



Advanced Mathematical Economics – 1st Semester - 2026/2027

First Test - 23rd of October 2026

Duration: $(60 + \varepsilon)$ minutes, $|\varepsilon| \leq 15$

Version B

Name:

Student ID #:

- Give your answers in exact form.
- In order to receive credit, you must show all of your work. If you do not indicate the way in which you solve a problem, you may get little or no credit for it, even if your answer is correct.

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1. Consider the initial value problem where y depends on x :

$$\begin{cases} (2xe^y + 2x) + (x^2e^y + y) y' = 0 \\ y(1) = 0. \end{cases}$$

Show that the differential equation is exact and determine implicitly the solution of the IVP.

2. Consider the following Initial Value Problem (IVP) where p is a function of $t \geq 0$:

$$\begin{cases} p' = p(5 - p) - 4 \\ p(0) = a \end{cases} \quad a \in \mathbb{R}_0^+.$$

- (a) Find all equilibria and classify them with respect to the Lyapunov stability.
- (b) Draw the phase line and determine the intervals where solutions are strictly decreasing.
- (c) If $a = 3$, solve explicitly the IVP. Write all details.

3. Consider the system of Ordinary Differential Equations (ODEs) where x and y depend on t :

$$\begin{cases} \dot{x} = x - y, \\ \dot{y} = y - x. \end{cases}$$

- (a) Compute the general solution of the system and sketch the phase portrait.
 (b) Consider the solution satisfying $x(0) = 1, y(0) = 0$. Compute $\lim_{t \rightarrow -\infty} (x(t), y(t))$.

4. Consider the differential equation where y depends on t :

$$y'' - 4y' + 4y = e^{2t}.$$

Determine the unique solution satisfying $y(0) = 1, y'(0) = 0$.

5. Consider the system where x and y depend on t :

$$\begin{cases} \dot{x} = x(1 - x) - y, \\ \dot{y} = -y. \end{cases}$$

- (a) Determine the equilibria and write the linearisation of the system at each equilibrium.
 (b) Sketch the phase portrait near each equilibrium. Justify.



Credits:

1	2(a)	2(b)	2(c)	3(a)	3(b)	4	5(a)	5(b)
1.5	1	1	1.5	1	0.5	1.5	1	1