First Exam
December 10, 2025

Advanced Macroeconomics
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Part 1

First, this is a closed-book examination. Only a pen or pencil and a single sheet of paper may be placed on the tabletop. **Second,** write your name on your exam. **Third,** this part of the exam has one question with multiple sub-questions. The sub-questions build on each other, so it is important to take your time and get a sub-question right before moving on to the next one. **Fourth,** please be sure to use your time wisely and show your work. Good luck.

Consider a variant of the standard model in which velocity can be greater than 1. Assume that the cash-in-advance constraint is given by

$$M_t \geq \kappa P_t (1+\tau_c) C_t,$$

where κ captures the extent to which cash is used in transactions and τ_c is the consumption tax. The remaining portion $(1-\kappa)P_t(1+\tau_c)C_t$ must be paid for in the asset market at the end of the period. The budget constraint is given by

$$P_t Z_t L_t + M_t - P_t(1+\tau_c)C_t + B_t + T_t - M_{t+1} - q_t B_{t+1} = 0.$$

All the other aspects of the model are standard, including the household's payoff which is given by

$$\sum_{t=1}^{T} \beta^{t-1} \left[u(C_t) - v(L_t) \right] + \beta^T V(M_{T+1}, B_{T+1}).$$

The market clearing conditions are also unchanged and given by

$$\begin{aligned} C_t &= Z_t L_t \\ B_{t+1} &= 0 \\ M_{t+1} &= \bar{M}_{t+1} \end{aligned}$$

where the aggregate money supply grows according to $\bar{M}_{t+1} = (1+\tau)\bar{M}_t$ and transfers are given by $T_t = P_t \tau_c C_t + \tau \bar{M}_t$. Assume also that productivity grows according to $Z_{t+1} = (1+g)Z_t$.

A) [10 pts.] Write down the Lagrangian for this problem and determine the first-order conditions for the optimal levels of consumption, labor, money and bonds. In your answer let λ_t be the Lagrange multiplier of the CIA constraint and μ_t be the Lagrange multiplier of the budget constraint.

B) [10 pts.] Assume that preferences take the forms

$$u(C_t) = \log(C_t), \quad v(L_t) = \frac{L_t^{1+\gamma}}{1+\gamma}.$$

Assume that the cash-in-advance constraint binds and that the goods market clearing condition holds. If labor L_t was constant what would this lead you to predict about the rate of change in the price level P_{t+1}/P_t?

Use your result here to discuss how lowering κ from our standard value of 1 can affect the endogenous variables in the standard empirical velocity equation (which we use to measure v):

$$Mv = PY.$$

C) [10 pts.] In the first-order conditions of the consumer's problem make a change-in-variables with respect to the multipliers, so that the unknowns are time invariant. Derive the expression for q_t in the steady state.

D) [10 pts.] We previously saw (in the model with $\kappa = 1$) that the solution to the social planner's problem boiled down to simply solving for L such that it maximized

$$\log(Z_t L_t) - v(L_t).$$

Denote this optimal level of labor by L^* and solve for it given our functional forms for u and v. If we guess that this is still the best we can do, does this still require that $q_t = 1$? And, if so, why?

E. [10 pts.] Compute the balanced growth path equilibrium for C_t and L_t. Let $L(\tau, \tau_c, \kappa)$ be the balanced growth path equilibrium for L_t. How does L depend upon these three variables? Also fixing τ and τ_c, what does reducing κ do to the gap between $L^* - L(\tau, \tau_c, \kappa)$? What does this say about the cost of inflation?