List of Exercises

Operational Research

Basic degree in Economics, Finance, and Management

2025-2026

Chapter 1 – Linear Programming

Exercise 1. Alfredo has a farm where he wants to raise chickens, rabbits, and goats. The price of each chicken, rabbit, and goat is 2, 5, and 40m.u., respectively.
To receive financial support for the farm, the sum of the number of legs of all animals on the farm cannot be less than 30, and the sum of animal heads cannot be less than 15. In addition, the number of chickens cannot exceed 20% of the number of the remaining animals, and the farm can only feed up to 800 animals.
It is estimated to obtain a profit of 1, 2, and 30 m.u. for each chicken, rabbit, and goat, and Alfredo wants to obtain a profit not lower than 500 m.u. The chicken's house is small and therefore can only accommodate up to 20 chickens.
There is a large stable on the farm reserved to the goats and rabbits. In this stable, there are 500 compartments, and each compartment can be empty or (when occupied) must contain exactly one goat and two rabbits, because the goats are afraid of being alone at night. There is no other place available for the goats on the farm, but there is an extra compartment with capacity for at most 50 rabbits.
Formulate the problem to determine the number of animals of each type that Alfredo should buy for his farm in order to minimize the total purchase cost of the animals (taking all constraints of the problem into account).

Prototype 1. The *Wyndor Glass Co.* produces high-quality glass products, including doors and windows. It has three Plants. Aluminum frames and hardware are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glasses and assembles products.
Because of declining earnings, top management has decided to restructure the company's product line. Unprofitable products are being discontinued, releasing production capacity to launch two new products having large sales potential:

Product 1: A glass door with aluminum framing
Product 2: A wood-framed window

Product 1 requires some of the production capacity of Plants 1 and 3, but none of Plant 2. Product 2 needs only Plants 2 and 3. The marketing division concluded that the company could sell as many products as could be produced by these Plants. However, because both products would be competing for the same production capacity in Plant 3, it is not clear which mix of the two products would be most profitable. Therefore, an OR team has been formed to study this question. After conducting some research, the OR team determined:

- The number of working hours (w.h.) available at each Plant
- The number of working hours required at each Plant to produce one batch of each product
- The unitary profit generated by one batch of each product

Such information was summarized in the following table:

	Working hours per batch (w.h.)		Available working hours
Plants	**Doors**	**Windows**	
P1	1	0	4
P2	0	2	12
P3	3	2	18
Profit per batch (m.u.)	3	5	

a) Formulate and solve the problem.

b) Consider the same set of feasible solutions and determine the set of optimal solutions if the objective is:

i. Max $z = 5x_1 + x_2$ ii. Min $z = -x_1 + x_2$ iii. Min $z = x_1 - x_2$

iv. Max $z = x_1$ v. Max $z = 4x_2$

c) Top management wants to know the consequences if a minimum of 50 m.u. of profit is required.

d) Solve the initial problem assuming that the capacity of Plants 2 and 3 is unlimited. Repeat b) with this new feasible region.

e) Consider that the total of 18 w.h. available at Plant 3 must be used. Keeping the remaining initial constraints, identify and explain what is the new feasible region and the optimal solution.

f) Solve the initial problem assuming that the number of windows cannot be smaller than the quadruple of the number of doors.

g) Solve the initial problem assuming that at least 2.5 batches of doors must be produced.

h) Solve the initial problem assuming unitary profits equal to 6 and 4 m.u. for doors and windows, respectively.

Prototype 2. The *Profit & Gambit Co.* produces cleaning products for home use. This is a highly competitive market, and the company continually struggles to increase its small market share. Management has decided to undertake a major new advertising campaign that will focus on the following three key products: a spray stain remover, a liquid laundry detergent, and a powder laundry detergent.

This campaign will use both television and print media. The liquid detergent will be the one featured on the TV advertisement. The advertisement for the print media will promote all three products. The general goal is to increase sales of each product. Specifically, management has set the following goals for the campaign:

- Sales of the stain remover should increase by at least 3%
- Sales of the liquid detergent should increase by at least 18%
- Sales of the powder detergent should increase by at least 4%

The percentage of sales of each product increased/decreased by each block of advertisement on TV and in the print media is displayed in the following table.

	TV	**Print Media**
Stain remover	0%	1%
Liquid detergent	3%	2%
Powder detergent	-1%	4%
Cost per block (m.u.)	1	2

The bottom row shows the cost per block of advertisement of each of the two outlets. The top management wants to determine the number of blocks of advertisement on TV and in the print media to meet the sales goals at a minimum total cost. Formulate this problem and solve it graphically and by using the solver.

Exercise 2. Solve by the graphical method and by the Solver the following LP problems.

a) $Max\ Z = x_1 + 2x_2$

$$s.t.\begin{cases} x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

b) $Max\ Z = 3x_1 + 4x_2$

$$s.t.\begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

c) $Max\ Z = x_1 + x_2$

$$s.t.\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

d) $Max\ Z = x_1 - x_2$

$$s.t.\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

e) $Max\ Z = -10x_1 - 5x_2$

$$s.t.\begin{cases} x_1 - x_2 \leq 5 \\ x_1 + \frac{8}{5}x_2 \geq -3 \\ x_1\ free, x_2 \leq 0 \end{cases}$$

f) $Min\ Z = x_1 + x_2$

$$s.t.\begin{cases} 2x_1 + x_2 \geq 4 \\ x_1 - x_2 \leq 2 \\ x_2 \geq 1 \\ x_1, x_2 \geq 0 \end{cases}$$

g) $Max\ Z = x_1 + x_2$

$$s.t.\begin{cases} 2x_1 + x_2 \geq 4 \\ x_1 - x_2 \leq 2 \\ x_1, x_2 \geq 0 \end{cases}$$

h) $Min\ Z = x_1 + x_2$

$$s.t.\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq -2 \\ x_1, x_2 \geq 0 \end{cases}$$

i) $Min\ Z = 3x_1 + 2x_2$

$$s.t.\begin{cases} x_1 + x_2 \geq 3 \\ 3x_1 + 2x_2 \leq 18 \\ 5x_1 + 2x_2 = 10 \\ x_1, x_2 \geq 0 \end{cases}$$

j) $Max\ Z = 3x_1 + 6x_2$

$$s.t.\begin{cases} x_1 + 2x_2 \leq 4 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

Exercise 3. A company has underused production capacity since the production of some products was discontinued. This production capacity should be used to produce at least one of the following three products: P1, P2, and P3. The available capacity on three machines, M1, M2, and M3, that might limit output is 500, 350, and 150, respectively. The number of working hours required for each machine for each of the products is the following:

	P1	**P2**	**P3**
M1	9	3	5
M2	5	4	0
M3	3	0	2

The sales department indicates that the potential sales for P1 are twice the sales potential of P2 and that it is not possible to sell more than 20 units of P3. The unit profit would be 50, 20, and 25 m.u. respectively. Formulate and solve this problem to maximize the total profit.

Exercise 4. A farm family owns 125 acres of land and has 4000 m.u. in funds available for investment. Its members can produce a total of 3500 person-hours worth of labor during the winter months and 4000 person-hours during the summer. If any of these person-hours are not needed, younger members of the family will use them to work on a neighboring farm for 0.5m.u./hour during the winter months and 0.6 m.u./hour during the summer. Cash income may be obtained from three crops and two types of livestock: dairy cows and laying hens. No investment funds are needed for the crops. However, each cow will require an investment outlay of 120 m.u., and each hen will cost 0.9 m.u. Each cow requires 1.5 acres of land, 100 person-hours of work during the winter months, and another 50 person-hours during the summer, and it is expected to result in a profit of 100 m.u./year for the family.
Each hen does not need land but requires 0.6 person-hours in the winter, 0.3 person-hours in summer and creates a profit of 0.5 m.u./year. The chicken's house can accommodate a maximum of 3000 hens and the size of the barn limits the herd to a maximum of 32 cows. Estimated person-hours and income per acre planted in each of the three crops are

	Soybeans	**Corn**	**Oats**
Winter person-hours	20	35	10
Summer person-hours	50	75	40
Net annual cash income (m.u.)	60	90	45

The family wishes to determine how much acreage should be planted in each of the crops and how many cows and hens should be kept to maximize its net cash income. Formulate the linear programming model for this problem and solve it by using the solver.

Exercise 5. A plant imports three types of thread (cotton, wool, and fiber) to produce three different types of cloth: **C1**, **C2**, and **C3**. The clothes should follow the specifications below:

Cloth	Composition	Selling price (m.u./kg)
C1	At least 60% of cotton & at most 20% of fiber	680
C2	at most 60% of fiber & At least 15% of wool	570
C3	at most 50% of fiber	450

The aim is to determine the production plan so that the profit is maximized, using the information about availabilities and costs given in the following table:

thread	availabilities (kg)	cost (m.u./kg)
Cotton	2 000	700
Wool	2 500	500
fiber	1 200	400

Formulate the problem and solve it by using the Solver.

Exercise 6. A company has at its service 100 skilled workers, 230 semi-skilled workers, and 80 unskilled workers. The semi-skilled workers can become skilled workers if they attend a one-year training course. These training courses cost 500 m.u. per worker. Unskilled workers may attend training courses to become semi-skilled workers. These one-year courses cost 350 m.u. per worker. The company intends to plan training of its staff over the next two years so that at the end of the planning period:

- unskilled workers can not represent more than 10% of the total;
- at least 40% of the workers should attend a training course;
- at least 35% of the amount spent on training should be on unskilled workers

How much should the company spend on training courses?

Exercise 7. A farmer is raising pigs for the market, and he wishes to determine the quantities of the available types of feed that should be given to each pig to meet certain nutritional requirements at a minimum cost. The number of units of each pig's basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional Ingredient	**kg of Corn**	**kg of Tankage**	**kg of Alfalfa**	Minimum Daily Requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (m.u./kg)	42	36	30	

Formulate and solve the linear programming model for this problem.

Exercise 8. An energetic company needs to make plans for the energy systems of a new building. The energy needs in the building fall into three categories: electricity, heating water, and heating space in the building. The daily requirements for these three categories are 30, 20, and 50 units, respectively.
The three possible sources of energy to meet needs are: electricity, natural gas, and solar panels. The size of the roof limits the number of solar panels to 40 units. Electricity needs can be met only by purchasing electricity (at a cost of 50 m.u./unit). Both other energy needs can be met by any source or combination of sources. The unit costs are:

	Electricity	**Natural Gas**	**Solar panels**
Heating water	150	110	70
Heating space	140	100	90

Formulate and solve the problem to satisfy the energetics needs at minimum cost.

Exercise 9. An investor has 60000m.u. that wishes to invest now to use the accumulation for purchasing a retirement annuity in 5 years. After consulting with his financial adviser, he has been offered four types of fixed-income investments, which we will label as investments A, B, C, and D.
Investments A and B are available at the beginning of each of the next 5 years. Each m.u. invested in A at the beginning of a year returns 1.4m.u. two years later, while each m.u. invested in B at the beginning of a year returns 1.7m.u. three years later.
Each investment C and D will be available once in the future. Each m.u. invested in C at the beginning of year 2, returns 1.9m.u. at the end of year 5. Each m.u. in D at the beginning of year 5 returns 1.3m.u. at the end of year 5. The investor wishes to know which investment plan maximizes the total money obtained at the beginning of year 6.

Exercise 10. This is your lucky day. You have just won 6000m.u. Upon hearing this news, two different friends, Joana and Pedro, have offered you an opportunity to become a partner in two different projects (a single project for each friend). In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a full partner in the project of Pedro would require an investment of 5000m.u. and 400 hours, and your estimated profit would be 4500. Similar features for the project of Joana are: 4000m.u. of investment, 500 hours, and a profit of 4500m.u.
However, both friends are flexible and would allow you to come in at any fraction of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership would be multiplied by this same fraction. Because you are looking for an interesting summer jog anyway (maximum of 600 hours), you have decided to participate in one or both projects. You now need to find the best combination that maximizes your profit. (Use the solver of Excel)

Chapters 2 – Simplex Method

Exercise 11. Consider the following LPP.

$$\text{Max } Z = 11.25x + 9y$$

$$\text{s.t.} \begin{cases} x + \quad y \leq 600 \\ 12.5x + 8y \leq 6000 \\ x \qquad\quad \leq 400 \\ \qquad\quad y \leq 500 \\ x, y \geq 0 \end{cases}$$

a) Represent the feasible region of the problem and compute the objective function value for each one of the extreme points. Obtain the optimal solution.
b) Identify the feasible basic solution associated with each extreme point by computing the slack variables. For each feasible basic solution, identify the basic and non-basic variables.
c) Indicate possible sequences of solutions that are obtained when solving the problem by using the simplex method.

Exercise 12. Solve the following problems by using the simplex method.

a) Max $Z = 2x_1 + 3x_2 + 2x_3$

$$\text{s.t.} \begin{cases} x_1 + 2x_2 + 3x_3 \leq 6 \\ x_1 + x_2 + x_3 \leq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

b) Min $Z = -3x_1 + x_2 - 2x_3$

$$\text{s.t.} \begin{cases} x_1 + x_2 \leq 3 \\ x_1 + 2x_2 + 2x_3 \leq 6 \\ 2x_1 + 2x_2 + x_3 \leq 8 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

c) Max $Z = 3x_1 + 4x_2 + x_3$

$$\text{s.t.} \begin{cases} x_1 + 2x_2 + x_3 \leq 5 \\ 2x_1 + 3x_2 + x_3 \leq 10 \\ 3x_1 + x_2 + x_3 \leq 8 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

d) Max $Z = 4x_1 + 5x_2 + 3x_3$

$$\text{s.t.} \begin{cases} 7x_1 + 4x_2 + x_3 \leq 10 \\ 2x_1 + 3x_2 + 2x_3 \leq 4 \\ 3x_1 + 4x_2 + x_3 \leq 11 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Exercise 13. Solve (if possible) all items of Exercise 2 by using the simplex method.

Exercise 14. A firm has a plant with a capacity to work 70 hours a week and can produce three products (P1, P2, and P3). Each unit of P3 requires one hour of that capacity, while the unit production of P1 and P2 needs, respectively, the double and the triple of that time. The three products, when finished, are stored in a warehouse with $100\, m^3$ available. Each unit of product (P1, P2, or P3) requires $1\, m^3$. The gross unit margin achieved by each product is 10 (P1), 15 (P2), and 5 (P3).

a) Formulate an LPP to maximize the total gross margin.

b) Find all the optimal solutions by using the simplex algorithm.

Exercise 15. Consider the following LPP:

$$\text{Max } Z = x_1 - 3x_2$$

$$\text{s.t.:} \begin{cases} \frac{1}{3}x_1 + x_2 \leq 8 \\ x_1 - x_2 \leq 8 \\ x_1 \geq 0 \end{cases}$$

a) Solve the problem by the graphical method.

b) Determine the solution with the first constraint binding and $x_2 = 0$. Classify it.

c) Write the problem in the *standard* form and in the augmented form.

d) Solve the problem by the simplex method.

Exercise 16. Consider the following simplex tableau of a maximization problem:

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	c	0	2	0	0	9
x_2	0	-1	1	a_1	0	0	3
x_4	0	a_2	0	-3	1	0	1
x_5	0	a_3	0	4	0	1	2

Determine the values of a_1, a_2, a_3, and c that make the following sentences true:

a) The current solution is optimal;

b) The current solution is optimal, and at least one more optimal alternative solution exists;

c) The objective function is unbounded.

d) The current solution is not optimal, and either x_4 or x_5 can leave the basis in the next iteration.

Chapters 3 – Duality and Sensitivity Analysis

Exercise 17. In factory Choco, three new types of chocolate bars are going to be made for the food industry. Each bar is made of sugar and chocolate only.

Bar	Quantity of sugar (*kg/bar*)	Quantity of chocolate (*kg/bar*)	Profit of each chocolate bar (*m.u.*)
Type 1	*1*	*2*	*3*
Type 2	*1*	*3*	*7*
Type 3	*1*	*1*	*5*
availabilities (*kg*)	*50*	*100*	

To formulate the problem, we defined variables x_j, representing the number of chocolate bars of type j to make, where $j = 1,2,3$. Answer the following questions using, when needed, the *Solver/Excel* to find the solution to the LP problems.

a) For which unitary profit values of chocolate bars of Type 2 does the current solution remain optimal? Which will be the optimal solution in case the unit profit is *13 m.u.*?

b) Is it worth considering an increase in the availability of sugar? For which amount of sugar is the set of basic variables in the optimal solution the same?

c) How much should the amount of sugar be increased to obtain a profit of 320?

d) The company can buy 2 extra kg of chocolate for 3m.u. Is it worth it?

e) If the amount of sugar available is *60 kg*, what would be the total profit obtained? What should be the production plan that Choco should apply in these conditions?

f) Repeat the previous question for an availability of sugar of *30 kg*.

g) Suppose that Choco plans to make a new type of chocolate bar, and that each bar requires two kg of sugar and one kg of chocolate. What is the minimum profit for this new chocolate bar to ensure it is worth it to produce?

Exercise 18. Write the dual of the following LPPs

a) $\text{Max } Z = 2x_1 + 3x_2 + x_3$

$$\text{s.t.}\begin{cases} x_1 + 2x_2 + 3x_3 \geq 6 \\ x_1 + x_2 + x_3 \leq 9 \\ x_1, x_2 \geq 0 \\ x_3 \leq 0 \end{cases}$$

b) $\text{Min } Z = -3x_1 + x_2$

$$\text{s.t.}\begin{cases} x_1 + x_2 = 3 \\ x_1 + 2x_2 \leq 6 \\ 2x_1 + 2x_2 \leq 8 \\ x_1 \; free \\ x_2 \geq 0 \end{cases}$$

c) $\text{Max } Z = 3x_1 + x_3$

$$\text{s.t.}\begin{cases} x_1 + 2x_2 + x_3 \geq 5 \\ 2x_1 + x_3 \leq 10 \\ x_2 + x_3 \leq 8 \\ x_1 \leq 0 \\ x_2 \; free \\ x_3 \geq 0 \end{cases}$$

d) $\text{Max } Z = 7x_1 + 5x_2$

$$\text{s.t.}\begin{cases} 8x_1 + x_3 \leq 10 \\ 2x_1 - 3x_2 + 2x_3 = -4 \\ 3x_1 - 4x_2 \geq 0 \\ x_1 \geq 0 \\ x_2, x_3 \leq 0 \end{cases}$$

e) $\text{Min } Z = x_1 - x_3$

$$\text{s.t.}\begin{cases} x_1 - 5x_2 \leq 6 \\ x_1 = 4x_2 - x_3 \\ x_1, x_2 \leq 0 \\ x_3 \geq 0 \end{cases}$$

Exercise 19. An individual strongly invested in real estate-based funds, and he has available 25 thousand *m.u.* (monetary units) which he intends to invest in two products. Based on his past experience, his goal is to minimize the risk; however, he would like to achieve a minimum return of 2 thousand *m.u*. The characteristics of the two financial products, which he ponders to include in his portfolio, made him formulate the following LP problem, where x_i represents the amount (in 10^3 *m.u.*) to be invested in product $i = 1,2$.

$$\min \quad z = x_1 + 2x_2$$
$$\text{s.t.}\begin{cases} x_1 + x_2 \leq 25 & \text{(budget constraint)} \\ 0.5x_1 + 0.8x_2 \geq 2 & \text{(return constraint)} \\ x_1, x_2 \geq 0 \end{cases}$$

a) Solve the given problem graphically. Present and interpret the optimum value of the decision and slack (auxiliary) variables.

b) Write the dual and determine its optimal solution (only the decision variables). Note that you can take advantage of the solution and resolution of a).

Exercise 20. An account manager criticized the approach to the above problem, arguing that this way, the return achieved would never be higher than the minimum required. According to him, the objective function should translate the maximization of the return and the risk might be controlled by constraints imposed on the portfolio composition. Besides, he suggested two additional financial products to take into account. Using the OR knowledge, he gathered the following information.

	x_1	x_2	x_3	x_4		RHS (thousand *m.u.*)
Budget	1	1	1	1	<=	25
Risk 1	1	1	0	0	<=	15
Risk 2	0	0	1	1	<=	15
Risk 3	1	0	1	0	>=	15
Return	0,50	0,80	0,75	0,90		

a) Give the LP formulation of the problem and solve it by S*olver/Excel*.

b) How much should be invested in each product, and what is the associated total return?

c) How much does the total return change if the Risk 2 constraint changes to allow a maximum of 14 thousand *m.u.* to invest?

d) Could you quantify the change in the total return, if it is required that the total invested in products 3 and 4 (RHS of constraint Risk 2) does not surpass 9 thousand *m.u.*?

e) How much does the total return change, if the return of product 1 increases from 0,5 to 0,6? Identify the optimal solution for this situation.

Exercise 21. Consider the following LP problem:

$$\max z = 3x_1 + 2x_2 \qquad (total\ profit, in\ m.u.)$$

$$s.t.\begin{cases} x_1 \leq 4 & (resource\ 1) \\ x_1 + 3x_2 \leq 15 & (resource\ 2) \\ 2x_1 + x_2 \leq 10 & (resource\ 3) \\ x_1, x_2 \geq 0 \end{cases}$$

a) Solve the problem by the graphical method, by the Simplex algorithm and by the *Solver/Excel*.

b) Write the dual of the problem formulated above.

c) Find the optimal solution of the dual with the help of the optimal solution of the primal, reading the output reports from the *Solver* obtained in a) and solving the dual itself by the *Solver*.

d) Assume that this is a problem for finding the amount to produce of two products sharing limited resources. Explain the economic meaning of the optimal solutions.

e) Determine the impact on the total profit of a reduction of the availability of resources 3 to 8 units.

Exercise 22. Write the dual associated with the following LP problem:

$$\max z = 6x_1 + 8x_2$$
$$s.t.\begin{cases} 5x_1 + 2x_2 \leq 20 \\ x_1 + 2x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{cases}$$

a) Solve by the graphical method the pair of dual problems.

b) Solve the primal problem by the simplex algorithm. Write and classify the solution associated with each simplex tableau and identify them in the graphic.

Exercise 23. Consider the following LP problem:

$$\max z = 2x_1 + 7x_2 + 4x_3$$
$$s.t.\begin{cases} x_1 + 2x_2 + x_3 \leq 10 \\ 3x_1 + 3x_2 + 2x_3 \leq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

a) Write its dual.

b) Solve the dual problem by the solver and then indicate the optimal solution of the primal.

Exercise 24. A firm wants to study the future production plan of products **P1**, **P2,** and **P3**. To maximize the global profit, the following LPP was formulated:

$$\max z = 3x_1 + 4x_2 + 2x_3$$
$$s.t.\begin{cases} x_1 + x_2 + 2x_3 \leq 10 \\ 2x_1 + 4x_2 + x_3 \leq 8 \\ 2x_1 + 3x_2 + 2x_3 \leq 20 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

where x_j represents the quantity of product $P_j, j = 1,2,3$ that should be produced. The first two constraints refer to the consumption of raw materials **rm1** and **rm2**, respectively, and the third constraint is associated with the limited availability of storage space in the warehouse. Assume that the optimal production plan indicates that only *2* units of **P1** and *4* units of **P3** should be produced.

a) Without solving the problem and using the information that the first shadow price is *1/3*, determine the internal values of the resources (raw materials and store space) and give the economic interpretation of those values.

b) Obtain the output *Solver/Excel* reports and find the increase in the actual unit profit of **P2** so that it becomes advantageous to include it in the production plan.

Exercise 25. Consider the following LPP:

$$\min z = x_1 + 3x_2$$
$$s.t. \begin{cases} x_1 + x_2 \geq 4 \\ -x_1 + x_2 \geq 0 \\ -x_2 \geq -6 \\ x_1, x_2 \geq 0 \end{cases}$$

a) Solve it with the graphical method and write the dual problem associated with this LPP.

b) Obtain the solution of the dual problem.

Exercise 26. Consider the following LP problem:

$$\min z = 3x_1 + 2x_2$$
$$s.t. \begin{cases} x_1 \leq 3 \\ 3x_2 \leq 12 \\ \alpha x_1 + x_2 \geq 6 \\ x_1, x_2 \geq 0 \end{cases}$$

a) Take $\alpha = 1$ and solve the problem by the graphical method. Solve the dual.

b) Find a value for α so that alternative optimal solutions can be found. (Use the graphic of a))

c) Find all values of α such that the problem is infeasible. (Use the graphic of a))

Exercise 27. Consider the following LP problem, formulated by the OR department of a company, which intends to optimize the total monthly revenue from the sale of four products (**P1**, **P2**, **P3**, and **P4**) produced using two types of raw material (***Rm1*** and ***Rm2***):

$$\max z = 2x_1 + 3x_2 + 7x_3 + 4x_4$$
$$s.t. \begin{cases} x_1 + x_2 + x_3 + x_4 \leq 9 & (\boldsymbol{Rm1}) \\ x_1 + 2x_2 + 4x_3 + 8x_4 \leq 24 & (\boldsymbol{Rm2}) \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

The department informs that only products **P1** and **P3** should be produced.

a) Which of the two products not included in the production plan would need a smaller increase in the unit selling price so that its production becomes profitable?

b) Assume that due to difficulties with importing next month, only *5* units of ***Rm1*** will be available. Determine the new optimal production plan and the total revenue associated with it.

c) Suppose that the company can produce a new product, with a unit selling price of *10 m.u.*, and a need of *2* units of each of the raw materials to produce one unit of the new product. What should be the new production plan?

d) Suppose that an external budget of *24 m.u.* is now available to spend on only one of the raw materials and that such a budget is lost if it is not used. Let *4 m.u.* and *8 m.u.* the unit price to acquire each extra unit of those raw materials, respectively. Which decision should be made if the company's management intends to keep the production of **P1** and **P3** only? Justify.

Exercise 28. Obtain the *output* from the *Solver* for the following problem, where x_1, x_2, and x_3 are the quantities to buy of products 1, 2, and 3, respectively, and answer the following questions.

$$\min z = 2x_1 + 3x_2 + 8x_3 \quad \textit{(Total cost)}$$

$$s.t. \begin{cases} x_1 + x_2 + x_3 \geq 90 & \textit{(Minimum quantity to buy)} \\ 5x_1 + 4x_2 + 3x_3 \leq 400 & \textit{(Maximum capacity)} \\ 3x_1 - 5x_2 = 0 & \textit{(Relation between purchases of products 1 and 2)} \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

a) Display the optimal solution of the primal and interpret the meaning of all its variables (including slack variables).

b) Display and interpret the shadow prices.

c) What are the consequences in the total cost if the third constraint changes to $3x_1 - 5x_2 = 100$?

d) Suppose that the unit cost of product 3 is now *4.5*, determine the consequences in the purchase plan and in the total cost.

Exercise 29. A humanitarian organization intends to plan a medicine distribution program in two regions located in the Great Lakes area of Africa. For strategic and security purposes, it is possible to use 3 airports from which the supply of the two regions will take place. Considering that the transportation cost of medicines to the airports should be minimized, ensuring, in each of the two regions, a minimum number of people is contemplated by the program, the following LP model was formulated:

$$\min z = 40x_1 + 18x_2 + 30x_3 \quad (in\ m.u.)$$

$$s.t. \begin{cases} 4x_1 + x_2 + x_3 \geq 250 & (thousands\ of\ people) \\ 4x_1 + 3x_2 + 6x_3 \geq 350 & (thousands\ of\ people) \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

where x_j is the tons of medicines to be shipped to airport j ($j = 1,2,3$).

a) Obtain the optimal solution for the problem by using the *Solver*.

b) Write a short report about the optimal solution. (Include the value of the dual decision variables as well as their meaning).

c) If the number of thousands of people to be contemplated in the first region is *350* instead of *250*, which will be the new cost for the program?

d) Determine the changes in the solution if airport 1 cannot receive more than *40 ton* of medicines.

e) The organization intends to reduce the total cost of the program by 4%, by reducing the minimum number of people in the second region contemplated with medicines. What should be the change in the previous LPP to achieve such a goal?

Exercise 30. Consider the following LPP referring to the production of **P1**, **P2**, and **P3**.

$$\max z = 10x_1 + 20x_2 + 15x_3$$
$$s.t.\begin{cases} x_1 + \ \ 3x_2 + 2x_3 \le 80 \\ 4x_1 + 10x_2 + 5x_3 \le 90 \\ 4x_1 + 10x_2 \ge 50 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

The objective function refers to the maximization of the total revenue, and the first and second constraints are related to the machine hours available in sections 1 and 2, respectively. The third constraint determines the minimum financial margin that should be achieved. The financial margin is the difference between the total revenue and the total variable costs.

a) Write and interpret the optimal solutions for the primal and dual problems.

b) The revenue of **P2** just increased by *20*%, although its financial margin is maintained. What are the changes in the production program?

c) How much should the revenue of **P2** increase (maintaining its financial margin) so that this product is included in the production plan?

d) Indicate a way of increasing the revenue by at least *2*% through the changes in the available quantities of the company´s resources.

e) What are the changes in the optimal solution arising from the new market requirements that obligate a minimum production for **P2** of *4* units?

Exercise 31. Consider the following LPP:

$$\max z = x_1$$
$$s.t.\begin{cases} 2x_1 + 2x_2 \ge 10 \\ -x_1 + \ \ x_2 \le 5 \\ 3x_1 + 2x_2 \le 30 \\ 2x_1 - \ \ x_2 \le 16 \\ x_1, x_2 \ge 0 \end{cases}$$

a) Solve it by the graphical method.

b) Without solving the dual, determine the optimal value of the dual variable associated with the first functional constraint.

c) Without using the graphic and without solving the resulting problem, check if the introduction of the constraint $x_1 + x_2 \ge 3$ changes the feasible region.

d) Determine the sensitivity interval to the right-hand side of the third functional constraint graphically.

e) Suppose now that the objective function is to minimize x_1. Determine the sensitivity interval to the right-hand side of the third functional constraint graphically.

Chapter 4 – Transportation and Assignment Problems

Prototype 3. One of the main products of the *P&T* company is canned peas. The peas are prepared in three canneries (Washington, **C1**; Oregon, **C2**; Minnesota, **C3**), and then shipped by truck to four distributing warehouses in the western United States (California, **W1**; Utah, **W2**; South Dakota, **W3**; New Mexico, **W4**). For the upcoming season, an estimate has been made of the output from each cannery, and a certain amount from the total supply of peas has been allocated to each warehouse. This information (in units of truckloads), along with the shipping cost per truckload for each cannery-warehouse combination, is given in the table below. Thus, there are a total of 300 truckloads to be shipped.

Canneries \ Warehouses	Shipping cost (*m.u.*) per truckload				Output
	W1	**W2**	**W3**	**W4**	
C1	464	513	654	867	75
C2	352	416	690	791	125
C3	995	682	388	685	100
Needs	80	65	70	85	

Because the shipping costs are a major expense, the management team aim to determine the cheapest way of assigning the shipments to the various cannery-warehouse.

a) Formulate the problem as an LPP.

b) Identify a feasible solution for this problem and determine its cost.

c) Find the optimal solution by using the Excel Solver.

d) Recently, a fire in cannery **C3** led to a reduction of the production capacity in 50%. Determine the new optimal solution and interpret it.

e) Suppose now that the need in **W4** is just 70 truckloads. Obtain the new solution.

f) Solve the problem assuming that all warehouses require a number of truckloads between 65 and 85.

Exercise 32. A *METRO WATER DISTRICT* is an agency that manages water distribution in a large geographic region. The main customers are four cities (Berdoo: **C1**; Los Devils: **C2**; Sam Go: **C3,** and Hollyglass: **C4**) and the water supply is from three rivers (Colombo: **R1**; Sacron: **R2,** and Calorie: **R3**). It is possible to supply any of the cities with water from any of the rivers, except **C4**, that cannot be supplied by **R3**. The costs (in *m.u.*) of sending one million *Kl* of water from the river **Ri** to city **Cj** are in the table below, as well as the availabilities and needs.

River \ City	**C1**	**C2**	**C3**	**C4**	availabilities (*millions of Kl*)
R1	16	13	22	17	50
R2	14	13	19	15	60
R3	19	20	23	—	50
Minimum needed (*millions of Kl*)	30	70	0	10	
Maximum requested (*millions of Kl*)	50	70	30	∞	

Management wishes to allocate <u>all available water</u> from the three rivers to the four cities to meet the essential needs (without exceeding the maximum requests), while minimizing the total cost.

Prototype 4. THE *JOB SHOP* company has purchased four new machines of different types. There are four locations available in the shop where a machine could be installed. Some of these locations are more desirable than others for particular machines due to their proximity to work centers with a heavy workflow to and from these machines. (There will be no workflow between the new machines.) Therefore, the objective is to assign the new machines to the available locations to minimize the total cost of materials handling. The estimated cost per hour of materials handling involving each of the machines is given in the table below for the respective locations. Location **L2** is not considered suitable for machine **M2**.

	Cost of materials handling			
Machine \ Location	**L1**	**L2**	**L3**	**L4**
M1	13	16	12	11
M2	15	–	13	20
M3	5	7	10	6
M4	12	20	15	13

a) Formulate and solve the problem by Solver.

b) Formulate and solve the problem assuming that the company does not buy **M4.**

Exercise 33. The company *Better Products* has decided to initiate the production of four new products (**P1**, **P2**, **P3**, and **P4**), using three factories (**F1**, **F2**, and **F3**) that currently have excess production capacity. The products require a comparable production effort per unit, thus the available production capacity of the factories is measured by the number of units of any product that can be produced per day, as given in the table below. The bottom row gives the required production rate per day to meet projected sales. Each factory can produce any of these products, except factory **F2,** which cannot produce product **P3**. However, the variable costs per unit of each product differ from factory to factory, as shown in the main body of the table:

	Product unit cost (*m.u.*)				
Product / **Factory**	**P1**	**P2**	**P3**	**P4**	Daily capacity available (*units*)
F1	41	27	28	24	75
F2	40	20	–	23	75
F3	37	30	27	21	45
Production rate per day (*units*)	20	50	30	50	

Management needs to decide how to obtain the minimum production cost.

a) Solve the problem assuming that the same product can be produced in more than one factory and each factory can produce more than one product.

b) Solve the problem assuming that each factory can only produce one product and each product can only be produced in one factory.

Exercise 34. The Childfair Company has three plants producing children's toys that have to be shipped to four distribution centers. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance (in miles) from each plant to each distributing center is given below. For geographical reasons, Plant 1 cannot serve Center 1, and Plant 2 cannot serve Center 4.

		Distribution Center			
		1	2	3	4
Plant	1	-	1300	400	700
	2	1100	1400	600	-
	3	600	1200	800	900

The freight cost for each shipment is 100m.u. plus 0.5m.u. per mile. How much should be shipped from each plant to each of the distribution centers to minimize the total shipping cost?

a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

b) Draw the network of this problem.

c) Obtain an optimal solution.

<u>Exercise 35</u>. A department has opened three vacancies for translators:

Vacancy 1: Portuguese/French;

Vacancy 2: Portuguese/German;

Vacancy 3: Portuguese/Greek.

Four candidates applied for the vacancies, and they achieved the grades below in the selection tests (scaled from a minimum of zero to a maximum of ten). Determine the assignment that provides the best service quality.

	Vacancy 1	**Vacancy 2**	**Vacancy 3**
Candidate 1	7	6	2
Candidate 2	8	8	4
Candidate 3	8	5	4
Candidate 4	9	7	6

<u>Exercise 36</u>. A company produces a product in two factories (**F1** and **F2**) and has three selling points (**S1**, **S2**, and **S3**). The maximum production for the next period is 400 *tons.* and 800 *tons.* in factories **F1** and **F2**, respectively. The potential sales in the three selling points are 400 *tons*, 500 *tons.* and 500 *tons,* respectively. The transportation cost, in *hundreds of m.u.* per ton transported, between each factory and each selling point are in the following table:

	S1	**S2**	**S3**
F1	10	20	25
F2	25	15	20

The product is sold by 15, 18, and 20 *thousand m.u. per ton* in selling points **S1**, **S2,** and **S3,** respectively, and the management of the company wants to maximize the total profit (revenue - cost). Determine the optimal solution.

<u>Exercise 37</u>. A couple wants to share some tasks to minimize the total time spent, but both should do the same number of tasks. The average weekly time (in *minutes*) needed for each one to do the tasks is the following:

	Shopping	Cooking	Dish washing	Laundry	House cleaning	Make bed
João	60	400	150	210	65	70
Ana	90	300	100	180	90	40

Determine the tasks that should be assigned to each one. How long, per week, will be spent by each one on the tasks assigned?

Exercise 38. A company has four vacancies: **V1**, **V2**, **V3,** and **V4** for 4 different candidates. According to the psychologist, vacancies should not be given to individuals with an I.Q. (intelligence quotient) lower than 150, 100, 80, and 75, respectively. Five candidates: **C1**, **C2**, **C3**, **C4**, and **C5** applied to all four vacancies, and the IQ tests performed determined I.Q. scores of 190, 160, 145, 100, and 85, respectively. The monthly wages asked by the candidates were 150, 80, 100, 100, and 70 *m.u.* What is the optimal assignment and respective cost, assuming that no more candidates applied and the first candidate was immediately hired?

Exercise 39. A company decided to produce three new products, **P1**, **P2**, and **P3**. Currently, the company has five factories with excess capacity. The production cost of each product in each factory, the sales forecast for each product, and the available capacity in factories are given in the table below. The goal is to minimize the total cost.

	F1	F2	F3	F4	F5	Sales
P1	7	10	13	11	12	3000
P2	5	4	6	3	4	3000
P3	9	7	9	-	-	2000
Capacity	2500	3000	2000	4000	5000	

Consider the following *output* from the *Solver* displaying the optimal solution for the problem:

Cell	Name	Original Value	Final Value
$N$12		0	45500

Cell	Name	Original Value	Final Value
$I$9	P1 F1	2500	2500
$J$9	P1 F2	500	500
$K$9	P1 F3	0	0
$L$9	P1 F4	0	0
$M$9	P1 F5	0	0
$I$10	P2 F1	0	0
$J$10	P2 F2	0	0
$K$10	P2 F3	0	0
$L$10	P2 F4	3000	3000
$M$10	P2 F5	0	0
$I$11	P3 F1	0	0
$J$11	P3 F2	2000	2000
$K$11	P3 F3	0	0
$L$11	P3 F4	0	0
$M$11	P3 F5	0	0

a) Write the optimal solution and explain it in economic terms.

b) For technical and logistical reasons, the management decided that each factory will either not produce any product or will produce only one, and that each product can only be produced by one factory. Which should be the new production plan?

Exercise 40. Consider the problem referring to the transportation of an item from three warehouses to three shops. The unit costs, supplies, demands, and *Solver output* are:

	Shop 1	**Shop 2**	**Shop 3**	Supply
Warehouse 1	4	6	8	40
Warehouse 2	2	4	2	20
Warehouse 3	6	-	4	30
Demand	20	50	40	

Target Cell (Min)

Name	Original Value	Final Value
Total cost	0	380

Adjustable Cells

Name	Original Value	Final Value
Warehouse 1 - Shop 1	0	10
Warehouse 1 - Shop 2	0	30
Warehouse 1 - Shop 3	0	0
Warehouse 2 - Shop 1	0	10
Warehouse 2 - Shop 2	0	0
Warehouse 2 - Shop 3	0	10
Warehouse 3 - Shop 1	0	0
Warehouse 3 - Shop 2	0	0
Warehouse 3 - Shop 3	0	30

Constraints

Name	Cell Value	Formula	Status	Slack
Warehouse 1	40	$F$13=$H$13	Binding	0
Warehouse 2	20	$F$14=$H$14	Binding	0
Warehouse 3	30	$F$15=$H$15	Binding	0
Shop 1	20	$C$16<=$C$18	Binding	0
Shop 2	30	$D$16<=$D$18	Not Binding	20
Shop 3	40	$E$16<=$E$18	Binding	0
Warehouse 3 - Shop 2	0	$D$15=0	Binding	0

a) Explain how the transportation should be done.

b) What changes should be introduced in the model defined in *Excel* and in the specifications file of *Solver* to ensure that **Shop 2** receives the quantity demanded and that **Warehouse 2** sends exactly 10 units to that shop?

<u>Exercise 41.</u> The following table displays the data of a problem that arose in a company that produces a product in four factories, **F1**, **F2**, **F3,** and **F4**, to be sold in four markets, **M1**, **M2**, **M3,** and **M4**. The demand to meet in each market (in *tons*) and the maximum capacity of each factory (in *tons*) are also displayed in such a table. The remaining values are the production and transportation costs of each ton of product (in *m.u.*).

	M1	**M2**	**M3**	**M4**	Supply
F1	5	8	4	7	23
F2	2	6	6	6	32
F3	3	7	5	7	38
F4	2	5	4	3	38
Demand	21	16	30	35	

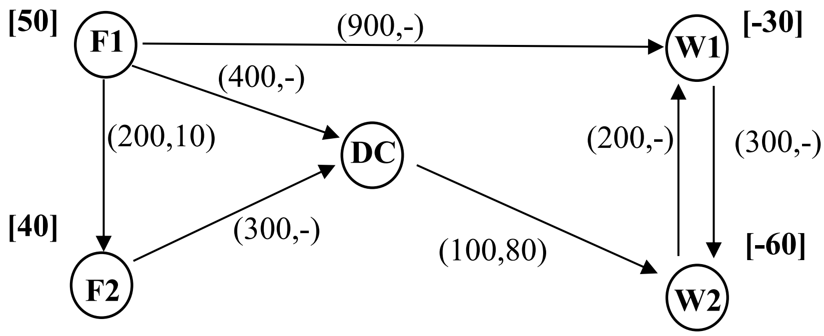
The conclusions of a study dictated that each market should be supplied by only one factory, provided it has enough capacity. On the other hand, it was decided that all factories must operate. Formulate the problem defining variables constraints and the objective function to optimize.

<u>Exercise 42</u>. A company that sells cars is going to open two new shops (**NA** and **NB**) with space for 30 cars each. For the moment, no cars are available at the factory, it was decided that the cars should be shipped from the four closest shops (**V1**, **V2**, **V3**, and **V4**). Each one of the shops offered no more than 20 vehicles to be transferred. The link between **V1**-**NA** is not available due to ongoing street repair. Knowing that the new shops should receive the maximum number of cars and that the transferring unit costs (in *m.u.*) are in the table below, fill the Excel sheet to solve the problem by *Solver*/*Excel* (write exactly what would be written in case a computer is available).

	NA	NB
V1	-	170
V2	230	140
V3	170	130
V4	200	150

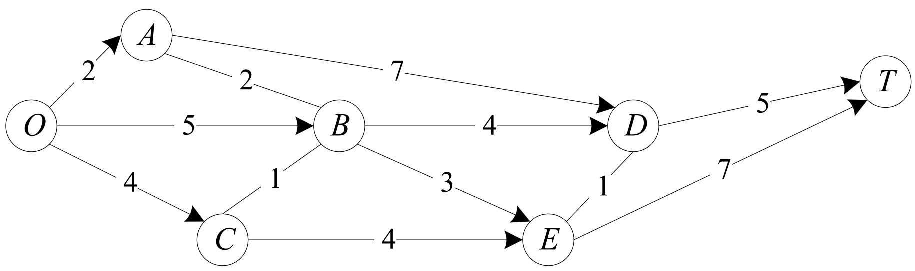
Chapter 5 – Network Optimization

Prototype 5. The *DISTRIBUTION UNLIMITED CO.* company will be producing the same new product at two different factories, and then the product must be shipped to two warehouses. The distribution network available for shipping this product is shown below, where F1 and F2 are the two factories, W1 and W2 are the two warehouses, and DC is a distribution center. The amounts to be shipped from F1 and F2 are shown on their left, and the amounts to be received at W1 and W2 are shown on their right. Each arrow represents a feasible shipping lane. The cost per unit shipped through each shipping lane is shown next to the arrow. Next to the lanes F1 → F2 and DC →W2 are also the maximum amounts that can be shipped through them. The other lanes have sufficient shipping capacity to handle everything these factories can send.



The decision to be made concerns how much to ship through each shipping lane. The objective is to minimize the total shipping cost. Formulate and solve the problem.

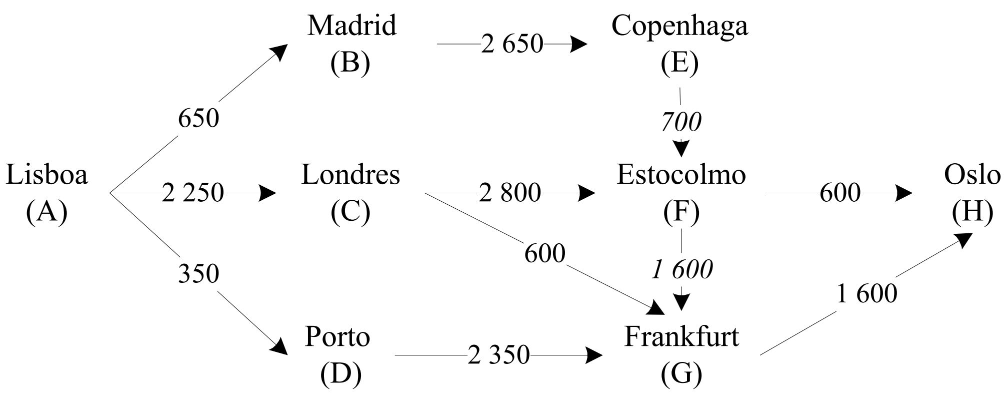
Prototype 6. The *SEERVADA PARK* has recently been set aside for a limited amount of sightseeing and backpack hiking. Cars are not allowed into the park, but there is a road system for the exclusive use of the cars driven by the park rangers and for trams transporting tourists. This road system is shown (without the curves) in the figure above, where location *O* is the entrance of the park, *T* is a scenic wonder station *T* and the other letters designate the locations of ranger stations (and other facilities). The numbers give the distances of these roads in miles. A small number of trams are used to transport tourists from the park entrance to station *T* and back.



a) The park management wants to know the route from the park entrance to station *T* with the smallest total distance for the operation of the trams.

b) Telephone lines must be installed over some roads to establish telephone communication among all locations. Because the installation is expensive, management wants to know where the lines should be installed to ensure that the total miles of telephonic line used are minimized.

Exercise 43. A manager needs to travel from Lisbon to Oslo. After consulting its travel agent, he noticed that the relevant connections are only the ones represented on the following graph, where the values in the arcs stand for the distances (in *km*) between cities:



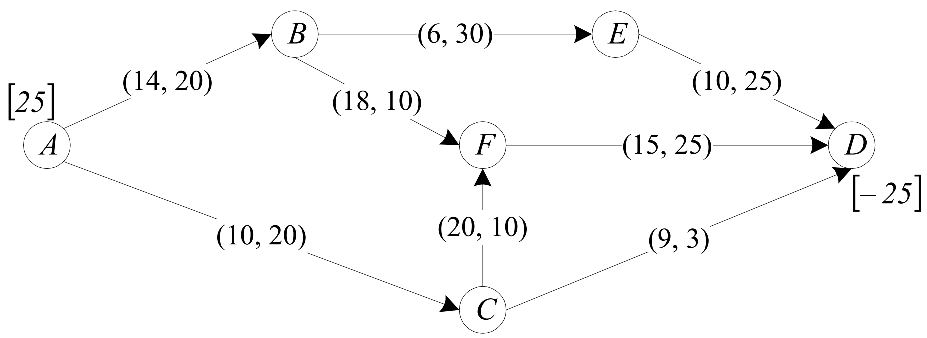
Assuming that the time spent on the trip can be considered proportional to the total distance travelled, find the minimum time connection between Lisboa and Oslo.

Exercise 44. A municipality wants to build a road network to ensure the connection of six locations. Given the scarcity of resources is intended to minimize the total number of *km* of road to be constructed; however, it should be possible to travel between any pair of locations. In the matrix, the distances in *km* between locations are:

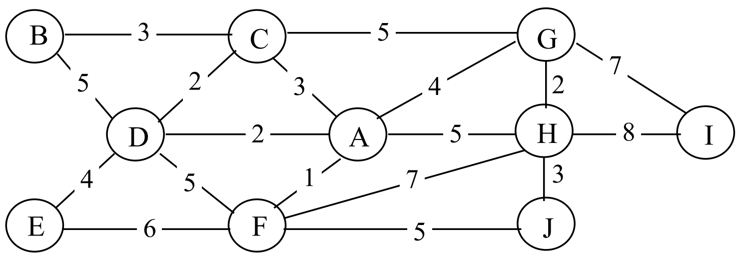
	A	B	C	D	E	F
A	–	9	6	11	10	14
B	9	–	5	6	15	23
C	6	5	–	5	10	18
D	11	6	5	–	9	17
E	10	15	10	9	–	8
F	14	23	18	17	8	–

Which pairs of locations should be linked directly?

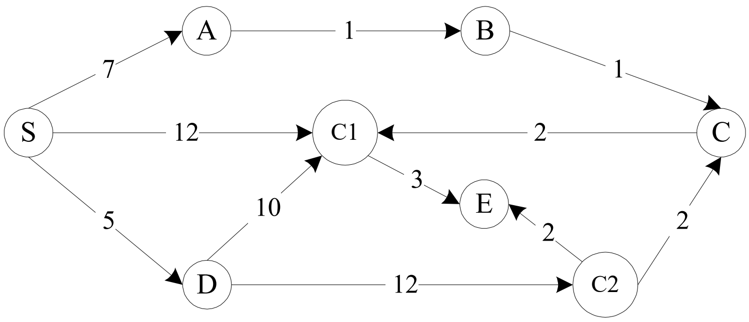
Exercise 45. Solve the Minimum Cost Flow Problem represented in the following network, the parameters (c_{ij}, u_{ij}) are close to the arcs and b_i near the nodes:



Exercise 46. A bank wants to link the computer terminals of its offices to the main office computer using special telephone lines. The telephone lines of each office do not need to be directly connected to the main office; however, it is necessary that communications can be established between any pair of computers. In the network below, the nodes represent the offices (B,…,J), and the main office is node A. The arcs represent the possible links, and the values on the arcs represent the distances in km. Knowing that the cost of a telephone line is proportional to the number of km involved, determine the links that should be selected to minimize the total cost.



Exercise 47. An individual is in S and wants to reach a post office as soon as possible, which can be either C1 or C2, as both are located nearby. In the network below, the information about the problem is displayed, and the numbers on the arcs represent the minutes needed to cross each link



Formulate the problem and determine which post office the individual should go.

Exercise 48. A company produces a product in two factories located in Lisboa and Porto, which is then shipped to warehouses in Coimbra and Évora or directly to the shops in Beja and Guarda. In the table below, the data about the problem is displayed, including the quantities supplied and demanded (in *truckloads*) as well as the distances (in *km*) of the available connections. Assuming that the transportation cost is proportional to the distance, formulate the problem in an Excel sheet and obtain the optimal solution.

Distances (*km*)

	Porto	Coimbra	Évora	Guarda	Beja	Supply (*truckload*)
Lisboa	310	200	130		180	1000
Porto		110		200		1500
Coimbra			290	150	340	
Évora					80	
Guarda					360	
Demand (truckload)				1100	1300	

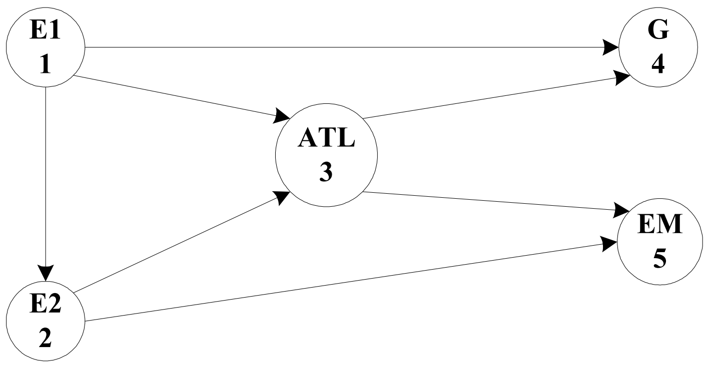
Exercise 49. Consider a computer system with a set of five terminals T1, T2, ..., T5 that must be connected to a central unit C. The connection of each terminal to C may be direct or through another terminal. The direct connection costs between terminals are in the following table (I represents an impossible link).

	C	T1	T2	T3	T4	T5
C	–	12	26	18	21	159
T1	12	–	400	I	I	112
T2	26	400	–	I	I	35
T3	18	I	I	–	22	23
T4	21	I	I	22	–	24
T5	159	112	35	23	24	–

a) Formulate the problem of finding the set of connections with minimal cost as a network optimization problem.

b) Find a set of direct connections to solve the problem.

Exercise 50. The company BUSE performs daily transportation of children from two schools (E1 and E2) to three local pre-school activities: an ATL, a gymnasium (G), and a music school (EM). To this end, the company has two types of vans, the largest ones with a capacity for 20 children that cost 10 *m.u.* per child, and those that carry up to 10 children for 25 *m.u.* per child. For logistical reasons, any of the links is only done by a van, and direct links between schools and activities in the gym and the school of music can only be done by smaller vans. The remaining connections are only done with the larger vans. The following figure presents all possible links, representing each facility by a vertex numbered from 1 to 5.



It is known that the number of children waiting to be transported from E1 and E2 is equal to 20 and 10, respectively. There are 15 children in ATL. Of these, 10 must be transported to the gym or to the school of music, and the other five must remain there. In turn, 25 children are expected in the gym and 15 in the music school.

a) Formulate the problem as a network optimization problem.

b) Formulate the problem as an LP problem.

c) Find the optimal solution by the Solver.

Exercise 51. A plant just bought equipment to produce a new product that has been patented for the next four years. Management wants to determine the best equipment replacement strategy during that period, assuming that at the end, the equipment is sold by its salvage value. The next table displays the equipment salvage value according to its use (age), as well as the other annual costs associated with its use and maintenance. The cost of new equipment is 100 *m.u.* (salvage value of year 0).

equipment age (in years)	1	2	3	4
salvage value (in *m.u.*)	50	25	10	5
use and maintenance costs (in *m.u.*)	40	50	70	100

a) Give a solution for the equipment replacement problem and the respective cost.

b) Formulate the problem as a network optimization problem.

c) Formulate the problem as an LP problem and solve it.

Chapter 6 – Integer Programming

Prototype 7. *TBA Airlines* is a small air company specialized in regional flights. The management is considering an expansion, and it has the possibility to buy small or medium-sized airplanes. Find the best strategy to increase the annual profits, knowing that no more than two small airplanes can be bought and that only \$100 *million* is available to invest. Consider also the values in the following table:

	small airplane	medium airplane
Annual Profit	\$1 *million*	\$5 *million*
Buying Cost	\$5 *million*	\$50 *million*

Prototype 8. The *CALIFORNIA MANUFACTURING COMPANY* is considering expansion by building a new factory in either Los Angeles (LA) or San Francisco (SF), or perhaps even in both cities. It is also considering building at most one new warehouse, but the choice of a location is restricted to a city where a new factory is being built. The *net present value* (total profitability considering the time value of money) of each of these alternatives is shown in the second column of the table below. The rightmost column gives the capital required (already included in the net present value) for the respective investments, where the total capital available is \$10 *million*. The objective is to find the feasible combination of alternatives that maximizes the total net present value.

	Net Value	Capital Required
Factory in LA	\$9 million	\$6 million
Factory in SF	\$5 million	\$3 million
Warehouse in LA	\$6 million	\$5 million
Warehouse in SF	\$4 million	\$2 million

<u>Exercise 52</u>. An oil company intends to select 5 out of 10 wells: **P1**, **P2**, … , **P10**. The associated (fixed) expected profits are 12, 14, 12, 8, 12, 10, 8, 12, 10, and 5, respectively. According to commitments with the local government, the company must comply with the following restrictions for regional development:

i. from **P5**, **P6**, **P7** and **P8** at most two can be selected;
ii. the selection of both **P1** and **P7** block selection of **P8**;
iii. the selection of **P3** or **P4** block selection of **P5**;
iv. the selection of **P1** forces selection of **P10**.

Formulate the problem and solve it.

<u>Exercise 53</u>. A real estate development firm, Peterson and Johnson, is considering five possible development projects. The following table shows the estimated long-run profit (net presented value) that each project would generate, as well as the amount of investment required to undertake the project, in units of millions of dollars.

	Development project				
	1	2	3	4	5
Estimated profit	1	1.8	1.6	0.8	1.4
Capital required	6	12	10	4	8

The owners of the firm have raised $20 million of investment capital for these projects and now they want to select the combination of projects that will maximize their total estimated long-run profit (net presented value) without investing more than $20 million.

a) Formulate a BIP model for this problem.
b) Fill an Excel spreadsheet to solve the problem by *Solver*/*Excel* (write exactly what would be written in case a computer is available).
c) Solve the problem by using the solver.

<u>Exercise 54</u>. A company has a portfolio with three investment projects with a lifetime of 4 years. The table below presents the *cash-flows* and the corresponding net present values (NPV) at 10% year rate:

Project	year 0	year 1	year 2	year 3	NPV (*m.u.*)
A	-100	50	300	-20	178
B	-300	100	150	200	65
C	0	-100	75	200	121

Project **B** can be deferred by one year and in that case the NPV is 59 *m. u.*, as it can be verified. There are budget constraints in the first two years of 300 and 200 *m. u.*, respectively. Formulate this problem and solve it by the *solver/excel.*

Exercise 55. The board of directors of General Wheels Co. is considering six large capital investments. Each investment can be made only once. The investments differ in the estimated long-run profit (net presented value) that they will generate as well as in the amount of capital required, as shown by the following table (in units of millions of dollars). The total capital available for these investments is $100 million. Investment opportunities 1 and 2 are mutually exclusive, and so are 3 and 4. Furthermore, neither 3 nor 4 can be undertaken unless one of the first two opportunities is undertaken. There are no such restrictions on investment opportunities 5 and 6. The objective is to select the combination of capital investments that will maximize the total estimated long-run profit (net presented value). Formulate the BIP model for this problem and determine the optimal solution.

	Investment Opportunity					
	1	2	3	4	5	6
Estimated profit	15	12	16	18	9	11
Capital required	38	33	39	45	23	27

Exercise 56. The research and development division of the Progressive Company has been developing four possible new product lines. Management must now decide which of these four products will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.

	Product			
	1	2	3	4
Start-up cost	$50 000	$40 000	$70 000	$60 000
Marginal revenue	$ 70	$ 60	$ 90	$ 80

A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the production mix that maximizes the total profit (total net revenue minus start-up costs). Let the continuous decision variable x_1, x_2, x_3 and x_4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

- No more than two products can be produced.
- Product 3 can be produced only if at least one of the products 1 or 2 is produced.
- Either $5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6000$ or $4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6000$
- Either product 4 is not produced, or the amount of product 4 produced is exactly 100 units.

Formulate a MIP model for this problem and solve it by using the solver.

Exercise 57. *Union Airways* is adding more flights to and from its hub airport, and so it needs to hire additional customer service agents. However, it is not clear how many more should be hired. Management recognizes the need for cost control while also consistently providing a satisfactory level of service to customers. Therefore, the OR team is studying how to schedule the agents to provide satisfactory service with the smallest personnel cost. Based on the new schedule of flights, an analysis has been made of the minimum number of customer service agents that need to be on duty at different times of the day to provide a satisfactory level of service. The rightmost columns of the following table show the number of agents needed for the time periods given in the first column.

	Time Periods Covered						Minimum number of Agents Needed
	Shift						
Time Period	1	2	3	4	5	6	
6:00 – 8:00	✓					✓	*35*
8:00 – 10:00	✓	✓				✓	*65*
10:00 – 12:00	✓	✓					*40*
12:00 – 14:00	✓	✓	✓				*55*
14:00 – 16:00		✓	✓				*40*
16:00 – 18:00			✓	✓		✓	*60*
18:00 – 20:00			✓	✓		✓	*75*
20:00 – 22:00				✓			*45*
22:00 – 24:00				✓	✓		*30*
0:00 – 6:00					✓		*10*
Daily cost per agent	$220	$180	$235	$200	$215	*320*	

Each agent works an 8-hour shift, five days per week. Checkmarks in the main body of the table show the hours covered by the respective shifts. For each shift the daily compensation for each agent is shown in the bottom row. Determine how many agents should be assigned to the respective shifts to minimize the total cost for agents while meeting the service requirements.

Exercise 58. A company is preparing its investment plan for the next three years. From the current activities, it predicts the availability of the following funds (in *m.u.*): 50 in year 1; 40 in year 2; 30 in year 3. Beyond this self-financing, the company may seek loans from short-term (one year) in each of the three years. The amount requested in each year shall be paid in the next year, along with interest, which is 6% per year. The company can also get in the 1st year a medium term loan to be paid at the end of the planning period, and each year the interest, which is 7% per year, must be paid. For financial reasons, the company cannot have both short-term loans in years 1 and 2. The funds available, including any loans, can be applied to the following investment projects:

	cash-flow			TPV*
	year 1	year 2	year 3	(end of year 3)
project 1	-100	20	30	100
project 2	-50	-40	50	90
project 3		-50	50	20

*TPV – Terminal Present Value, that is, the present value at the end of year 3 of the *cash-flows* that will occur in the 4 coming years.

The company intends to maximize the net assets of liabilities at the end of the planning period, that is, intends to maximize the TPV projects less debt at the end of the planning period. Knowing that the projects can only be done in whole, establish a mode.