Lisbon School of Economics and Management

OPERATIONAL RESEARCH

Regular Period - Version B

Duration: 2h

12/12/2025

Note: Justifications are as important as final answers!

Justify all your answers and present all the relevant calculations.

1. Santa Claus's team intends to determine the number of working hours for the upcoming season needed to produce each of three types of similar toys. The three types of toys are manufactured in two sections. To meet the incoming orders, Santa was advised to produce at least 220 thousand toys in section 1 and 340 thousand in section 2, considering all three toys together. Each hour used in manufacturing the toys has an associated cost of 20, 30, and 25 euros for toy types 1, 2, and 3, respectively. To solve the problem, the following linear programming model was formulated, in which constraints R3 and R4 correspond to work requirements of the gnomes. This problem was solved using the Excel solver, leading to the sensitivity report below.

$$\begin{aligned}
&\min\ 20x_1 + 30x_2 + 25x_3 \\
&\text{s.t. } 3x_1 + 5x_2 + 2x_3 \geq 220 \quad \text{(R1)} \\
&\quad 4x_1 + x_2 + 2x_3 \geq 340 \quad \text{(R2)} \\
&\quad x_1 - x_2 - 2x_3 \leq 0 \quad \text{(R3)} \\
&\quad x_2 \geq 10 \quad \text{(R4)} \\
&\quad x_1, x_2, x_3 \geq 0
\end{aligned}$$

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
x1	68	0	20	30	32,5
x2	10	0	30	1E+30	17,5
x3	29	0	25	35	15

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
R1	312	0	220	92	1E+30
R2	340	6,5	340	1E+30	115
R3	0	-6	0	72,5	340
R4	10	17,5	10	58	10

(a) *(5 pts)* Interpret the meaning of the coefficient of x_1 in constraint R1.

(b) *(15 pts)* Indicate the optimal solution (main variables and slack variables) of the given problem. Interpret the meaning of the main variables and of the first two slack variables in the context of the problem.

(c) *(15 pts)* Write the optimal solution of the dual (main variables only) and interpret the meaning of the dual variables associated with constraints R1 and R4 (y_1 and y_4) in the context of the problem.

(d) *(20 pts)* Suppose now that it is required to produce 2 thousand additional toys, and that all of them must be produced in a single section. In which section should the additional toys be produced?

(e) *(15 pts)* Analyze the consequences on the optimal solution and on the optimal value of a decrease of 5 euros in the hourly production cost of type 2 toys.

(f) *(20 pts)* Due to a gnomes' strike, Santa was forced to close one of the production sections. Knowing that the minimum number of required toys is 500 thousand, make the necessary changes to the model to identify which section should be closed and to model the additional condition that either no hours are dedicated to producing toys of type 1, or the number of hours used for producing this toy type is at least 70.

2. *(30 pts)* Consider the following LP problem and perform one iteration of the Simplex method. Write down the solution obtained and classify it, justifying.

$$\begin{aligned} \max\ & z = 2x_1 + 4x_2 + x_3 \\ \text{s.t. } & 2x_1 - x_2 + 2x_3 \leq 2 \\ & 2x_1 - 2x_2 - x_3 \geq 0 \\ & -x_1 + 2x_2 + x_3 \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

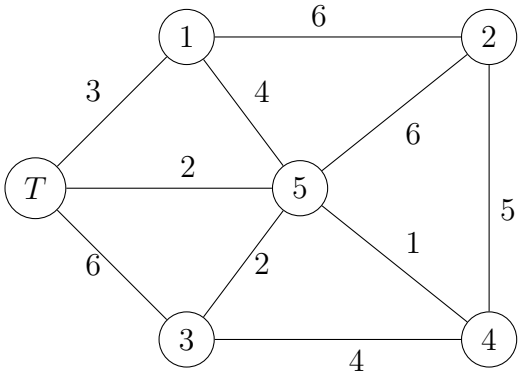
3. The distribution of toys in a given area is carried out from Santa's warehouse (P) and two garages ($G1$ and $G2$) rented for this purpose. The table below shows the unitary cost of distributing toys from these locations to four towns ($L1$, $L2$, $L3$, and $L4$) in the area, along with the expected number of toys (in hundreds) required in each town. It is possible to distribute 10 hundred toys from $G1$, 12 hundred from $G2$, and 20 hundred from P.

	$L1$	$L2$	$L3$	$L4$
P	12	10	8	12
$G1$	10	8	15	12
$G2$	12	12	10	20
Expected number of toys	12	4	6	10

(a) *(20 pts)* Suppose you want to determine the LP model that allows for determining how the distribution should be carried out, minimizing the total cost. Define the decision variables and their domain, the objective function, and the constraints associated with P and $L3$.

(b) *(20 pts)* Assume that the distribution is carried out only by gnomes. From each garage, the delivery is made by a single gnome, who can travel to at most one location. At Santa's warehouse, there are two other gnomes ($G3$ and $G4$). Each of these gnomes can deliver 10 hundred toys to at most one location. Formulate this new problem to identify which gnome delivers the toys to each location, ensuring that all locations receive the expected number of toys and that the total cost is minimized.

4. *(20 pts)* Upon arriving at a location, Santa uses his magical powers to park the sleigh and set up a network of pipes that delivers the toys to all the chimneys, represented by the nodes of the following network. Point T marks the place where the sleigh is parked. The values on the connections correspond to the length of each pipe that can be installed. Santa wants to use a network with the minimum total length. In the network shown, apply one of the algorithms studied to determine the minimum total length pipe network that Santa should establish



5. Verify whether the following statements are true or false and justify your answer.

(a) *(10 pts)* In a Minimum Cost Flow Problem, if the supplies and demands are integers, we can guarantee that there exists an optimal solution in which all variables take only integer values.

(b) *(10 pts)* The optimal value of the linear relaxation of any integer linear programming problem is an upper bound for the optimal value of the given problem.