INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

OPERATIONAL RESEARCH – 1º Semestre 2025/26

12 de dezembro de 2025 **Regular Period** Duração: 2h

Solution topics

1.

a) x_1 is the number of hours used to produce toys of type 1; thus, the coefficient 3 means that each hour dedicated to toys of type 1 makes it possible to produce 3000 toys in section 1.

b) $x^* = (68, 10, 29, 92, 0, 0, 0)$: $s_1^* = |220 - 312| = 92, s_2^* = |340 - 340| = 0,\ s_3^* = |0 - 0| = 0, s_4^* = |10 - 10| = 0.$

To satisfy the orders at minimum cost, 68 hours (x_1^*) must be used in the production of type 1 toys, 10 hours (x_2^*) in the production of type 2 toys, and 29 hours (x_3^*) in the production of type 3 toys.

Section 1 produces 92 thousand toys (s_1^*) more than the required minimum, while section 2 produces exactly the minimum number of toys required ($s_2^* = 0$).

c) $y^* = (0;\ 6,5; -6; 17,5).$
$y_1^* = 0$: if the minimum number of toys required in section 1 increases or decreases (within the sensitivity interval) the total cost does not change.

$y_4^* = 17,5$: If the minimum number of hours required to produce toys of type 2 (10) increases by one hour, the total cost increases 17,5 euros (the variation occurs in the sensitivity interval).

d) $b_1^{new} = 222 \in]-\infty, 312]$. Then, $\Delta z = \Delta b_1 \times y_1 = 0$.

$b_2^{new} = 342 \in [225, +\infty[$. Then, $\Delta z = \Delta b_2 \times y_2 = 2 \times 6,5 = 13$.

Option 1 is the best because it does not lead to extra costs.

e) $\Delta c_2 = -5 \in [-17,5, +\infty[$. Then, the optimal solution remains the same and $\Delta z = \Delta c_2 \times x_2 = -5 \times 10 = -50$. The total cost is reduced by 50 euros.

f) M is a sufficiently large constant and:

$$y = \begin{cases} 1 \text{ if section 1 is closed} \\ 0 \text{ otherwise} \end{cases} ;\ u = \begin{cases} 1 \text{ if toys of type 1 are produced} \\ 0 \text{ otherwise} \end{cases}$$

Changes to the model:

$$3x_1 + 5x_2 + 2x_3 \geq 500 - M(1 - y)$$
$$4x_1 + x_2 + 2x_3 \geq 500 - My$$
$$x_1 \geq 70u$$
$$x_1 \leq Mu$$
$$y, u \in \{0,1\}$$

2. Augmented form:

$$\text{Max } z = 4x_1 + 2x_2 + 3x_3$$

$$\text{s. t:} \begin{cases} 2x_1 - x_2 + 2x_3 + s_1 = 2 \\ -2x_1 + 2x_2 + x_3 + s_2 = 0 \\ -x_1 + 2x_2 + x_3 + s_3 = 4 \\ \\ x_j \geq 0 \quad j = 1,2,3,4,5,6 \end{cases}$$

VB	Z	x_1	x_2	x_3	s_1	s_2	s_3	TI	Entering and leaving criteria
Z	1	-2	-4	-1	0	0	0	0	EC: Mín $\{-2;-4;-1\} = -4 \leftarrow x_2$
s_1	0	2	-1	2	1	0	0	2	LC:
s_2	0	-2	**2**	1	0	1	0	0	$0/2$ $\xrightarrow{\text{Mín}} x_5$
s_3	0	-1	2	1	0	0	1	4	$4/2$
Z	1	-6	0	1	0	2	0	0	
s_1	0	1	0	$5/2$	1	$1/2$	0	2	
x_2	0	-1	1	$1/2$	0	$1/2$	0	0	
s_3	0	1	0	0	0	-1	1	4	

$\mathbf{x} = (0, 0, 0, 2,0,4)$ is a BFS not optimal because there are negative coefficients in the first row.

3.

a) Define x_{ij} = hundreads of toys to send from i ($i = \text{P}, 1(\text{G1}), 2(\text{G2})$) to Lj ($j = 1,2,3,4$)

$$Min\ Z = 100 \times (12x_{P1} + 10x_{P2} + 8x_{p3} + 12x_{P4} + 10x_{11} + 8x_{12} + 15x_{13} + 12x_{14} + 12x_{21} + 12x_{22} + 10x_{23} + 20x_{24})$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 20 \quad (\text{P})$$

$$x_{P3} + x_{13} + x_{23} = 6 \quad (\text{L3})$$

$$x_{ij} \geq 0 \ i = P, 1, 2; j = 1,2,3,4$$

b) Define $x_{ij} = \begin{cases} 1 \text{ if Gnome } \text{G}i \text{ go to } \text{L}j \\ 0 \text{ otherwise} \end{cases}, \ i, j = 1,2,3,4.$

Consider the following cost matrix of an assignment problem, with origins (gnomes) and destinations (locations).

	L1	L2	L3	L4
G1	-	8×4	15×6	12×10
G2	12×12	12×4	10×6	20×10
G3	-	10×4	8×6	12×10
G4	-	10×4	8×6	12×10

$$Min\ Z = 100 \times (144 + 32x_{12} + 90x_{13} + 120x_{14} + 48x_{22} + 60x_{23} + 200x_{24} + 40x_{32} + 48x_{33} + 120x_{34} + 40x_{42} + 48x_{43} + 120x_{44})$$

$$\text{s.a:} \begin{cases} \sum_{j=2}^{4} x_{ij} = 1 & i = 1,3,4 \\ x_{1j} + x_{3j} + x_{4j} = 1 & j = 2,3,4 \\ x_{ij} \geq 0 & i = 1,3,4;\ j = 2,3,4 \end{cases}$$

Note that $x_{21} = 1$.

4. Minimum spanning tree problem.

Iteração	Vértice na árvore	Vértice adjacente $\notin$ árvore + perto	Comprimento da ligação	Ligação a juntar
1	T	5	2	$(T, 5)$
2	T	1	3	
	5	4	1 ←mín	$(4,5)$
3	T	1	3	
	5	3	2 ←mín	$(3,5)$
	4	3	4	
4	T	1	3←mín	$(T, 1)$
	5	1	4	
	4	2	5	
	3	-	-	
5	T	-	-	$(1,2)$
	5	2	6	
	4	2	5 ←mín	
	3	-	-	
	1	2	6	

MST: $\{(T, 5); (4,5); (3,5); (T, 1); (1,2)\}$ total length: 13.

5.

a) False. To ensure that there is at least one optimal integer solution, the capacities must also be integer (and the problem must be feasible).

b) False. This is only true for maximization problems.