For each of the following case studies, formulate an appropriate linear programming model (or, if necessary, a binary/integer or mixed-integer linear programming model).

1. An association of young environmental activists decided to create four open-air markets to sell fruit from small producers on the first Saturday of each month. The table below shows the distances, in kilometres, between the farms of the eight producers who joined the initiative and the markets created by the group.

   The estimated monthly sales at the markets are 120, 150, 90, and 130 kg/month, respectively. The quantities of fruit supplied by the producers for this initiative are 50, 65, 80, 75, 95, 60, 80, and 65 kg/month, respectively.

   The organizers decided that no sale involving transportation over 15 kilometres will be allowed, and they wish the producers to feel that they are treated fairly. What should the organizers propose to each of the producers?

	P1	P2	P3	P4	P5	P6	P7	P8
Market 1	13	12	19	3	8	2	9	17
Market 2	18	6	2	16	7	5	11	10
Market 3	5	14	11	5	9	18	13	4
Market 4	9	7	10	8	14	19	5	7

2. The administrations of three hospitals requested the opening of a call for the recruitment of non-specialized nurses (N) and two specialties (E1 and E2) to fill the vacancies indicated in Table I.

   For budgetary reasons, the authority only authorized the hiring of 140 nurses.

   The three administrations agreed on the following rules to hire the 140 authorized nurses:

   - Hospital 3, in a pre-shortage situation, will hire nurses for all vacancies.
   - Each of the other two hospitals will hire non-specialized nurses to fill at least 60% of the vacancies indicated by their administrations and at least 4 for each specialty.
   - At least 35 specialized nurses will be hired.
   - Whenever possible, nurses will be hired for the hospital of their preference.

   A total of 302 non-specialized nurses, 30 from specialty E1, and 38 from specialty E2 applied. The preferences of the applicants are presented in Table II.

   Table I: Vacancies indicated by the hospital administrations.

Hospital 1			Hospital 2			Hospital 3		
N	E1	E2	N	E1	E2	N	E1	E2
80	22	34	42	8	8	12	4	2

   Table II: Number of applicants preferring each hospital.

	Hospital 1	Hospital 2	Hospital 3
Non-specialized	280	18	4
Specialty E1	20	9	1
Specialty E2	36	2	0

How should the hires be made?

3. A foundation launched a call for grants for projects in the areas of theatre, music, and dance, with a total budget of 150,000 euros. Out of the 39 submitted projects, the jury selected 10. The scores assigned by the jury to the 10 projects and the amounts requested (in thousands of euros) were as follows:

Area	Theatre					Music		Dance		
Project	A	B	C	D	E	F	G	H	I	J
Score	97	89	87	85	85	88	85	95	91	85
Amount requested (€k)	40	30	50	40	60	15	20	20	30	25

The foundation's board decided that the grants to be awarded shall follow the rules below:

- The theatre area will receive between 40% and 60% of the total budget, and the music and dance areas between 10% and 30% each.
- The top-ranked project in each area will receive at least 60% of the amount requested.
- The percentage of the requested amount allocated to each project will respect the ranking established by the jury in each area.
- No project will receive less than 20% of the requested amount.
- The average percentage of grants allocated relative to the requested amounts should be as high as possible.

Develop a final decision proposal to submit for approval by the foundation's board.

4. An investor has US$ 60,000 that he wishes to invest to create a retirement plan five years from now. His financial advisor proposed four types of fixed-income investments, called A, B, C, and D.

- Investments A and B are available at the beginning of each of the 5 years (years 1 to 5).
  - Each dollar invested in A at the beginning of a year returns US$ 1.04 two years later (4% profit), and becomes immediately available for reinvestment.
  - Each dollar invested in B at the beginning of a year returns US$ 1.07 three years later.
- Investments C and D are available only once:
  - Each dollar invested in C at the beginning of year 2 returns US$ 1.09 at the end of year 5.
  - D is only available at the beginning of year 5, and each dollar invested will return 1.03 at the end of that year.

The investor wants to determine the investment plan that maximizes the amount accumulated at the beginning of year 6.

5. Next academic year, a college will host many international students. For this reason, the director decided that during the first five weeks, from Monday to Friday, between 8 a.m. and 6 p.m., there will always be a person in the entrance hall able to provide information in English.

Since many students currently face financial difficulties, the director decided to offer the job to these students, with a payment of 5 or 6 euros per hour, depending on whether the student is enrolled in the first or second cycle.

At the end of the selection process, six students were recruited, four from the first cycle and two from the second, who were asked to indicate the number of hours they can work each day of the week (see table).

Student	Mon	Tue	Wed	Thu	Fri	Cycle
Alice	6	0	6	0	6	2nd
Bruna	0	6	0	6	0	1st
César	4	8	4	0	0	1st
Diogo	5	5	5	0	5	1st
Elsa	3	0	3	8	0	1st
Fábio	0	0	6	6	4	2nd

For fairness, each of the six students was promised a schedule of at least seven hours per week.

What is the minimum budget required to cover the costs of this initiative?

6. A company won a tender to organize five scientific conferences (see Table I). After making several contacts, it received responses from three universities interested in hosting them.

Universities A and B can host at most three conferences, provided they are held in different months. University C can host at most two, in different months, and cannot accommodate conferences with more than 350 participants.

University B will only agree to host the national conference if it is also assigned an international one.

Due to the external visibility of the conferences, the universities will subsidize the conferences they host (see Table II).

Table I - Conferences

CONFERENCE	C1	C2	C3	C4	C5
Scope	International	International	International	International	National
Expected participants	420	310	750	130	150
Date	Jun 7-9	Jul 4-5	Jul 14-17	Sep 1-3	Nov 2-3

Table II - Subsidies (in thousand euros)

	C1	C2	C3	C4	C5
University A	7	5	20	4	2
University B	6	6	20	3	2
University C	–	7	–	5	3

The organizing company wants to select which university should host each conference.

7. A library received a donation of 4000 euros. According to the donor's wishes, at least 50% of this money must be spent on works by Portuguese-speaking authors.

The library director wants to strengthen the audiobook and DVD collections, deciding that at least 20% of the donation will be spent on audiobooks and at least 30% on DVDs.

As the donor was a Philosophy teacher, the director also decided that at least 10% of the books purchased will be essays and at least 15% of the money will be spent on children's books.

What should be the purchasing plan using this donation?

Table I - Average price per item (in euros)

Item type	Average price (€)
Books - Essays	22.50
Books - Fiction	15.00
Books - Children	16.00
Audiobooks	28.00
DVDs	18.00

8. A construction company has 4 projects that it wants to assign to subcontractors. After contacting several potential subcontractors, it received 6 proposals. The table shows the durations (in days) submitted by the subcontractors for each project. All projects must start on the same day. The company does not want to assign more than one project to each subcontractor and aims to complete all projects in the shortest possible time. How should the assignments be made?

Project / Subcontractor	P1	P2	P3	P4	P5	P6
Project 1	56	52	57	59	55	53
Project 2	54	58	55	57	57	56
Project 3	59	55	57	51	59	50
Project 4	55	56	56	54	56	55

9. An organic farming cooperative plans to sell directly to consumers each month four types of baskets called Complete, Green, Fruit, and Mini (see Table I). Sales will be made at the cooperative's headquarters and at four partner associations. Each association indicated the fixed number of baskets for which they already have committed customers and the number of baskets they would like for occasional customers (see Table II).

The cooperative management wants to reserve at least 10% of the total baskets for sale at its headquarters and wants at least two Mini baskets in each association every month to attract new buyers.

Due to adverse weather conditions in the region, the cooperative cannot provide more than 400 kg of vegetables, 320 kg of leafy greens, and 450 kg of fruit per month.

How should the cooperative manage this initiative to offer the maximum possible amount of vegetables, leafy greens, and fruit directly to consumers each month?

Table I – Quantity per basket (Kg)

Basket	Vegetables	Leafy greens	Fruit
Complete	5	4	6
Green	4	3	-
Fruit	-	-	5
Mini	2	1	2

Table II – Baskets with committed customers and for other customers

Association	Complete	Green	Fruit	Mini	Other customers
Association 1	10	8	4	8	2–5
Association 2	6	6	3	4	2–4
Association 3	5	12	7	2	3–4
Association 4	6	7	8	9	2–3

10. A hostel received 5 reservation requests for the first half of August from foreign students (E1, ..., E5) attending a summer course in Lisbon. For that period, the hostel has only 2 vacant double rooms.

   The manager is committed to guest satisfaction, in order to maintain the #1 ranking among Lisbon hostels. For this reason, the following table was prepared showing a compatibility indicator for sharing a room, based on the students' ages and the cultural backgrounds of their countries:

Table I – Compatibility between students for sharing a room (%)

Students	E1 & E2	E1 & E3	E1 & E4	E1 & E5	E2 & E3
Compatibility (%)	80	75	80	55	90
Students	E2 & E4	E2 & E5	E3 & E4	E3 & E5	E4 & E5
Compatibility (%)	95	45	55	75	85

   It is desirable to achieve a 100% occupancy rate, provided that the 2 vacant rooms can be allocated with an average compatibility score of at least 85%.

   The hostel manager wants to make a well-informed decision on which reservation requests to accept.

11. In three neighborhoods on the outskirts of a large city (A, B, and C) there are 122, 74, and 86 youths, respectively, who will attend technical-vocational education for the first time in the next school year.

   In neighborhood A there are two schools for new students; one can accommodate 70 and the other 40 students. The school in neighborhood B has 80 spots and the one in neighborhood C has 70 spots. There is also a school in the city center that can accept students from the outskirts.

   All students placed outside their home neighborhood will receive a transportation allowance, calculated based on distances (see Table I).

Table I – Distances for calculating the transportation allowance (in km)

	Bairro B	Bairro C	City Center
Bairro A	120	100	250
Bairro B	-	140	90
Bairro C	-	-	80

   How should the students be assigned to schools in order to minimize the total amount of transportation allowances to be paid to the youths annually?