Regular Period Duration: **2h**

Note: Justify all your answers and present all the calculations 18/1/2023

1. Following a disaster, donations have been collected in medicines (M), food (F) and clothing (C). The goods need to be selected and packed, and each of these tasks is carried out by a team of volunteers. The selection team - if it only selects clothes - has the capacity to process 20 tons. The same happens for the food. Selecting a ton of medicines takes three times the time of the clothing selection. The packaging team has the capacity to process 10 tons of clothing. The packaging of medicinal products and food takes three times and twice the packaging of clothing (in tons), respectively. At least 2 tons of medicines should be available in this shipment. To decide on which goods to prepare to ship in the next transport an LP problem was formulated:

$$\begin{aligned} \max z = {} & 2x_M + 2x_F + x_C \\ \text{s.t.} \quad & 3x_M + x_F + x_C \leq 20 \\ & 3x_M + 2x_F + x_C \leq 10 \\ & x_M \geq 2 \\ & x_M, x_F, x_C \geq 0 \end{aligned}$$

where x_j represents tons of donation type j (j = M, F, C) to prepare for shipment. The objective function translates the total utility considering that a ton of medicines has the same utility as a ton of food which in turn is double that of clothing. If necessary, use the Solver/Excel reports below to answer the following questions.

a) (*20 points*) Write and interpret the optimal solution of the problem (main variables and slack variables) and the value of the objective function.

b) (*15 points*) Write the dual problem.

c) (*25 points*) Determine the optimal solution of the dual problem (main variables and slack variables) by the complementary slackness relations.
Remark: *If you are not able to solve this question and need to know the dual solution to answer the next questions use* $y_1 = 0, y_2 = 2, y_3 = -2$ *in what follows.*

d) (*10 points*) Interpret the meaning of the first dual variable.

e) (*15 points*) Determine the change in total utility if it was not mandatory to carry medicines.

f) (*15 points*) What would be the new optimal solution and optimal value if the utility of a ton of medicines decreases to 1?

Answer Report

Variables Cell

Name	Original Value	Final Value	Integer
Medicines	2	2	Contin
Food	2	2	Contin
Clothing	0	0	Contin

Constraints

Name	Cell Value	Formula	Status	Slack
Selection	8		Not Binding	
Packing	10		Binding	
Min Medicines	2		Binding	

Sensitivity Report

Variable Cell

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Medicines	2	0	2	1	1E+30
Food	2	0	2	1E+30	0
Clothing	0	0	1	0	1E+30

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Selection	8		20	1E+30	12
Packing	10		10	24	4
Medicines	2		2	1,33	2

2. Three available volunteers with adequate skills to perform any of the three urgent tasks have been identified. The time required to perform the tasks (T1, T2, and T3) by the volunteers (V1, V2, and V3) is in the following table:

	T1	T2	T3
V1	1	3	4
V2	2	1	3
V3	1	4	4

It is intended to assign a task to each of the volunteers in order to minimize the time needed in the execution of the set of tasks.

a) (*10 points*) Propose a feasible solution to the problem and indicate the time that corresponds to it.

b) (*10 points*) Indicate, justifying, an upper bound for the optimal value of the problem.

c) (*20 points*) Suppose that there is an extra volunteer (V4) that can only perform two of the tasks: task T2 with time 3 and task T3 with time 2. Write the ILP formulation of the problem as well as constraints to ensure that:

- V4 is assigned to one of the tasks
- V2 can only be assigned to task T2 if V3 is assigned to task T1.

3. In the affected area it is urgent to re-establish some of the links between sites A, B, C, D, E, and F, so that it is possible to reach any of these locations from A. With this purpose, a team will be sent to the area. The time (in hours) that the team needs to reconnect each pair of locations is indicated in the following table:

	A	**B**	**C**	**D**	**E**	**F**
A	*	3	5	7	2	10
B		*	3	4	7	1
C			*	5	12	18
D				*	2	9
E					*	5

a) (*5 points*) Identify the problem that makes it possible to determine the set of links that should be repaired to achieve the goal and minimizing the total time.

b) (15 *points*) Without determining the optimal solution of the problem, justify if the set of links { (A,B) (A,C) (A,D) (A,E) (A,F)} can be an optimal solution for the problem.

c) (25 *points*) Determine an optimal solution to the problem by using an algorithm studied in the course. Start the algorithm by node A.

4. (15 *points*) Explain why, in the leaving criterion of the simplex method, only the positive coefficients in the pivotal column are considered to determine the basic variable that will be non-basic in the next iteration.