1. The manager of a company intends to determine a new production plan for one of the sections of its factory, in order to guarantee the jobs of the 36 employees assigned to that section.

   In this section, three different products can be produced; however, due to the age of the equipment, there are severe technical limitations on production.

   The estimates carried out indicate that the three products can be sold, respectively, at 4, 5, and 12 thousand euros per ton.

   Based on this information, the manager's technical advisor constructed and solved the following Linear Programming (LP) model, with the objective of finding the best solution to the problem. The decision variables x_j, $j = 1, 2, 3$, represent the quantities produced of each product, expressed in tons.

   The Linear Programming model considered is presented below, as well as the optimal tableau obtained through the application of the simplex method.

$$
\begin{aligned}
\max\ z &= 4x_1 + 5x_2 + 12x_3 \\
\text{s.t.} \quad & x_1 + 2x_2 + 4x_3 \leq 38 \qquad \text{(Limit 1)} \\
& 2x_1 + 2x_2 + 6x_3 \leq 56 \qquad \text{(Limit 2)} \\
& x_1 + 2x_2 + 2x_3 \geq 36 \qquad \text{(Job guarantee)} \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

BV	x_1	x_2	x_3	x_4	x_5	x_6	R	RHS
z (max)	0	0	1	1	1.5	0	–	122
x_6	0	0	2	1	0	1	-1	2
x_1	1	0	2	-1	1	0	0	18
x_2	0	1	1	1	-0.5	0	0	10

Tabela 1: Final table of the simplex method.

   a) Present the optimal production plan and the corresponding revenue.
   b) Present the optimal basis matrix and its inverse.
   c) Write the dual problem and its optimal solution.
   d) Show that the optimal solution of the original problem and the one obtained in item (c) satisfy the complementary slackness conditions.

2. Consider the following Linear Programming problem:

$$
\begin{aligned}
(\text{P}) \quad \max \quad & z = 4x_1 + 10x_2 + 5x_3 + 6x_4 \\
\text{s.a.} \quad & 2x_1 - x_2 + x_3 \le 40, \\
& x_2 + 2x_3 + 5x_4 \le 60, \\
& 4x_1 + 2x_2 + 2x_4 \le 70, \\
& x_1, x_2, x_3, x_4 \ge 0.
\end{aligned}
$$

The table corresponding to the optimal solution obtained by applying the simplex method is as follows:

BV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	11	0	0	14	0	5/2	15/4	825/2
x_5	5	0	0	-1	1	-1/2	3/4	125/2
x_3	-1	0	1	2	0	1/2	-1/4	25/2
x_2	2	1	0	1	0	0	1/2	35

a) Write the optimal solution of problem (P) and the corresponding optimal value.

b) Present the optimal basis matrix and its inverse.

c) Write the dual problem of problem (P).

d) Extract the optimal solution of the dual of problem (P) from the tableau provided.

e) Show that the solutions you presented for problem (P) and its dual satisfy the complementary slackness conditions.

3. Consider the following Linear Programming problem:

$$
\begin{aligned}
(\text{P}) \quad \max \quad & z = x_1 + x_2 + x_3 \\
\text{s.a.} \quad & x_1 + 2x_2 + x_3 \le 10, \\
& -2x_1 + 3x_2 + 2x_3 \ge 3, \\
& x_1, x_2, x_3 \ge 0.
\end{aligned}
$$

a) Determine a basic feasible solution of the problem using the two-phase method.

b) Calculate the value of z at the solution obtained in part (a) and verify whether it is optimal.

c) Write the dual problem of (P).

d) Provide a lower bound for the optimal value of the dual problem.

4. Consider the following Linear Programming problem:

$$
\begin{aligned}
\min \quad & z = -2x_1 + x_2 \\
\text{subject to} \quad & x_1 + x_2 \leq 4, \\
& x_1 + 2x_2 \geq 2, \\
& -x_1 + x_2 \geq 1, \\
& x_1, x_2 \geq 0.
\end{aligned}
$$

Solve the following questions:

a) Solve the problem graphically.

b) Write its dual.

c) Using the solution obtained in part (a) and the complementary slackness conditions, determine the optimal solution of the dual problem.

5. The manager of a company observed that 2600 kg of raw materials and 3200 machine hours were unused. For this reason, he decided to increase the production of three products by 900 kg, maintaining the proportion between the quantities produced of the first two products and the third one.

To determine the most profitable way to implement this decision, the following Linear Programming model was used:

$$
\begin{aligned}
\max \quad & z = 14x_1 + 16x_2 + 12x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 = 900, \\
& 2x_1 + +3x_2 + 2.5x_3 \leq 2600, \\
& 5x_1 + 4x_2 + 3x_3 \leq 3200, \\
& x_1 + x_2 - x_3 \leq 0, \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}
$$

The optimal tableau obtained using the simplex method is as follows:

BV	x_1	x_2	x_3	x_4	x_5	x_6	R	RHS
z (max)	2.00	0.00	0.00	0.00	0.00	2.00	-	12600.00
x_2	1.00	1.00	0.00	0.00	0.00	0.50	0.50	450.00
x_4	-1.00	0.00	0.00	1.00	0.00	-0.25	-2.75	125.00
x_3	0.00	0.00	1.00	0.00	0.00	-0.50	0.50	450.00
x_5	1.00	0.00	0.00	0.00	1.00	-0.50	-3.50	50.00

Answer the following questions:

a) Prepare a report for the company manager including the information you consider relevant to evaluate the consequences of the decision taken.

b) Write the dual problem and provide an economic interpretation of its optimal solution.