Mathematical Programming I

BSc in Applied Mathematics for Economics and Management (MAEG)

2025-2026 **Problem Set 2**

1. Solve the following problems graphically.

(a)
$$\begin{aligned} \max\ & z = -2x_1 - x_2 \\ \text{s.t.}\ & -x_1 + x_2 \leq 1 \\ & x_1 + x_2 \geq -3 \\ & x_1 \leq 0 \end{aligned}$$

(b)
$$\begin{aligned} \max\ & z = 4x_1 + 3x_2 \\ \text{s.t.}\ & 3x_1 + 4x_2 \leq 12 \\ & x_1 + x_2 \geq 4 \\ & 4x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(c)
$$\begin{aligned} \max\ & z = 3x_1 + 9x_2 \\ \text{s.t.}\ & x_1 + 4x_2 \leq 8 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(d)
$$\begin{aligned} \max\ & z = 2x_1 + x_2 \\ \text{s.t.}\ & x_1 - x_2 \leq 10 \\ & 2x_1 - x_2 \leq 40 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(e)
$$\begin{aligned} \min\ & z = 2x_2 - 6x_1 \\ \text{s.t.}\ & 2x_1 - x_2 \leq 2 \\ & x_1 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(f)
$$\begin{aligned} \max\ & z = 4x_1 + 14x_2 \\ \text{s.t.}\ & 2x_1 + 7x_2 \leq 21 \\ & 7x_1 + 2x_2 \leq 21 \\ & x_1, x_2 \geq 0 \end{aligned}$$

2. Solve the problems in Exercise 1 using the simplex method.

3. A factory produces three types of products: A, B, and C. Each unit of product A generates a profit of 8 euros, product B generates 4 euros, and product C generates 8 euros. Each product consumes two limited resources: raw materials and labor hours. The factory has 20 kg of raw materials and 90 labor hours available. Each unit of product A consumes 2 kg of raw materials and 1 hour of labor; for product B, these values are 1 kg and 4 hours, and for C, they are 2 kg and 2 hours. Formalize and solve an LP problem that allows you to maximize total profit while respecting the availability of resources.

4. Resolva graficamente (apenas os problemas (a) e (c)) e pelo método simplex.

(a)
$$\begin{aligned} \max\ & z = x_1 + 2x_2 \\ \text{s.t.}\ & x_1 + x_2 \leq 4 \\ & x_1 + 2x_2 \geq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(b)
$$\begin{aligned} \max\ & z = x_1 - 3x_2 + 4x_3 \\ \text{s.t.}\ & x_1 - x_2 + x_3 = 4 \\ & 2x_2 \leq 6 \\ & -x_2 + 2x_3 \geq 0 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(c)
$$\begin{aligned} \min\ & z = 4x_1 + x_2 \\ \text{s.t.}\ & 3x_1 + x_2 = 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$