only obligations of the defaulted reference credit.

The basket of a FtD tipically comprises 4 to 12 reference credits.

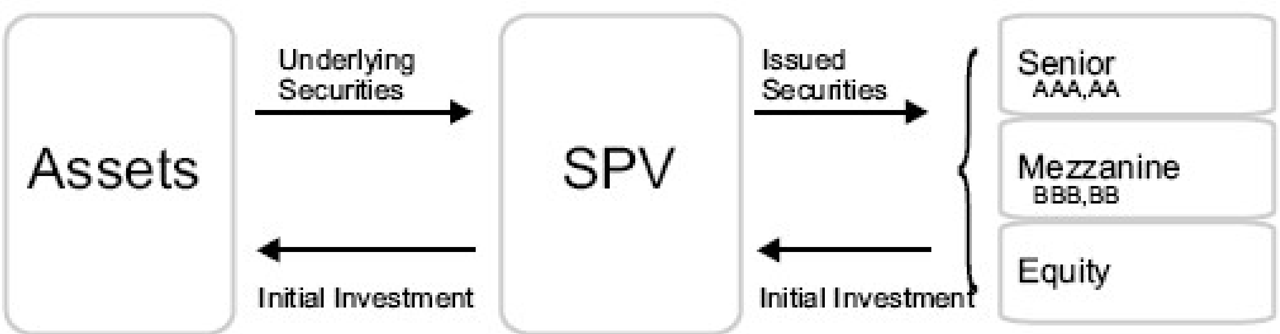
A natural extension of the first-to-default concept is the introduction of second-to-default (StD) and nth-to-default (ntD) basket credit derivatives. Such credit derivatives only differ in the specification of the default event, the basic structure remains the same. While FtD credit derivatives are a common structure, second- and higher-order ntD structures are rarer.

COLLATERALIZED BOND OBLIGATIONS

- underlying portfolio of defaultable bonds
- the portfolio is transferred to an SPV
- the SPV issues notes
    - an equity (or first loss) tranche
    - several mezzanine tranches
    - a senior tranche

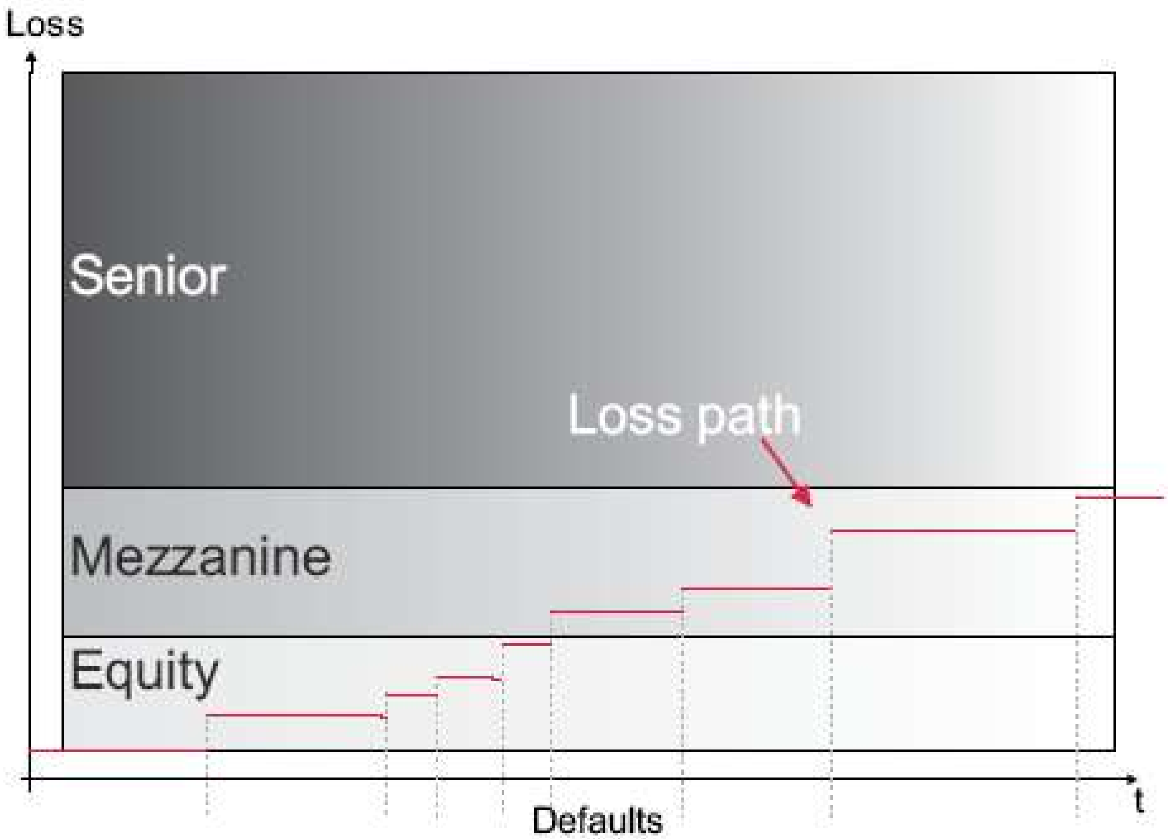
These notes are collateralized by the bonds sold to the SPV

Similar to RMBS but with bonds instead of residential mortgage loans



- if during the life of the CBO one of the bonds defaults, the recovery payments are reinvested in default–free securities

- at maturity of the CBO, the portfolio is liquidated and the proceeds distributed to the tranches, according to their seniority ranking



In this case, no losses will be suffered by the senior bonds, while equity bonds will get a total loss.

COLLATERALIZED DEBT OBLIGATIONS

- Designed exactly in the same way as CBOs. The main difference is that the underlying assets can be defaultable bonds or any other credit related instruments.

- Cash CDO – when the underlying assets are bonds

- Synthetic CDOs – when the underlying bonds are replaced by credit derivatives, e.g.:
  - CLOs – when the underlying assets are loans.
  - CDS are often used as underlying assets.

CREDIT-LINKED NOTES

- **CLNs are a combination of a medium-term note with a credit derivative, where the underlying note pays a given reference rate plus a spread, that is related to the issuer and the underlying entity risks.**
- In its simplest form, a CLN is just a note (bond or loan) with an embedded credit feature.
- The issuer is typically a bank, with a high rating, but can also be issued by a non-financial entity.
- Sometimes CLNs have principal protection, i.e. only the coupon payments of the note are at risk if a credit event occurs.
- If no principal protection is provided, a CDS is embedded in the CLN and the issuer of the CLN is buying protection on the risk of a given underlying entity.
- In that case, the investor is exposed to 2 risks:
  - The counterparty risk
  - The underlying entity risk

CREDIT-LINKED NOTES

Example of a CLN with no principal protection:

Example 2.8 (Wal-Mart credit-linked note) *Issuer: JPMorgan, September 1996 (via an AAA trust). The buyers of the CLN receive:*

- *Coupon (fixed or floating);*
- *Principal if no default of reference credit (Wal-Mart) until maturity;*
- *Only the recovery rate on the reference obligation as final repayment if a default of reference credit occurs.*

The buyers of the note now have credit exposure to Wal-Mart which is largely equivalent to the direct purchase of a bond issued by Wal-Mart. They also have some residual exposure to the credit risk of the AAA-rated trust set up to manage the note. From JPMorgan's point of view the investors of the CLN have sold them a CDS and posted 100% collateral.

Source: Schonbucher, Philipp J. (2003), "Credit Derivatives Pricing Models – Models, Pricing and Implementation", Wiley.

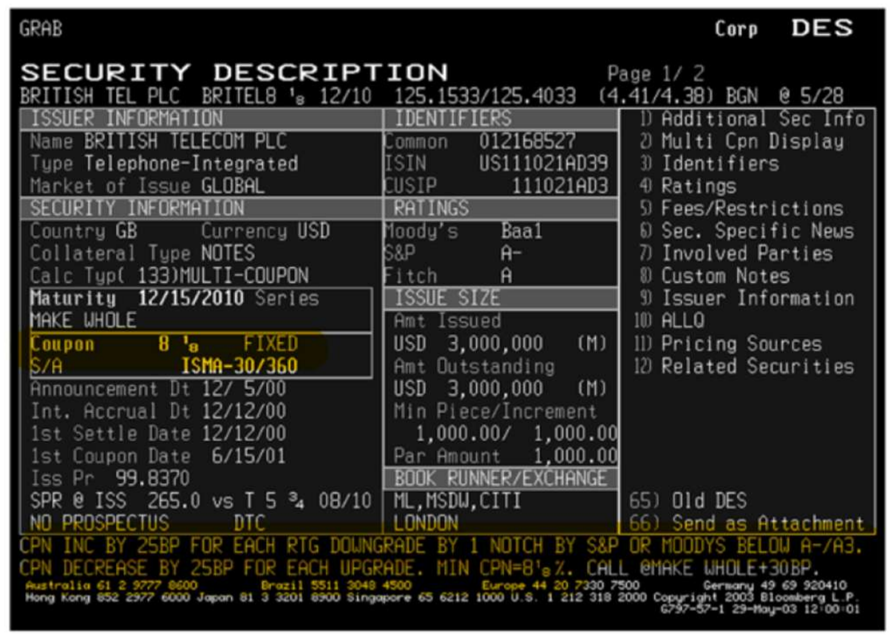
CREDIT-LINKED NOTES

- If the CLN is a 100% principal protected note, with an embedded CDS, the coupon of the note terminates following a credit event and the note redeems at par on its maturity date.
- The cost of the protection is usually a loss or reduction in the coupon on the note following the credit event.
- In this case, the only principal exposure that the investor has is to the issuer of the note, just like in a plain vanilla bond, and the CLN may be structured as a risk-free bond + a call option on the credit risk of the reference entity.

Example of a CLN with principal protection:

- Bond issued by British Telecom, with a minimum coupon of 8.125%, increasing by 25 bps for each one-notch rating downgrade below A–/A3 suffered by the issuer during the life of the note and decreasing by 25 bps for each ratings upgrade.

EXHIBIT 6.5 Bloomberg Screen DES for British Telecom plc 8.125% 2010 Credit-Linked Note Issued on December 5, 2000



Source: © Bloomberg L.P. Used with permission.

Source: Anson, Mark J.P., Frank J. Fabozzi, Moorad Choudhry, Ren-Raw Chen (2004), "Credit Derivatives: Instruments, Applications, and Pricing Credit Derivatives: Instruments, Applications, and Pricing", Wiley.