Repeat Period Duration: **2h**

Note: Justify all your answers and present all the calculations 3 / 2 / 2023

1. A company wants to plan the weekly production of three products (P1, P2, and P3) to maximize its revenue. The company has available 40 working hours (w.h.) and 35 machine hours (m.h.) per week. Let x_j be the number of tons of product P_j to produce, $j = 1, 2, 3$. The following linear programming model was proposed to solve the problem:

$$\begin{aligned} \max z &= 20x_1 + 30x_2 + 10x_3 \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 \le 40 \\ & x_1 + 2x_2 + x_3 \le 35 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

In the optimal production plan, only products P1 and P2 are produced, and all the available work hours and machine hours are used.

a) (*15 points*) Without solving the problem by the simplex method, and taking into account only the information provided, indicate the optimal solution of the problem (main variables) and its revenue.

b) (*15 points*) Write the dual problem and indicate if the **first dual constraint** is binding or not in the optimal dual solution.

c) The original problem was solved in the Excel/Solver and the following sensitivity report was obtained. Taking it into account, answer to the following questions:

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
P1		0	20	10	5
P2		0	30	10	10
P3	0	-5	10	5	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
w.h	40	5	40	30	5
m.h	35	10	35	5	15

i) (*20 points*) The company can buy either 10 extra working hours by 20m.u. or 4 extra machine hours by 10m.u. Which option is the best?

ii) (*15 points*) In the optimal production plan, P3 is not produced. What is the minimum increase in the unitary revenue of P3 to make it worth producing?

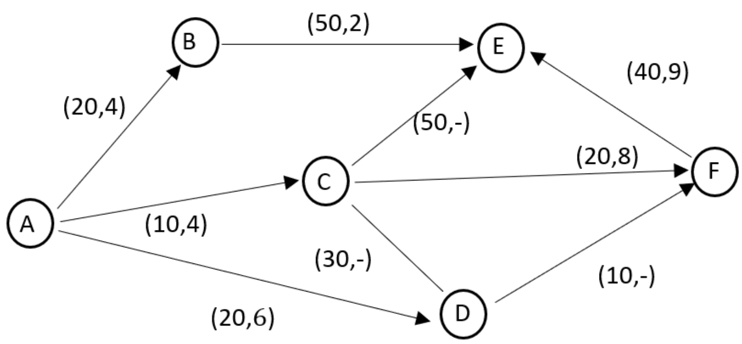
iii) (*15 points*) Comment the sentence: *Considering only the sensitivity report, we can say that if the availability of working hours decreases from 40 to 30, then the revenue decreases 50m.u.*

d) (*20 points*) Suppose now that the company must pay 40m.u. to produce product P1 (regardless of the quantity produced) and that product P2 can only be produced if product P3 is produced. Formulate the problem to cover these new conditions.

2. (*20 points*) Consider the following simplex tableau. Execute one iteration of the simplex method, write the solution obtained and classify it.

BV	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	RHS
z	1	-1	0	-3	0	0	0	0	0
s_1	0	1	-1	6	1	0	0	0	18
s_2	0	3	5	-3	0	1	0	0	1
s_3	0	2	1	3	0	0	1	0	6
s_4	0	0	1	0	0	0	0	1	4

3. The given network represents a road map. Each arc has two values (c_{ij}, u_{ij}) associated with it. The notation "-" indicates an unlimited capacity. The first value c_{ij} is the cost of crossing arc (i,j) with a truck and the value u_{ij} is the maximum number of trucks that can cross that arc. There are 10 trucks available in location A. Locations E and F are destinations, and they must receive 3 and 7 trucks, respectively. The problem consists of sending all trucks from A to E and F at the minimum cost.



a) (10 points) Indicate a feasible solution for the problem and compute its cost.

b) (20 points) Define the decision variables of the problem and write **all** the constraints related to arcs and nodes involving locations A, C, and F.

c) (15 points) Suppose that this problem is solved by using the Excel/Solver. Fill the Excel sheet with all the information needed to solve the problem.

d) (15 points) Ignoring the orientation of the arcs and their capacities in the network presented, determine a spanning tree for the resulting network and compute its cost.

4. *HappyBeans* is a new company dedicated to the distribution of beans. The company has m warehouses (origins) that must be used to supply n markets (destinations). The availability in each warehouse is k ton. The demand at each market is l ton.

a) (*10 points*) Suppose that $k = 2l$. What is the relation between m and n to ensure that the problem is balanced?

b) (*15 points*) Suppose now that $k = l$ and $n = m$. The cost of transporting one ton of beans from origin O_i to destination D_j is denoted by c_{ij} and is equal to $c_{ij} = i + j,\ i,j = 1, \dots, n$. Determine the cost of a feasible solution of this problem.