Mathematical Programming I

BSc in Applied Mathematics for Economics and Management (MAEG)

2025-2026 **Mock Exam**

Duration: 2h

Question	1a	1b	1c	1d	1e	2a	2b	2c	2d	3	4
Points	1.5	2.5	2	2	2	1.5	1	2	1	2	2.5

Present all calculations and appropriately justify all answers.

1. A company is studying the production of three new products, P_1, P_2, and P_3. The objective is to maximize revenue, without exceeding the 210 available labor hours, consuming at least 150 units of a substance that is close to its expiration date, and complying with the marketing department's guidelines.

   For this purpose, the following Linear Programming problem was formulated:

$$
\begin{array}{rll}
\max & z = 8x_1 + 6x_2 + 5x_3 & \\
\text{subject to:} & 5x_1 + 3x_2 + 4x_3 & \leq 210 \\
 & 4x_1 + 2x_2 + 3x_3 & \geq 150 \\
 & x_2 + x_3 & \geq 50 \\
 & x_1, x_2, x_3 & \geq 0
\end{array}
$$

   where x_i represents the quantity produced of product P_i, $i = 1, 2, 3$.

   Solving the model led to the following optimal tableau:

VB	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	1	0	0	0	12	13	4	370
x_2	0	0	1	0	4	5	1	40
x_1	0	1	0	0	1	1	1	10
x_3	0	0	0	1	-4	-5	-2	10

   where x_4, x_5, and x_6 are the auxiliary variables associated with the first, second, and third constraints, respectively.

   a) Write the dual of the given problem and indicate one of its optimal solutions.

   b) Prepare a brief report containing all the relevant information that can be extracted from the optimal tableau.

   c) The manager does not understand how it is possible to produce both a large quantity of the cheapest product and a large quantity of the most expensive product. Therefore, they intend to increase the selling price of the cheapest product, provided that this change does not alter the optimal production plan. Determine the maximum increase allowed for this price.

   d) The inclusion of a new product is being studied. This product consumes, per unit produced, 2 labor hours and 2 units of the aforementioned substance, and is sold for 1 monetary unit. Determine how much of the new product should be produced.

   e) The marketing department intends to withdraw its guideline regarding the minimum production quantities of P_2 and P_3. Based on the optimal tableau presented, determine the exact consequences of this decision.

2. A company owns three grain warehouses, A_1, A_2, and A_3, which will be used to supply two of its customers, C_1 and C_2. The company has committed to delivering 35 tons to customer C_1 and 45 tons to customer C_2.

   The contracts also specify that the company will pay a penalty of 50 monetary units for each ton not delivered to customer C_1 and 60 monetary units for each ton not delivered to customer C_2.

   The following table presents the transportation costs per ton from the warehouses to the customers, as well as the quantities available at each warehouse.

Warehouse	Transportation cost (m.u./ton) C_1	C_2	Supply (tons)
A_1	100	120	20
A_2	110	105	28
A_3	95	90	25

   The company intends to ship all available grain to its customers in order to minimize total transportation and penalty costs.

   a) Formulate this problem as a Transportation Problem.

   b) Determine a basic feasible solution using the Northwest Corner Method.

   c) Solve the problem using Dantzig's algorithm.

   d) Is there an optimal solution in which customer C_2 receives equal quantities from warehouses A_2 and A_3? Justify your answer.

3. A company opened a recruitment process for three different positions, F_1, F_2, and F_3. The candidates were evaluated on a scale from 0 to 100 (with 100 being the maximum score), and the four best candidates, C_1, C_2, C_3, and C_4, were selected to become employees of the company.

   The scores obtained by each candidate for each position are shown below:

	F_1	F_2	F_3
C_1	84	86	91
C_2	92	90	87
C_3	90	91	86
C_4	91	88	85

   Each of the four employees must be assigned to exactly one position. One worker is required for each of the positions F_1 and F_3, while two workers are required for position F_2.

   Formulate and solve an optimization problem that assigns the four workers to the positions in such a way as to obtain the best possible average score.

4. Consider the following statements:

   (i) If the feasible region of a Linear Programming problem is unbounded, then its dual problem does not have a finite optimum.

   (ii) The introduction of a new variable into a Linear Programming problem may not affect the feasible region of the corresponding dual problem.

   Indicate whether the statements are true or false. Carefully justify your answers, presenting a counterexample whenever a statement is false.