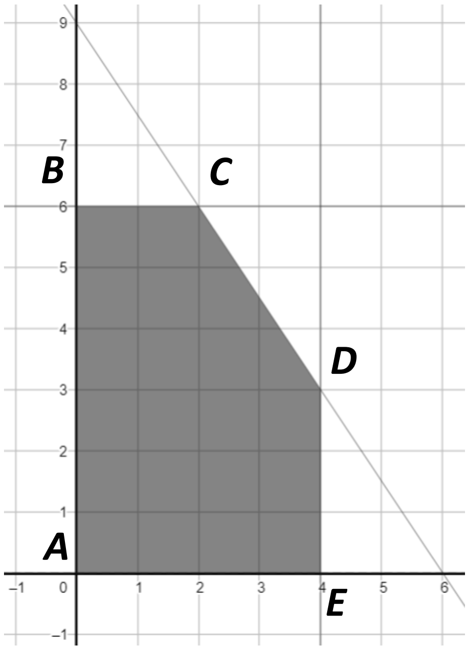
Chapter 1 – Linear Programming

Prototype 1



a) x_1- "Number of batches of doors to produce"

x_2- "Number of batches of windows to produce"

$$\begin{array}{ll} \max & 3x_1 + 5x_2 \\ \text{s.t.} & x_1 \leq 4 \\ & 2x_2 \leq 12 \\ & 3x_1 + 2x_2 \leq 18 \\ & x_1, x_2 \geq 0 \end{array}$$

$A(0,0) \rightarrow f(0,0) = 0$

$B(0,6) \rightarrow f(0,6) = 30$

$C(2,6) \rightarrow f(2,6) = 36$

$D(4,3) \rightarrow f(4,3) = 27$

$E(4,0) \rightarrow f(4,0) = 12$

The optimal solution is (2,6), and the optimal value is 36.

b)

i) The optimal solution is point (4,3)

ii) The optimal solution is point (4,0)

iii) The optimal solution is point (0,6)

iv) The optimal solutions belong to the segment connecting (4,3) and (4,0)

v) The optimal solutions belong to the segment connecting (0,6) and (2,6)

c) Impossible

d) Unbounded.

e) The feasible region is just the segment connecting (2,6) and (4,3). The optimal solution is still point (2,6).

f) The optimal solution is the point (3/2,6)

g) The optimal solution is the point (2.5 , 5.25)

h) The optimal solution is the segment connecting (2,6) and (4,3).

Prototype 2

The optimal solution is the point B(4,3), and the optimal value is 10.

Exercise 2

a) Solution: (0,3), $z^* = 6$

b) Infeasible problem

c) Unbonded problem

d) Semi-line segment starting on (2,0), $z^* = 2$

e) Solution: (-3,0), $z^* = 30$

f) Solution: (3/2,1), $z^* = 5/2$

g) Unbonded problem
h) Solution: (0,0), $z^* = 0$
i) Solution: (4/3,5/3), $z^* = 22/3$
j) Solution: A segment connecting two points, $z^* = 12$

Exercise 3

Sol: $(47.6\,,23.8\,,0) \rightarrow z^* = 2857.143$

or (Integer solution)

Sol: $(46\,,23\,,3) \rightarrow z^* = 2835$

Exercise 4

The optimal solution is: $(a_s = 56.25\,, a_c = 0\,, a_o = 0\,, c = 23.75, h = 0\,, t_w = 0\,, t_s = 0) \rightarrow z^* = 5750$

Or (integer solution: cows and hens as integers)

$(a_s = 53.3\,, a_c = 0\,, a_o = 3.3\,, c = 24, h = 0\,, t_w = 0\,, t_s = 0) \quad \rightarrow z^* = 5750$

Exercise 5

The optimal solution is: $3333.3\ kg\ of\ C_1\ (2000 cotton + 1333.3 wool)$
$2366.7\ kg\ of\ C_2\ (1166.7 wool + 1200 fiber)$
$0\ kg\ of\ C_3$

Profit: 485666.7

Exercise 6

The optimal solution is: $(x, y, z) = (84,80,0)$, the optimal value is 70 000.

Exercise 7

The optimal solution is: $(c, t, a) = (1.14\,, 0\,, 2.43)$, the optimal value is 120.86

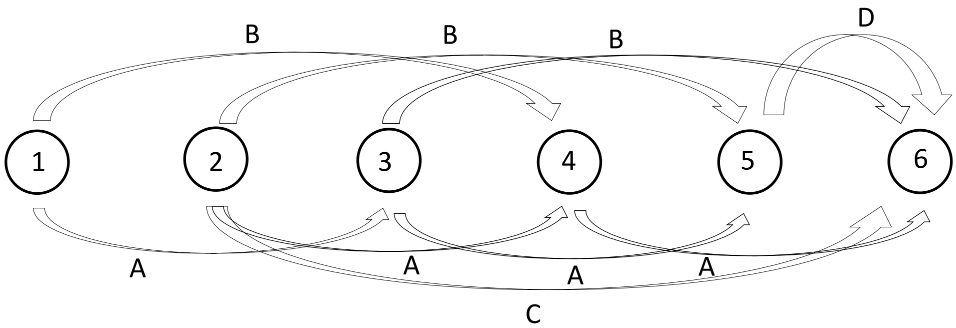
Exercise 8

The optimal solution is: 30 units of energy (from energy)
20 units of heating water (from solar panels)
50 units of heating space (30 from natural gas and 20 from solar panels)
Cost: 7700

Exercise 9

A_i- "Amount invested in A at the beginning of year i", $i = 1,2,3,4$
B_i- "Amount invested in B at the beginning of year i", $i = 1,2,3$
C- "Amount invested in C at the beginning of year 2"
D- "Amount invested in D at the beginning of year 5"

The optimal solution is:
$A_1 = 60\,000$
$A_3 = 84\,000$
$D = 117.600$
$Profit = 152\,880$



Exercise 10

Solution: (266.7,333.3) z* = 6000

Chapter 2 – Simplex Algorithm

Exercise 11

a)

$$
\begin{aligned}
&A(0,500) &&\rightarrow f(0,500) &&= 4500\\
&B(100,500) &&\rightarrow f(100,500) &&= 5625\\
&C(266.7,333.3) &&\rightarrow f(266.7,333.3) &&= 6000\\
&D(400,125) &&\rightarrow f(400,125) &&= 5625\\
&E(400,0) &&\rightarrow f(400,0) &&= 4500\\
&F(0,0) &&\rightarrow f(0,0) &&= 0
\end{aligned}
$$

The optimal solution is the point C(266.7 , 333.3) and the optimal value is 6000.

b) -

c) F $\rightarrow A \rightarrow B \rightarrow C$ or $F \rightarrow E \rightarrow D \rightarrow C$

Exercise 12

a) The optimal solution is (6,0,0,0, 4) and $z^* = 12$.

b) The optimal solution is (3, 0, 3/2, 0, 0, 1/2) and $z^* = -12$.

c) The optimal solution is (11/5, 7/5, 0, 0, 7/5, 0) and $z^* = 61/5$.

d) The optimal solution is (14/13, 8/13, 0, 0, 0, 69/13) and $z^* = 96/13$.

Exercise 13

a) The optimal solution is (0, 3, 9, 0) and $z^* = 6$

b) Out of the scope of this course

c) Unbounded

d) The solution $(2, 0, 0, 2)$ with optimal value $z^* = 2$ is optimal but there are alternative solutions.

e) The optimal solution is (0, 3, 0, 8,0) and $z^* = 30$.

f) Out of the scope of this course

g) Out of the scope of this course

h) The optimal solution is $(0, 0, 2, 2)$ and $z^* = 0$.

i) Out of the scope of this course

j) Optimal solutions in segment connecting points (4/3, 4/3, 0, 0) and (4,0,0,4). $z^* = 12$.

Exercise 14

a) -

b) The solution (0, 70/3, 0, 0, 230/3) with value 350 is optimal.
The solutions (35, 0, 0, 0, 195/3) and (0, 0, 70, 0, 30) are alternative optimal solutions.

Exercise 15

a) The optimal solution is point $(0, -8)$ and the optimal value is 24.

b) The obtained solution is $(24, 0, 0, -16)$ and it is a basic non-feasible solution.

c) –

d) The optimal solution is (0, 0, 8, 16, 0) with value 24 is optimal.

Exercise 16

a) $c \geq 0$.

b) $c = 0$.

c) $c < 0$ and $a_2, a_3 \leq 0$.

d) $c < 0$ (to ensure not-optimality), and $a_3 = 2a_2$ (to ensure a tie) with $a_2, a_3 > 0$.

Chapter 3 – Duality and Sensitivity Analysis

Exercise 17

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Type 1	0	-3	3	3	1E+30
Type 2	25	0	7	8	2
Type 3	25	0	5	2	2,666666667

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Sugar	50	4	50	50	16,66666667
Chocolate	100	1	100	50	50

Objective Cell (Max)

Name	Original Value	Final Value
Profit	0	300

Variable Cells

Name	Original Value	Final Value	Integer
Type 1	0	0	Contin
Type 2	0	25	Contin
Type 3	0	25	Contin

Constraints

Name	Cell Value	Formula	Status	Slack
Sugar	50		Binding	0
Chocolate	100		Binding	0

a) $\mathrm{SI}_{c_2} = [5,15]$. When $c_2 = 13$, the optimal solution is the same (0, 25, 25) and the optimal value is changed to $z_{new} = 450$.

b) It is worth to increase the availability of sugar if the price per kg is lower than 4m.u.. The interval in which the set of basic variables is the same is $\mathrm{SI}_{b_1} = [33.333,100]$.

c) $b_1 = 55$.

d) It is not worth buying the extra 2kg at the proposed price.

e) $z_{new} = 340$. The new solution is: (0, 20, 40).

f) It is impossible to know what the new solution and its associated value are without solving the problem. By using the solver, the new solution is (0,30,0), and the profit is 210.

g) The profit should be greater than 9 m.u.

Exercise 18

a)
$$\begin{aligned} \max\ z &= 2x_1 + 3x_2 + x_3 \\ \text{s.t.}\quad & x_1 + 2x_2 + 3x_3 \geq 6 \\ & x_1 + x_2 + x_3 \leq 9 \\ & x_1, x_2 \geq 0 \\ & x_3 \leq 0 \end{aligned}$$

⬇

$$\begin{aligned} \min\ w &= 6y_1 + 9y_2 \\ \text{s.t.}\quad & y_1 + y_2 \geq 2 \\ & 2y_1 + y_2 \geq 3 \\ & 3y_1 + y_2 \leq 1 \\ & y_1 \leq 0 \\ & y_2 \geq 0 \end{aligned}$$

b)
$$\begin{aligned} \min\ z &= -3x_1 + x_2 \\ \text{s.t.}\quad & x_1 + x_2 = 3 \\ & x_1 + 2x_2 \leq 6 \\ & 2x_1 + 2x_2 \leq 8 \\ & x_1\ free \\ & x_2 \geq 0 \end{aligned}$$

⬇

$$\begin{aligned} \max\ w &= 3y_1 + 6y_2 + 8y_3 \\ \text{s.t.}\quad & y_1 + y_2 + 2y_3 = -3 \\ & y_1 + 2y_2 + 2y_3 \leq 1 \\ & y_1\ free \\ & y_2, y_3 \leq 0 \end{aligned}$$

c)
$$\begin{aligned} \max\ z &= 3x_1 + x_3 \\ \text{s.t.}\quad & x_1 + 2x_2 + x_3 \geq 5 \\ & 2x_1 + x_3 \leq 10 \\ & x_2 + x_3 \leq 8 \\ & x_1 \leq 0 \\ & x_2\ free \\ & x_3 \geq 0 \end{aligned}$$

⬇

$$\begin{aligned} \min\ w &= 5y_1 + 10y_2 + 8y_3 \\ \text{s.t.}\quad & y_1 + 2y_2 \leq 3 \\ & 2y_1 + y_3 = 0 \\ & y_1 + y_2 + y_3 \geq 1 \\ & y_1 \leq 0 \\ & y_2, y_3 \geq 0 \end{aligned}$$

d) $\max\ z = 7x_1 + 5x_2$

$$\begin{aligned} \text{s.t.} \quad & 8x_1 + x_3 \le 10 \\ & 2x_1 - 3x_2 + 2x_3 = -4 \\ & 3x_1 - 4x_2 \ge 0 \\ & x_1 \ge 0 \\ & x_2, x_3 \le 0 \end{aligned}$$

⬇

$\min\ w = 10y_1 - 4y_2$

$$\begin{aligned} \text{s.t.} \quad & 8y_1 + 2y_2 + 3y_3 \ge 7 \\ & -3y_2 - 4y_3 \le 5 \\ & y_1 + 2y_2 \le 0 \\ & y_1 \ge 0 \\ & y_2 free \\ & y_3 \le 0 \end{aligned}$$

e) $\min\ z = x_1 - x_3$

$$\begin{aligned} \text{s.t.} \quad & x_1 - 5x_2 \le 6 \\ & x_1 - 4x_2 + x_3 = 0 \\ & x_1, x_2 \le 0 \\ & x_3 \ge 0 \end{aligned}$$

⬇

$\max\ w = 6y_1$

$$\begin{aligned} \text{s.t.} \quad & y_1 + y_2 \ge 1 \\ & -5y_1 - 4y_2 \ge 0 \\ & y_2 \le -1 \\ & y_1 \le 0 \\ & y_2\ free \end{aligned}$$

Exercise 19

a) The optimal solution is point (4,0) and its values is 4. This means that … The BFS is (4, 0, 21, 0).

b)

$$\begin{aligned} \min\ z = \ & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le 25 \\ & 0.5x_1 + 0.8x_2 \ge 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

➡

$$\begin{aligned} \max\ w = \ & 25y_1 + 2y_2 \\ \text{s.t.} \quad & y_1 + 0.5y_2 \le 1 \\ & y_1 + 0.8y_2 \le 2 \\ & y_1 \le 0 \\ & y_2 \ge 0 \end{aligned}$$

From the complementary slackness relations we have $(y_1^*, y_2^*) = (0,2)$ and the optimal value is 4.

Exercise 20

Objective Cell (Max)

Name	Original Value	Final Value
Return total	19,25	19,25

Variable Cells

Name	Original Value	Final Value	Integer
X1	0	0	Contin
X2	10	10	Contin
X3	15	15	Contin
X4	0	0	Contin

Constraints

Name	Cell Value	Formula	Status	Slack
Risk 3 total	15		Binding	0
Budget total	25		Binding	0
Risk 1 total	10		Not Binding	5
Risk 2 total	15		Binding	0

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
X1	0	0	0,5	0,15	1E+30
X2	10	0	0,8	1E+30	0,15
X3	15	0	0,75	1E+30	0,15
X4	0	-0,15	0,9	0,15	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Budget total	25	0,8	25	5	10
Risk 1 total	10	0	15	1E+30	5
Risk 2 total	15	0,25	15	0	5
Risk 3 total	15	-0,3	15	10	0

a) –

b) The optimal solution is (0, 10, 15, 0) and the optimal value is 19.25. This mean that …

c) $\Delta_z = -0.25$.

d) $9 \notin SI_{b_3} = [10, 15]$ and therefore, it is not possible to quantify the change in the total return without resolve the problem.

e) $\Delta_z = 0$ (no variation). The optimal solution is the same.

Exercise 21

a) The optimal solution is point (3,4) and the optimal value is 17.

b)

$$\begin{array}{ll} \max\ z = 3x_1 + 2x_2 & \\ \text{s.t.} & x_1 \leq 4 \\ & x_1 + 3x_2 \leq 15 \\ & 2x_1 + x_2 \leq 10 \\ & x_1, x_2 \geq 0 \end{array} \quad \Rightarrow \quad \begin{array}{ll} \min\ w = 4y_1 + 15y_2 + 10y_3 & \\ \text{s.t.} & y_1 + y_2 + 2y_3 \geq 3 \\ & 3y_2 + y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{array}$$

c) The optimal solution of the dual is $(y_1^* = 0, y_2^* = \frac{1}{5}, y_3^* = \frac{7}{5})$

d) –

e) $\Delta_z = -2.8$.

Exercise 22

a) The pair of dual problems is:

$$\begin{array}{ll} \max\ z = 6x_1 + 8x_2 & \\ \text{s.t.} & 5x_1 + 2x_2 \leq 20 \\ & x_1 + 2x_2 \leq 10 \\ & x_1, x_2 \geq 0 \end{array} \quad \Rightarrow \quad \begin{array}{ll} \min\ w = 20y_1 + 10y_2 & \\ \text{s.t.} & 5y_1 + y_2 \geq 6 \\ & 2y_1 + 2y_2 \geq 8 \\ & y_1, y_2 \geq 0 \end{array}$$

The primal solution is (5/2,15/4) and the dual solution is (1/2,7/2). The optimal value of both problems is 45.

b) –

Exercise 23

a) The pair of dual problems is

$$\begin{array}{ll} \max\ z = 2x_1 + 7x_2 + 4x_3 & \\ \text{s.t.} & x_1 + 2x_2 + x_3 \leq 10 \\ & 3x_1 + 3x_2 + 2x_3 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{array} \quad \Rightarrow \quad \begin{array}{ll} \min\ w = 10y_1 + 10y_2 & \\ \text{s.t.} & y_1 + 3y_2 \geq 2 \\ & 2y_1 + 3y_2 \geq 7 \\ & y_1 + 2y_2 \geq 4 \\ & y_1, y_2 \geq 0 \end{array}$$

b) The solution of the dual problem is (0, 7/3) and the optimal value is 70/3. The optimal solution of the primal problem is (0,10/3,0).

Exercise 24

a) The pair of dual problems is

$$\begin{aligned} \max\ z &= 3x_1 + 4x_2 + 2x_3 \\ \text{s.t.}\quad & x_1 + x_2 + 2x_3 \leq 10 \\ & 2x_1 + 4x_2 + x_3 \leq 8 \\ & 2x_1 + 3x_2 + 2x_3 \leq 20 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \min\ w &= 10y_1 + 8y_2 + 20y_3 \\ \text{s.t.}\quad & y_1 + 2y_2 + 2y_3 \geq 3 \\ & y_1 + 4y_2 + 3y_3 \geq 4 \\ & 2y_1 + y_2 + 2y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

The solution of the dual problem is $\left(\frac{1}{3}, \frac{4}{3}, 0\right)$.

b) To be profitable to include product 2 in the production plan, its unitary profit must increase in at least 1.67.

Exercise 25

a) The solution of the primal problems is (2,2). The dual problem is:

$$\begin{aligned} \max\ w &= 4y_1 \qquad - 6y_3 \\ \text{s.t.}\quad & y_1 - y_2 \qquad \leq 1 \\ & y_1 + y_2 - y_3 \leq 3 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

b) The optimal dual solution is (2,1,0).

Exercise 26

a) The solution of the primal problems is (2,4). The dual problem is:

$$\begin{aligned} \max\ w &= 3y_1 + 12y_2 + 6y_3 \\ \text{s.t.}\quad & y_1 + \qquad y_3 \leq 3 \\ & 3y_2 + y_3 \leq 2 \\ & y_1, y_2 \leq 0 \\ & y_3 \geq 0 \end{aligned}$$

The optimal dual solution is (0, -1/3, 3).

b) $\alpha = 3/2$.

c) $\alpha < 2/3$.

Exercise 27

a) Product 2 is the one that requires a lower increase in profit to be included in the production plan.

b) It is not possible to determine the new obtained solution and its value without resolving the problem. From the solver, the new optimal solution is (0, 0, 5, 0) and its value is 35.

c) The optimal solution can only be determined by using the solver. Such a solution is: (0, 0, 5, 0, 2) and its cost is 55.

d) The extra 24 m.u. should be invested in rm2 since the increase in profit is larger.

Exercise 28

a) The optimal solution of the primal problem is: (50, 30, 10, 0, 0, 0) and the optimal value is 270. The optimal solution indicates that …

b) The shadow prices are (18.38, -3.46, 0.31). The interpretation is …

c) $\Delta_z = 31$

d) $\Delta_z = -35$

Exercise 29

a) The solution is $(57.5, 0, 20)$ and the minimum cost is 2900.

b) –

c) $\Delta_z = +600 m.u.$

d) The new solution is: (40, 90, 0) and the minimum cost is 3220

e) $b_2 = 321$

Exercise 30

a) The optimal primal solution is (12.5, 0, 8) and the optimal value is 245. This means that … The optimal solution of the dual problem is (0, 3, -0.5). This means that…

b) The current solution remains optimal. The optimal value is also the same.

c) P2 will be included in the production plan if its revenue is higher than 25.

d) Increase the capacity in section 2. $b_2 = 91.63$

e) The solution is (2.5, 4, 8) with $z^* = 225$.

Exercise 31

a) The optimal solution is point (62/7, 12/7) and the optimal value is 62.7.

b) $y_1^* = 0$

c) The constraint $x_1 + x_2 \geq 3$ will not change the feasible region.

d) $SI_{b_3} = [24, 115]$

e) $SI_{b_3} = [10, +\infty[$

Chapter 4 – Transportation and Assignment Problems

Prototype 3

a) Consider the following variables:

x_{ij} – "truckloads to send from cannery i to the warehouse j", $i = 1,2,3$, $j = 1,2,3,4$.

$$\begin{aligned}
\min \quad & 464x_{11} + 513x_{12} + \cdots + 388x_{33} + 685x_{34} \\
s.t. \quad & x_{11} + x_{12} + x_{13} + x_{14} = 75 \\
& x_{21} + x_{22} + x_{23} + x_{24} = 125 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 100 \\
& x_{11} + x_{21} + x_{31} = 80 \\
& x_{12} + x_{22} + x_{32} = 65 \\
& x_{13} + x_{23} + x_{33} = 70 \\
& x_{14} + x_{24} + x_{34} = 85 \\
& x_{ij} \geq 0, \quad i = 1,2,3, \quad j = 1,2,3,4
\end{aligned}$$

b) –

c) The optimal solution has the cost 152 535.

d) The optimal solution has the cost 119 965.

e) The optimal solution has the cost 139 530.

f) The optimal solution has the cost 125 610.

Exercise 32

The optimal solution value is 2 460.

Prototype 4

a) Consider the following binary variables:

$$x_{ij} = \begin{cases} 1 & \text{if machine } M_i \text{ goes to location } L_j \\ 0 & \text{otherwise} \end{cases}, \quad i,j = 1,2,3,4.$$

The formulation is:

$$\min \quad 13x_{11} + 16x_{12} + \cdots + 15x_{43} + 13x_{44}$$

$$\begin{aligned}
s.t. \quad & x_{11} + x_{12} + x_{13} + x_{14} = 1 \\
& x_{21} \phantom{+ x_{22}} + x_{23} + x_{24} = 1 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 1 \\
& x_{41} + x_{42} + x_{43} + x_{44} = 1 \\
& x_{11} + x_{21} + x_{31} + x_{41} = 1 \\
& x_{12} \phantom{+ x_{22}} + x_{32} + x_{42} = 1
\end{aligned}$$

Solution:							
	L1	L2	L3	L4			
M1				1	1	==	1
M2			1		1	==	1
M3		1			1	==	1
M4	1				1	==	1
	1	1	1	1	43		
	==	==	==	==			
	1	1	1	1			

$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$
$$x_{ij} \in \{0,1\},\ i,j = 1,2,3,4$$

b) The new solution has the value 29.

Exercise 33

a) The optimal solution value is 3 680.
b) The optimal solution value is 2 580.

Exercise 34

a) min $100 \times 40 + 650x_{12} + \cdots + 400x_{33} + 450x_{34})$

$$
\begin{aligned}
s.t.\quad & x_{12} + x_{13} + x_{14} = 12 \\
& x_{21} + x_{22} + x_{23} = 17 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 11 \\
& x_{21} + x_{31} = 10 \\
& x_{12} + x_{22} + x_{32} = 10 \\
& x_{13} + x_{23} + x_{33} = 10 \\
& x_{14} + x_{34} = 10 \\
& x_{ij} \geq 0,\ i = 1,2,3, j = 1,2,3,4
\end{aligned}
$$

		Distribution Center				
		1	2	3	4	Production
	1	-	650	200	350	12
Plant	2	550	700	300	-	17
	3	300	600	400	450	11
Demand		10	10	10	10	

b) –
c) The optimal solution value is 20 200.

Exercise 35

The optimal solution value is 22.

Exercise 36

The optimal solution value is 199 500 (× 100)

Exercise 37

The optimal solution value is 775

Exercise 38

The optimal solution is: C1→ V1, C2→ V3, C3→ V2, and C5→ V4. The total cost is 400.

Exercise 39

a) The optimal solution is:

	F1	**F2**	**F3**	**F4**	**F5**	Sales
P1	2500	500				3000
P2				3000		3000
P3		2000				2000
Capacity	2500	3000	2000	4000	5000	

b) The optimal solution value is 57 000.

Exercise 40

a) The optimal solution value is 380.

b) We must impose the constraint $x_{22} = 10$. Hence, since W3 cannot supply S2, and the demand on this shop must be satisfied we should also impose $x_{12} = 40 \Leftrightarrow x_{12} + x_{22} = 50$.

Exercise 41

Consider the variables:

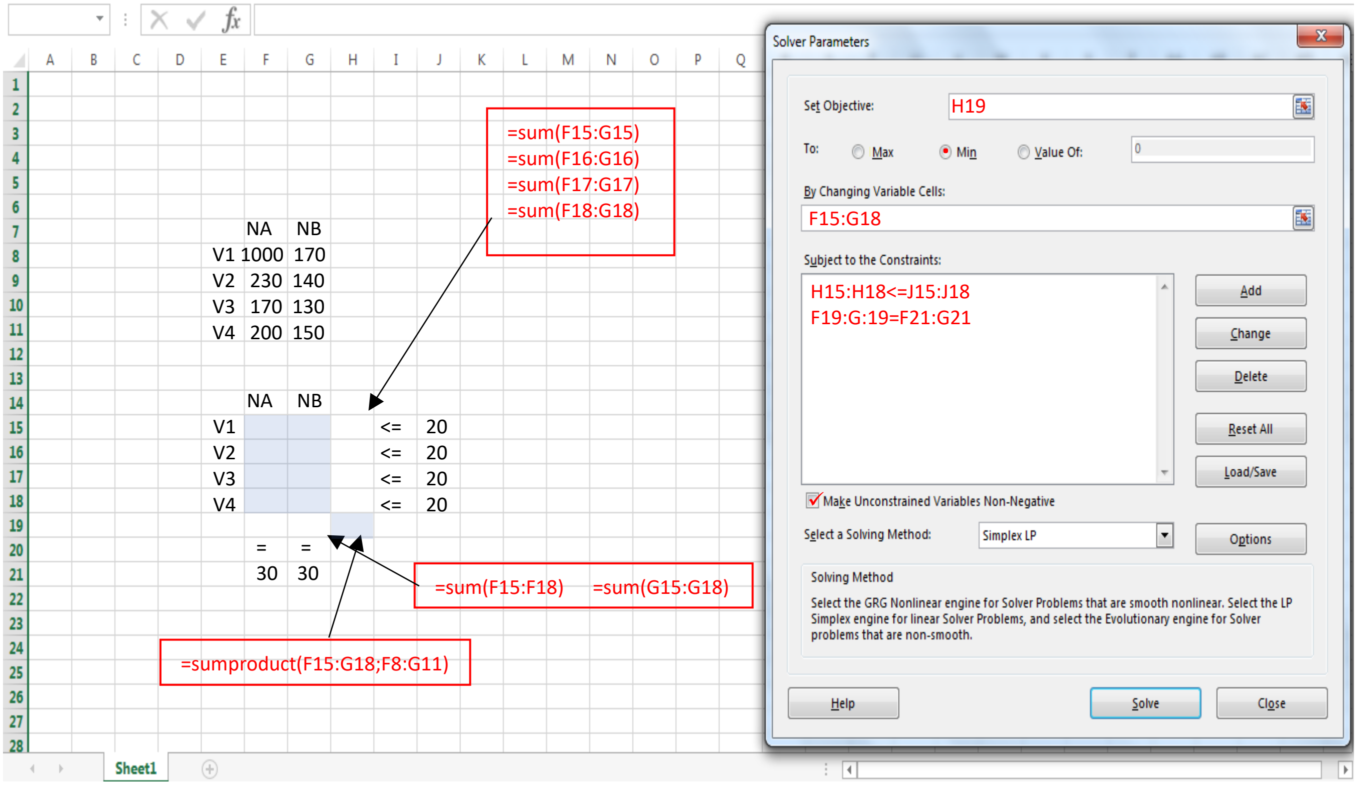
$$x_{ij} = \begin{cases} 1 & \text{if factory } i \text{ serves market } j \\ 0 & \text{otherwise} \end{cases}, \quad i,j = 1,2,3,4.$$

The problem is formulated as following

$$\begin{aligned} \min\ & 21(5x_{11} + 2x_{21} + 3x_{31} + 2x_{41}) + 16(8x_{12} + 6x_{22} + 7x_{32} + 5x_{42}) \\ & + 30(3x_{23} + 5x_{33} + 4x_{43}) + 35(7x_{34} + 3x_{44}) \end{aligned}$$

$$\begin{aligned} s.t.\quad & x_{11} + x_{12} = 1 \\ & x_{21} + x_{22} + x_{23} = 1 \\ & x_{31} + x_{32} + x_{33} + x_{34} = 1 \\ & x_{41} + x_{42} + x_{43} + x_{44} = 1 \\ & x_{11} + x_{21} + x_{31} + x_{41} = 1 \\ & x_{12} + x_{22} + x_{32} + x_{42} = 1 \\ & x_{23} + x_{33} + x_{43} = 1 \\ & x_{34} + x_{44} = 1 \\ & x_{ij} \in \{0,1\}, \quad i,j = 1,2,3,4. \end{aligned}$$

Exercise 42



Chapter 5 – Network Optimization

Prototype 5

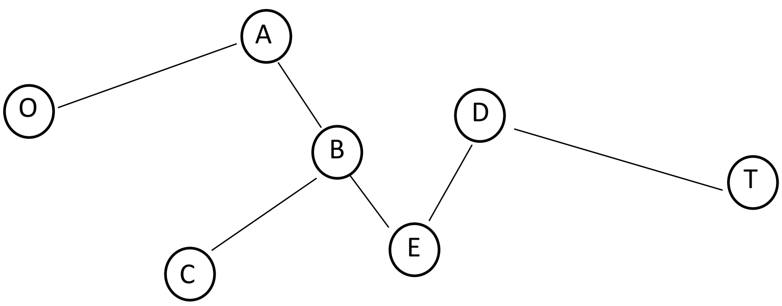
The optimal solution has the cost 49 000.

From	To	Flow
F1	F2	0
F1	W1	10
F1	DC	40
F2	DC	40
DC	W2	80
W1	W2	0
W2	W1	20

Prototype 6

a) The optimal solution is $O \to A \to B \to D \to T$, and the distance is 13.

b) The minimum spanning tree has a total distance of 14 and is:

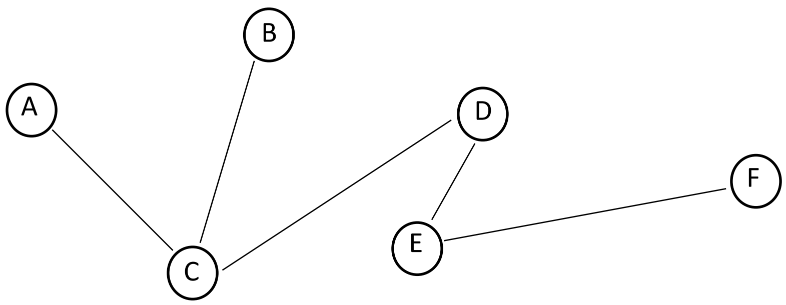


Exercise 43

The optimal solution is $Lisboa \rightarrow Porto \rightarrow Frankfurt \rightarrow Oslo$. The total distance is 4300 km.

Exercise 44

The minimum spanning tree has a total distance of 33 and is:



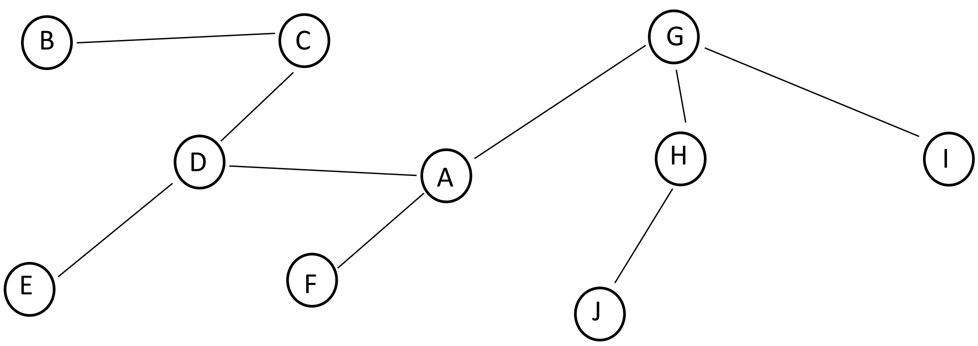
Exercise 45

The cost of the optimal solution is 747 and the solutions is as follows:

From	To	Flow
A	B	20
A	C	5
B	E	20
B	F	
C	D	3
C	F	2
E	D	20
F	D	2

Exercise 46

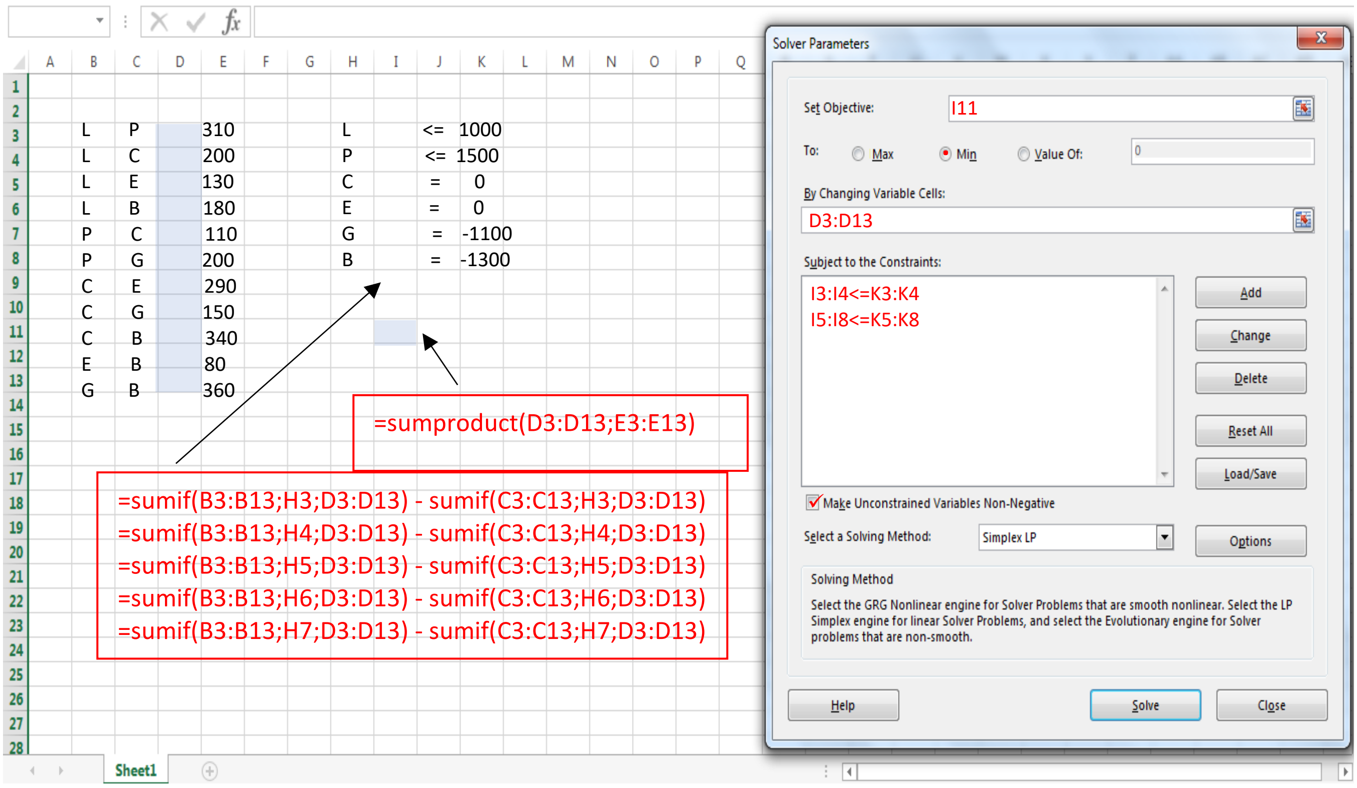
The minimum spanning tree has the total distance 28 and is:



Exercise 47

The destination node is C1. The solution is $S \rightarrow A \rightarrow B \rightarrow C \rightarrow C1$ (Time = 11)

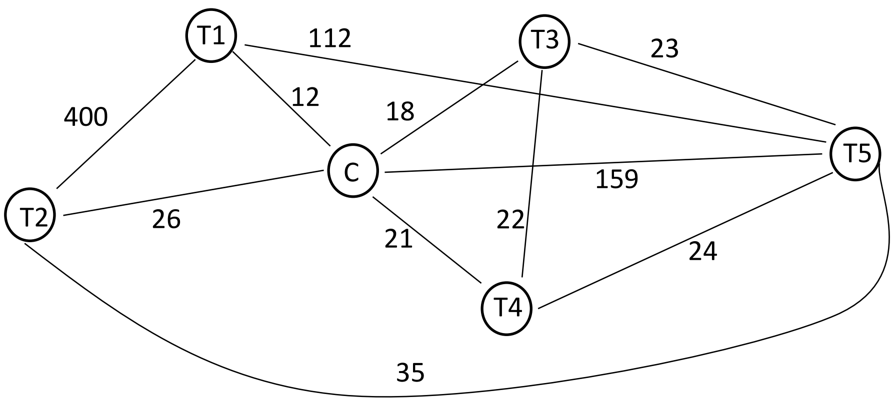
Exercise 48



$$x_{LB} = 1000, x_{PC} = 300, x_{PG} = 1100, x_{CB} = 300, \qquad Cost = 535\ 000$$

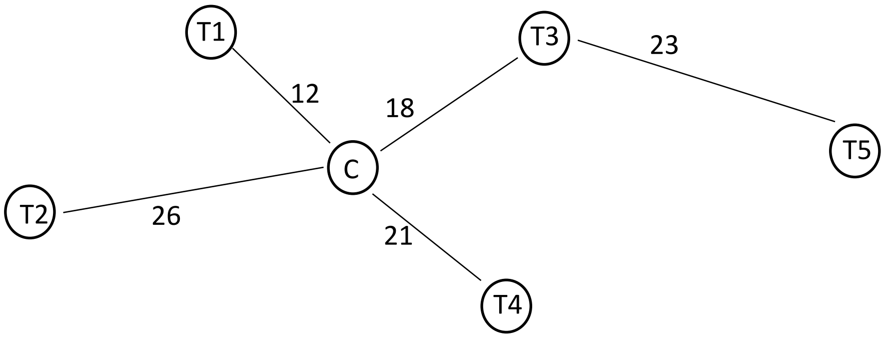
Exercise 49

a) The corresponding network is as follows.



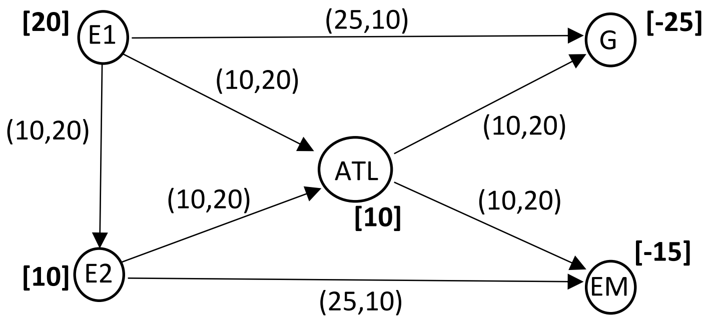
Incomplete!

b) The minimum spanning tree has a total distance of 100 and is:



Exercise 50

a) The network formulation of the problem is as follows:

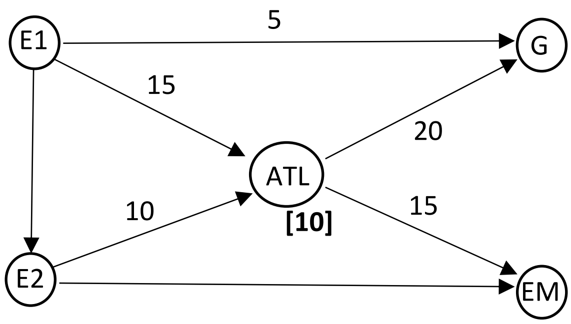


Label:
(c_{ij}, u_{ij}) – (unitary cost, capacity)
$[b_i]$ – Generated flow at node i

This is a MCFP with sources E1, E2, and ATL and destinations G and EM. The decision variables are x_{ij} and represent the number of children transported from place i to place j, where $(i, j) \in A = \{(1,2), (1,4), (2,3), (2,4), (3,4), (3,5)\}$. The goal of the problem is to determine how to transport the children from the sources to the destinations at the minimum cost.

b) -

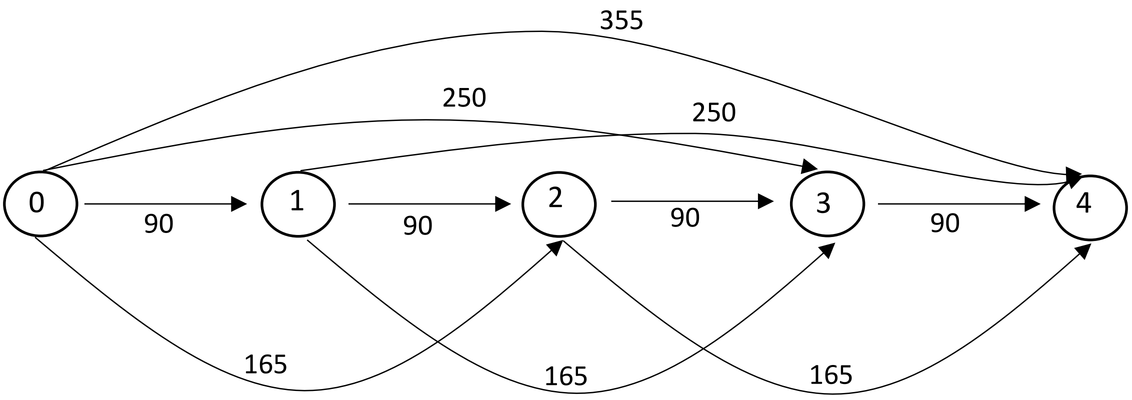
c) The optimal solution of this problem has a cost of 725 and is as follows:



Exercise 51

a) -

b) The problem can be formulated as a SPP in the following network (where each node represents the end of each year and the arcs represent a replacement action).



c) The optimal solution is $x_{02} = x_{24} = 1$ and the optimal value is 330.

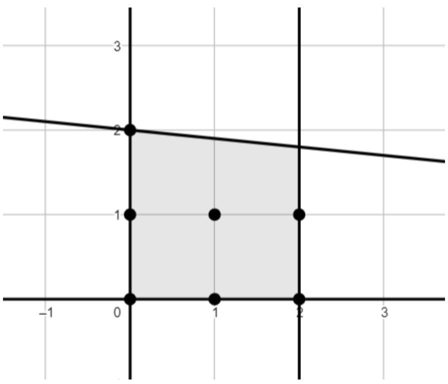
Chapter 6 – Integer Linear Programming

Prototype 7

Let us define the following variables:

- x_s: "Number of small airplanes to buy"
- x_m: "Number of medium airplanes to buy"

$$\begin{aligned} \max \ & x_s + 5x_m \\ s.t. \ & 5x_s + 50x_m \leq 100 \\ & x_s \leq 2 \\ & x_s, x_m \in \mathbb{Z}_0^+ \end{aligned}$$



The feasible region is the set of points $\{(0,0), (0,1), (0,2), (1,0), (1,1), (2,0), (2,1)\}$. Calculating the objective function value at each point, we conclude that the optimal solution is point (0, 2) and the optimal value is 10.

Prototype 8

Let us define the following variables:

$$x_1 = \begin{cases} 1 & \text{if a factory in LA is built} \\ 0 & \text{otherwise} \end{cases}$$

$$x_2 = \begin{cases} 1 & \text{if a factory in SF is built} \\ 0 & \text{otherwise} \end{cases}$$

$$y_1 = \begin{cases} 1 & \text{if a warehouse in LA is built} \\ 0 & \text{otherwise} \end{cases}$$

$$y_2 = \begin{cases} 1 & \text{if a warehouse in SF is built} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \max \quad & 9x_1 + 5x_2 + 6y_1 + 4y_2 \\ s.t. \quad & 6x_1 + 3x_2 + 5y_1 + 2y_2 \leq 10 \\ & y_1 \leq x_1 \\ & y_2 \leq x_2 \\ & y_1 + y_2 \leq 1 \\ & x_1, x_2, y_1, y_2 \in \{0,1\} \end{aligned}$$

The optimal solution is $(x_1^* = 1, x_2^* = 1, y_1^* = 0, y_2^* = 0)$ and the optimal value is 14. This means that only the new factories in LA and SF are built.

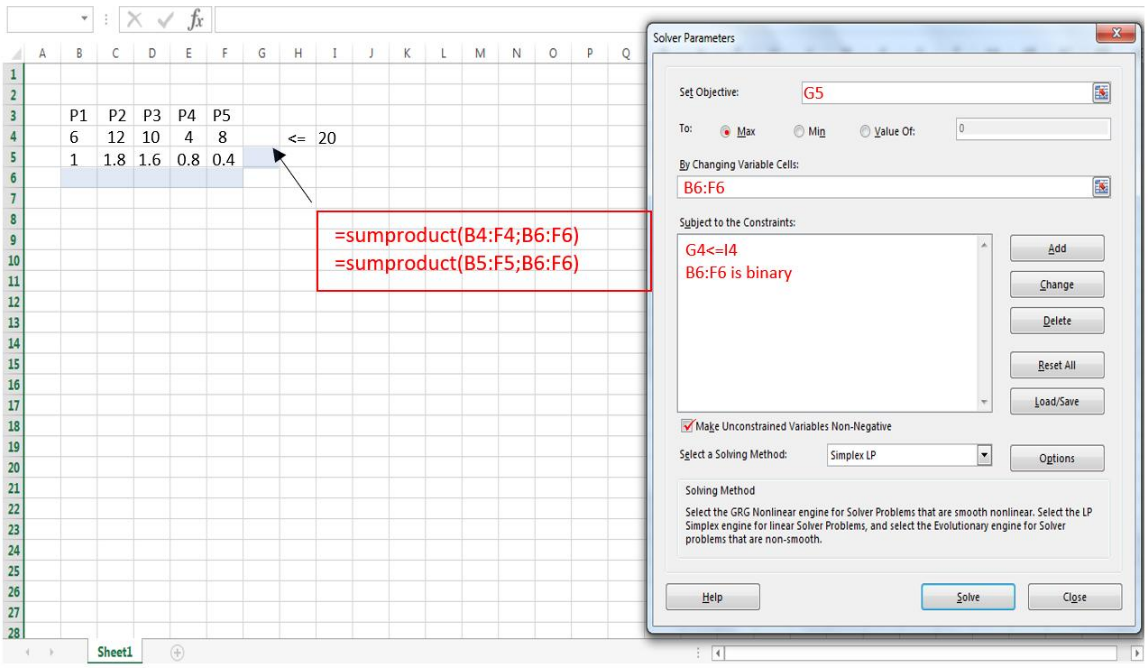
Exercise 52

The optimal solution is to produce P2, P3, P6, P8, and P9 and the total profit is 58.

Exercise 53

a) -

b)



c) The optimal solution is to invest in projects 1, 3, and 4 and the total profit is 3.4.

Exercise 54

The optimal solution is to invest in A, B' and C and its value is 1213.

Exercise 55

The optimal solution is to invest in 1, 3, and 5. The total profit is 40.

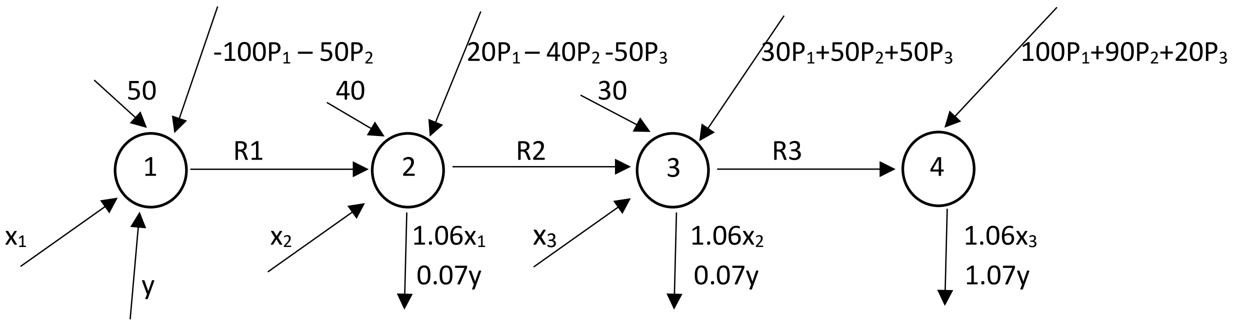
Exercise 56

The optimal solution is to produce only 2000 units of product 2. The total profit is 80 000.

Exercise 57

The optimal solution is (15, 30, 10, 45, 10, 20) and the optimal value is 28600.

Exercise 58



The optimal solution has the optimal value 216,78 and is:

$$(R_1 = 0,\ R_2 = 0,\ R_3 = 113{,}78,\ R_4 = 216{,}78,\ x_1 = 0,\ x_2 = 37,\ x_3 = 0, y = 100,\ P_1 = P_2 = P_3 = 1)$$