# Mathematical Programming I

BSc in Applied Mathematics for Economics and Management (MAEG)

2025-2026 **Problem Set 5**

1. Consider a transportation problem with three origins, $O_1$, $O_2$, and $O_3$, and four destinations, $D_1$, $D_2$, $D_3$, and $D_4$. The available supplies (in tons) at the origins are 50, 25, and 25, respectively, and the demands at the destinations are 30, 30, 20, and 20. The unit transportation costs are given in Table 1.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $O_1$ | 3 | 7 | 6 | 4 |
| $O_2$ | 2 | 6 | 3 | 7 |
| $O_3$ | 9 | 3 | 8 | 5 |

Tabela 1: Unit transportation costs.

   a) Determine a feasible solution using the least-cost method.
   b) Determine an optimal solution of the problem using Dantzig's algorithm. Are there alternative optimal solutions?

2. Consider the transportation problem with the following data:

$$\mathbf{a}^T = \begin{bmatrix} 12 & 26 \end{bmatrix}, \qquad \mathbf{b}^T = \begin{bmatrix} 10 & 14 & 8 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 13 & 17 & 7 \\ 12 & 16 & 14 \end{bmatrix}.$$

   a) Determine a basic feasible solution of the problem.
   b) Without solving the problem, provide a nonzero upper bound and a nonzero lower bound for the optimal value of the problem. Justify your choices.
   c) Explain how the dual problem can be used to answer the question in part (b).

3. In three districts on the outskirts of a large city, denoted by $A$, $B$, and $C$, there are, respectively, 122, 74, and 86 young people who will attend, for the first time, vocational-technical education in the next academic year.

   In district $A$, there are two schools with available spots for new students, one with a capacity of 70 students and another with 40 students. In district $B$, there is a school with 80 spots, and in district $C$, a school with 70 spots. There is also a school in the city center that can accommodate students from the peripheral districts.

   All students placed outside their district of residence receive a transportation subsidy, whose annual value depends on the distance between the district of residence and the school attended.

   The relevant distances (in monetary units associated with the subsidy) are shown in Table 2.

| | District $B$ | District $C$ | City |
|---|---|---|---|
| District $A$ | 120 | 100 | 250 |
| District $B$ | | 140 | 90 |
| District $C$ | | | 80 |

Tabela 2: Relevant distances for calculating the transportation subsidy.

a) Determine the allocation of students to schools that minimizes the total annual transportation subsidy to be paid.

b) The local youth association argues that the number of students assigned to schools outside their district of residence should be minimized. What would be the cost of satisfying this requirement? How many students would benefit?

4. A company operates three factories, $F_1$, $F_2$, and $F_3$, whose production is distributed to two distribution centers, $C_1$ and $C_2$.

   Given the high unemployment rate in the municipality where factory $F_2$ is located, the company's management has committed to the mayor to keep this factory operating at 100% of its production capacity whenever the company has sufficient orders to do so.

   For the next planning period, the distribution centers have signed contracts corresponding to orders of 180 and 140 tons, respectively.

   The company aims to plan its operations in order to fulfill all commitments already undertaken at the minimum total cost.

   Table 3 presents the available data regarding unit transportation costs, production capacities of the factories, and unit production costs.

| | Unit transportation costs | | Production | Unit production |
|---|---|---|---|---|
| | $C_1$ | $C_2$ | capacity | costs |
| $F_1$ | 11 | 12 | 200 | 6 |
| $F_2$ | 11 | 12 | 100 | 9 |
| $F_3$ | 9 | 14 | 150 | 7 |

Tabela 3: Transportation costs, production capacities, and production costs.

a) Formulate a transportation problem whose optimal solution corresponds to the company's desired operating plan.

b) Determine the company's optimal plan using Dantzig's algorithm.

   **Suggestion:** Use the least-cost method to obtain an initial solution.

c) Assess the additional costs incurred by the company as a result of the commitment made to the mayor of the municipality where factory $F_2$ is located.

5. A public inspection service has 4 inspectors available to carry out 5 possible inspection tasks for 5 different companies. The tasks are individual, and each inspector can inspect only one company.

   Each inspector was asked to indicate the number of calendar days they commit to completing each task. The data are presented in Table 4.

   Task $A_3$ must be performed, as there is a complaint regarding the corresponding company.

a) Propose an immediate work schedule for the inspectors that minimizes the total working time while completing the maximum number of tasks possible. Justify the methodology adopted.

b) Suppose there are exactly 26 days left until the inspection deadline. Verify if it is possible to arrange a work schedule so that each inspector completes one task within the deadline.

| | | Task | | | | |
|---|---|---|---|---|---|---|
| | | A1 | A2 | A3 | A4 | A5 |
| **Inspector** | T1 | 20 | 28 | 26 | 22 | 27 |
| | T2 | 28 | 24 | 27 | 20 | 27 |
| | T3 | 39 | 27 | 27 | 22 | 26 |
| | T4 | 25 | 31 | 29 | 27 | 28 |

Tabela 4: Number of days each inspector commits to completing each task.

6. To participate in a tango competition, 3 girls and 4 boys from a dance school rehearsed and danced in pairs in front of the instructors.

   The scores given by the instructors during the rehearsals were as follows:

| | David | Eduardo | Filipe | Gil |
|---|---|---|---|---|
| Ana | 15 | 18 | 12 | 19 |
| Beatriz | 11 | 17 | 13 | 14 |
| Carla | 14 | 12 | 16 | 15 |

Tabela 5: Scores assigned to the pairs during rehearsals.

   Knowing that all participants will compete, forming 4 fixed pairs, determine how these pairs should be arranged based on the rehearsal scores in order to achieve the "`highest total score for the school`".

7. To participate in the 4 events of a competition, an arts school will form a team of 4 students, selected based on the grades obtained in the corresponding subjects. With 5 candidates, the following rules are defined:

   (A) Each student can participate in only one event;

   (B) A student can be selected for an event only if they scored at least 14 in the corresponding subject;

   (C) Graduated students have priority over the others.

   The grades of the candidates are presented in Table 6. Additionally note that Daniela is a graduate.

| | Events | | | |
|---|---|---|---|---|
| **Students** | Musical Performance | Children's Theater | Puppets | Clowns |
| Arnaldo | 14 | 15 | 17 | 20 |
| Bruno | 16 | 11 | 14 | 15 |
| Cátia | 12 | 16 | 16 | 18 |
| Daniela | 14 | 13 | 15 | 16 |
| Edgar | 17 | 14 | 18 | 12 |

Tabela 6: Grades of students applying for the 4 events.

   a) Formulate an optimization model whose solution identifies the team with the highest average grade the school can send to the competition, respecting the three rules.

   b) Solve the model formulated in the previous item and analyze the consequences of rule (C) on the team selection.