



Microeconomics
Fall 2025-2026
Midterm
November 2025

Duration: 1 hour (60 minutes)

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General Guidelines

- You may use a calculator;
- You may **not** use a programmable calculator;
- You may **not** use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up to 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.

1. Production (5 points)

Consider the production function $y = \frac{1}{2}x_1^{1/2}x_2^{1/2}$, where y is the output and x_1, x_2 are the two inputs.

1.1 Find the technical rate of substitution (TRS).

1.2. Sketch the isoquant and the input requirement set corresponding to the output level $= \bar{y}$:

$$\{ (x_1, x_2) \text{ in } R_+^2 \mid \frac{1}{2}x_1^{1/2}x_2^{1/2} = \bar{y} \}$$

1.3. Using the formula of the TRS, and/or the figure of the isoquant, discuss how the TRS depends on the levels of x_1 and x_2 . Provide an economic interpretation of this relationship.

2. Profit maximization (8 points)

A firm produces output y using one input x . The output price is p , and the input price is w . The table below reports the firm's choices and prices over two years. Suppose the firm forgot to record its output in Year 2, so that y^2 is missing in your data. You observe the following:

Year	p	w	y	x
1	6	3	8	2
2	4	2	a	3

2.1. For which value(s) of a the firm's behavior **does not violate** WAPM?

2.2. Briefly explain why WAPM is only a necessary (but not sufficient) condition for profit maximization.

2.3. Now assume differentiability and $y = f(x)$. Consider the firm produces one output y and multiple inputs $\mathbf{x}$, solving

$$\pi(p, w) = py - wx$$

Write the first order condition with respect to input x_1 . Then suppose that expression is positive. Should the firm increase or decrease x_1 to raise profits? Explain.

3. Constrained optimization (4 points)

A firm uses inputs x_1 and x_2 to produce output y . Imagine that the prices for x_1 and x_2 are $w_1 = 4$ and $w_2 = 1$ respectively. The production function is given by $y = x_1^{1/3}x_2^{2/3}$.

3.1. Calculate the minimum costs to produce $y = 12$.

5. Cost functions (3 points)

5.1. Provide a brief but rigorous (mathematical) proof to show the short-run marginal cost curve (SMC) is equal to short-run average cost (SAC) at the point of minimum SAC. That is, let $y = y^*$ be the point of minimum SAC, and show that:

$$\frac{\partial c(y^*)}{\partial y} = \frac{c(y^*)}{y^*}$$

Hint: Recall that $SAC = \frac{c(y)}{y}$ and $MC = \frac{\partial c(y)}{\partial y}$.